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Mixed cycle-E-super magic decomposition of complete bipartite graphs

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ABSTRACT

An H-magic labeling in a H-decomposable graph G is a bijection $f : V(G) \cup E(G) \to \{1, 2, ..., p + q\}$ such that for every copy H in the decomposition, $\sum_{v \in V(H)} f(v) + \sum_{e \in E(H)} f(e)$ is constant. f is said to be H-E-super magic if $f(E(G)) = \{1, 2, \cdots, q\}$. A family of subgraphs $H_1, H_2, \cdots, H_h$ of G is a mixed cycle-decomposition of G if every subgraph H_i is isomorphic to some cycle C_k, for $k \geq 3$, $E(H_i) \cap E(H_j) = \emptyset$ for $i \neq j$ and $\cup_{i=1}^{h} E(H_i) = E(G)$. In this paper, we prove that $K_{2m,2n}$ is mixed cycle-E-super magic decomposable where $m \geq 2$, $n \geq 3$, with the help of the results found in [1].

Keyword: H-decomposable graph, H-E-super magic labeling, mixed cycle-E-super magic decomposable graph.

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1 Introduction

In this paper we consider only finite and simple undirected bipartite graphs. The vertex and edge sets of a graph G are denoted by $V(G)$ and $E(G)$ respectively and we let

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$|V(G)| = p$ and $|E(G)| = q$. For graph theoretic notations, we follow [2, 3]. A labeling of a graph G is a mapping that carries a set of graph elements, usually vertices and/or edges into a set of numbers, usually integers. Many kinds of labeling have been studied and an excellent survey of graph labeling can be found in [5].

Although magic labeling of graphs was introduced by Sedlacek [18], the concept of vertex magic total labeling (VMTL) first appeared in 2002 in [9]. In 2004, MacDougall et al. [10] introduced the notion of super
vertex magic total labeling (SVMTL). In 1998, Enomoto et al. [4] introduced the concept of super edge-magic graphs. In 2005, Sugeng and Xie [19] constructed some super edge-magic total graphs. The usage of the word "super" was introduced in [4]. The notion of a V-super vertex magic labeling was introduced by MacDougall et al. [10] as in the name of super vertex-magic total labeling and it was renamed as V-super vertex magic labeling by Marr and Wallis in [15] after referencing the article [12]. Most recently, Tao-ming Wang and Guang-Hui Zhang [20], generalized some results found in [12].

A vertex magic total labeling is a bijection f from $V(G) \cup E(G)$ to the integers $1, 2, ..., p+q$ with the property that for every $u \in V(G)$, $f(u) + \sum_{v \in N(u)} f(uv) = k$ for some constant k, such a labeling is V-super if $f(V(G)) = \{1, 2, ..., p\}$. A graph G is called V-super vertex magic if it admits a V-super vertex labeling. A vertex magic total labeling is called E-super if $f(E(G)) = \{1, 2, ..., q\}$. A graph G is called E-super vertex magic if it admits a E-super vertex labeling. The results of the article [12] can also be found in [15]. In [10], MacDougall et al., proved that no complete bipartite graph is V-super vertex magic. An edge-magic total labeling is a bijection f from $V(G) \cup E(G)$ to the integers $1, 2, ..., p+q$ with the property that for any edge $uv \in E(G)$, $f(u) + f(uv) + f(v) = k$ for some constant k, such a labeling is super if $f(V(G)) = \{1, 2, ..., p\}$. A graph G is called super edge-magic if it admits a super edge-magic labeling.

Most recently, Marimuthu and Balakrishnan [13], introduced the notion of super edge-magic graceful graphs to solve some kind of network problems. A (p, q) graph G with p vertices and q edges is edge magic graceful if there exists a bijection $f : V(G) \cup E(G) \to \{1, 2, ..., p+q\}$ such that $|f(u) + f(v) - f(uv)| = k$, a constant for any edge uv of G. G is said to be super edge-magic graceful if $f(V(G)) = \{1, 2, ..., p\}$.

A covering of G is a family of subgraphs $H_1, H_2, ..., H_h$ such that each edge of $E(G)$ belongs to at least one of the subgraphs H_i, $1 \le i \le h$. Then it is said that G admits an $(H_1, H_2, \cdots, H_h)$ covering. If every H_i is isomorphic to a given graph H, then G admits an H-covering. A family of subgraphs $H_1, H_2, \cdots, H_h$ of G is a H-decomposition of G if all the subgraphs are isomorphic to a graph H, $E(H_i) \cap E(H_j) = \emptyset$ for $i \ne j$ and

$\cup_{i=1}^{h} E(H_i) = E(G)$. In this case, we write $G = H_1 \oplus H_2 \oplus \cdots \oplus H_h$ and G is said to be H-decomposable.

The notion of H-super magic labeling was first introduced and studied by Gutiérrez and Lladó [6] in 2005. They proved that some classes of connected graphs are H-super magic. Suppose G is H-decomposable. A total labeling $f : V(G) \cup E(G) \to \{1, 2, \cdots, p+q\}$ is called an H-magic labeling of G if there exists a positive integer k (called magic constant) such that for every copy H in the decomposition, $\sum_{v \in V(H)} f(v) + \sum_{e \in E(H)} f(e) = k$. A graph G that admits such a labeling is called a H-magic decomposable graph. An H-magic labeling f is called a H-V-super magic labeling if $f(V(G)) = \{1, 2, \cdots, p\}$. A graph that admits a H-V-super magic labeling is called a H-V-super magic decomposable graph. An H-magic labeling f is called a H-E-super magic labeling if $f(E(G)) = \{1, 2, \cdots, q\}$. A graph that admits a H-E-super magic labeling is called a H-E-super magic decomposable graph. The sum of all vertex and edge labels on H is denoted by $\sum f(H)$.

In 2007, Lladó and Moragas [8] studied the cycle-magic and cyclic-super magic behavior of several classes of connected graphs. They gave several families of C_r-magic graphs for each $r \geq 3$. In 2010, Ngurah, Salman and Susilowati [17] studied the cycle-super magic labeling of chain graphs, fans, triangle ladders, graph obtained by joining a star $K_{1,n}$ with one isolated vertex, grids and books. Maryati et al. [16] studied the H-super magic labeling of some graphs obtained from k isomorphic copies of a connected graph H. In 2012, Mania Roswitha and Edy Tri Baskoro [11] studied the H-super magic labeling for some trees such as a double star, a caterpillar, a firecracker and banana tree. In 2013, Toru Kojima [21] studied the C_4-super magic labeling of the Cartesian product of paths and graphs. In 2012, Inayah et al. [7] studied magic and anti-magic H-decompositions and Zhihe Liang [22] studied cycle-super magic decompositions of complete multipartite graphs. They are all called a H-magic labeling as a H-super magic if the smallest labels are assigned to the vertices. Note that an edge-magic graph is a K_2-magic graph.

All the articles mentioned here which are dealing with H-magic or H-super magic decomposition of a graph G consider a fixed subgraph H. But we discuss the following.

A family of subgraphs $H_1, H_2, \cdots, H_h$ of G is said to be mixed cycle- decomposition if every subgraph H_i is isomorphic to some cycle C_k, for $k \geq 3$. Suppose G is mixed cycle-decomposable. A total labeling $f : V(G) \cup E(G) \to \{1, 2, \cdots, p+q\}$ is called an mixed cycle-magic labeling of G if there exists a positive integer k (called magic constant) such that for every cycle H_i in the decomposition, $\sum_{v \in V(H_i)} f(v) + \sum_{e \in E(H_i)} f(e) = k$. A graph G that admits such a labeling is called a mixed cycle-magic decomposable graph. A mixed cycle-magic labeling f is called a mixed cycle-V-super magic labeling if

$f(V(G)) = \{1, 2, \cdots, p\}$. A graph that admits a mixed cycle-V-super magic labeling is called a mixed cycle-V-super magic decomposable graph. A mixed cycle-magic labeling f is called a mixed cycle-E-super magic labeling if $f(E(G)) = \{1, 2, \cdots, q\}$. A graph that admits a mixed cycle-E-super magic labeling is called a mixed cycle-E-super magic decomposable graph.

Let $K_{m,n}$ and C_k denote the complete bipartite graph and the elementary cycle of length k. If a graph G can be decomposed into p_1 copies of C_4, q_1 copies of C_6 and r_1 copies of C_8, then we write $G = p_1C_4 + q_1C_6 + r_1C_8$. We assume throughtout this paper that $p_1, q_1, r_1 \in N \cup \{0\}$, the set of nonnegative integers. In [1], Chao-Chih Chou et al. introduced the notions $D(G)$ and S_i. $D(G) = \{(p_1, q_1, r_1) : p_1, q_1, r_1 \in N \cup \{0\}$ and $G = p_1C_4 + q_1C_6 + r_1C_8\}$ and $S_i = \{(p_1, q_1, r_1) : p_1, q_1, r_1 \in N \cup \{0\}$ and $4p_1 + 6q_1 + 8r_1 = i\}$, for each positive integer i. Then clearly $D(G) \subseteq S_q$. In [14], Marimuthu and Stalin Kumar studied the Mixed cycle-V-super magic decomposition of complete bipartite graphs. This idea helps us to study about the mixed cycle-E-super magic decomposition of complete bipartite graphs.

We shall make use of the following results from [1].

Theorem 1.1. $D(K_{2,2t}) = \{(t, 0, 0)\}$, *for each* $t \in N$.

Theorem 1.2. $D(K_{4,4}) = \{(4, 0, 0), (1, 2, 0), (0, 0, 2)\} = S_{16} - \{(2, 0, 1)\} = S^*_{16}$.

Theorem 1.3. *If* m *and* n *are integers such that* $m \geq 2$, $n \geq 3$ *then* $D(K_{2m,2n}) = S_{4mn}$.

2 C_4-E-super magic decomposition of $K_{2,2t}$ and $K_{4,4}$

In this section, we prove that the graphs $G \cong K_{2,2t}$ where $t \in N$ and $G \cong K_{4,4}$ are C_4-E-super magic decomposable.

Theorem 2.1. *Suppose that* $G \cong K_{2,2t}$ *is* C_4*-decomposable. Then* G *is a* C_4*-*E*-super magic decomposable graph with magic constant* $26t + 10$.

Proof. By Theorem 1.1, we have $D(K_{2,2t}) = \{(t, 0, 0)\}$, for each $t \in N$.
Let $U = \{u_1, u_2\}$ and $W = \{v_1, v_2, \cdots, v_{2t}\}$ be two stable sets of G. Let $\{H_1, H_2, \cdots, H_t\}$ be a C_4-decomposition of G, such that $V(H_i) = \{u_1, u_2, v_{2i-1}, v_{2i}\}$ and
$E(H_i) = \{u_1v_{2i-1}, v_{2i-1}u_2, u_2v_{2i}, v_{2i}u_1\}$, for $1 \leq i \leq t$. Clearly $p = 2 + 2t$ and $q = 4t$. Define a total labeling $f : V(G) \cup E(G) \to \{1, 2, \cdots, 2 + 6t\}$ by $f(u_i) = 4t + i$ for all $i = 1, 2$ and

$$f(v_j) = \left\{ \begin{array}{ll} (4t+3) + \left\lfloor \frac{j}{2} \right\rfloor, & if \;\; j = 1, 3, \cdots, 2t-1 \\ \\ (6t+3) - \left\lfloor \frac{j}{2} \right\rfloor, & if \;\; j = 2, 4, \cdots, 2t \end{array} \right\}$$

Table 1: The edge label of a C_4-decomposition of $K_{2,2t}$ if t is odd.

f	v_1	v_2	v_3	v_4	...	v_t	v_{t+1}	...	v_{2t-1}	v_{2t}
u_1	1	2	3	4	...	t	$t+1$	...	$2t-1$	$2t$
u_2	$4t$	$4t-1$	$4t-2$	$4t-3$	...	$3t+1$	$3t$	...	$2t+2$	$2t+1$

Table 2: The edge label of a C_4-decomposition of $K_{2,2t}$ if t is even.

f	v_1	v_2	v_3	v_4	...	v_{t-1}	v_t	...	v_{2t-1}	v_{2t}
u_1	1	2	3	4	...	$t-1$	t	...	$2t-1$	$2t$
u_2	$4t$	$4t-1$	$4t-2$	$4t-3$	...	$3t+2$	$3t+1$	...	$2t+2$	$2t+1$

It remains only to label the edges.

Case 1. t is odd.
Now the edges of G can be labeled as shown in Table 1.
From Table 1, $\sum f(E(H_i)) = 8t+2$ for $1 \leq i \leq t$. By the definition of f,

$$\begin{aligned}\sum f(H_1) &= f(u_1)+f(u_2)+f(v_1)+f(v_2)+\sum f(E(H_1))\\ &= ((4t+1)+(4t+2)+(4t+3)+(6t+2))+(8t+2)\\ &= 26t+10.\end{aligned}$$

In a similar way,

$$\begin{aligned}\sum f(H_2) &= f(u_1)+f(u_2)+f(v_3)+f(v_4)+\sum f(E(H_2))\\ &= ((4t+1)+(4t+2)+(4t+4)+(6t+1))+(8t+2)\\ &= 26t+10.\end{aligned}$$

Thus, $\sum f(H_2) = \sum f(H_1) = 26t+10$. In general,

$$\begin{aligned}\sum f(H_t) &= f(u_1)+f(u_2)+f(v_{2t-1})+f(v_{2t})+\sum f(E(H_t))\\ &= ((4t+1)+(4t+2)+(5t+2)+(5t+3))+(8t+2)\\ &= 26t+10.\end{aligned}$$

so, $\sum f(H_1) = \sum f(H_2) = \sum f(H_3) = \cdots = \sum f(H_t) = 26t+10$.
Thus the graph G is a C_4-E-super magic decomposable graph.

Case 2. t is even.
The edges of G can be labeled as shown in Table 2.
Here also, we have $\sum f(E(H_i)) = 8t+2$ for $1 \leq i \leq t$, and as in case 1,

Table 3: The edge label of a C_4-decomposition of $K_{4,4}$.

f	v_1	v_2	v_3	v_4
u_1	16	5	13	4
u_2	11	6	14	3
u_3	10	7	15	2
u_4	9	8	12	1

$\sum f(H_1) = \sum f(H_2) = \sum f(H_3) = \cdots = \sum f(H_t) = 26t + 10$. Thus G is a C_4-E-super magic decomposable graph. □

Theorem 2.2. *The graph $G \cong K_{4,4}$ is C_4-E-super magic decomposable under the decomposition* $(4,0,0) \in D(K_{4,4})$.

Proof. By Theorem 1.2, we have $D(K_{4,4}) = \{(4,0,0),(1,2,0),(0,0,2)\}$. Let $U = \{u_1,u_2,u_3,u_4\}$ and $W = \{v_1,v_2,v_3,v_4\}$ be two stable sets of G. Define a total labeling $f : V(G) \cup E(G) \to \{1,2,\cdots,24\}$ by $f(u_i) = 16 + i$ and $f(v_i) = 20 + i$, for all $i = 1,2,3,4$. consider the decomposition of G, $(4,0,0) \in D(K_{4,4})$.
Let $H = \{H_1 = (u_1v_1u_2v_2), H_2 = (u_1v_3u_2v_4), H_3 = (u_3v_1u_4v_2), H_4 = (u_3v_3u_4v_4)\}$ be a C_4-decomposition of G. Now the edges of G can be labeled as shown in Table 3. Using this, we have

$$\begin{aligned}\sum f(H_1) &= f(u_1)+f(u_2)+f(v_1)+f(v_2)+\sum f(E(H_1))\\ &= (17+18+21+22)+(16+5+11+6)\\ &= 78+38\\ &= 116.\end{aligned}$$

$$\begin{aligned}\sum f(H_2) &= f(u_1)+f(u_2)+f(v_3)+f(v_4)+\sum f(E(H_2))\\ &= (17+18+23+24)+(13+4+14+3)\\ &= 82+34\\ &= 116.\end{aligned}$$

$$\begin{aligned}\sum f(H_3) &= f(u_3)+f(u_4)+f(v_1)+f(v_2)+\sum f(E(H_1))\\ &= (19+20+21+22)+(10+7+8+9)\\ &= 82+34\\ &= 116.\end{aligned}$$

Table 4: The edge label of a C_8-decomposition of $K_{4,4}$.

f	v_1	v_2	v_3	v_4
u_1	1	2	15	16
u_2	4	5	14	11
u_3	7	8	9	10
u_4	6	3	12	13

$$\begin{aligned}\sum f(H_4) &= f(u_3)+f(u_4)+f(v_3)+f(v_4)+\sum f(E(H_1))\\ &= (19+20+23+24)+(15+2+12+13)\\ &= 86+30\\ &= 116.\end{aligned}$$

Thus $\sum f(H_1) = \sum f(H_2) = \sum f(H_3) = \sum f(H_4) = 116$.
Hence the graph $K_{4,4}$ is C_4-E-super magic decomposable under the decomposition $(4,0,0) \in D(K_{4,4})$. □

3 C_8-E-super magic decomposition of $K_{4,4}$

In this section, we prove that the graph $G \cong K_{4,4}$ is C_8-E-super magic decomposable.

Theorem 3.1. *The graph $G \cong K_{4,4}$ is C_8-E-super magic decomposable under the decomposition $(0,0,2) \in D(K_{4,4})$.*

Proof. By Theorem 1.2, we have $D(K_{4,4}) = \{(4,0,0),(1,2,0),(0,0,2)\}$. Let $U = \{u_1,u_2,u_3,u_4\}$ and $W = \{v_1,v_2,v_3,v_4\}$ be two stable sets of G. Define a total labeling $f : V(G) \cup E(G) \to \{1,2,\cdots,24\}$ by $f(u_i) = 16+i$ and $f(v_i) = 20+i$, for all $i = 1,2,3,4$. consider the decomposition of G, $(0,0,2) \in D(K_{4,4})$.
Let $H = \{H_1 = (u_1v_1u_2v_2u_3v_3u_4v_4), H_2 = (u_1v_2u_4v_1u_3v_4u_2v_3)\}$ be a C_8-decomposition of G. Now the edges of G can be labeled as shown in Table 4.
Thus,

$$\begin{aligned}\sum f(H_1) &= f(u_1)+f(u_2)+f(u_3)+f(u_4)+f(v_1)+f(v_2)+f(v_3)+f(v_4)\\ &\quad +\sum f(E(H_1))\\ &= (17+18+19+20+21+22+23+24)\\ &\quad +(1+4+5+8+9+12+13+16)\\ &= 164+68\\ &= 232.\end{aligned}$$

$$\begin{aligned}
\sum f(H_2) &= f(u_1)+f(u_2)+f(u_3)+f(u_4)+f(v_1)+f(v_2)+f(v_3)+f(v_4)\\
&\quad +\sum f(E(H_2))\\
&= (17+18+19+20+21+22+23+24)\\
&\quad +(2+3+6+7+10+11+14+15)\\
&= 164+68\\
&= 232.
\end{aligned}$$

So $\sum f(H_1) = \sum f(H_2) = 232$.
Thus the graph $K_{4,4}$ is C_8-E-super magic decomposable under the decomposition $(0,0,2) \in D(K_{4,4})$. □

4 Mixed Cycle-E-super magic decomposition of Complete bipartite graphs

Many authors studied H-(super)magic labeling for a fixed graph H, for example see [6, 7, 8, 11, 16, 17, 21, 22]. In this section, we introduce the concept of mixed cycle decomposition of a graph G after refering [1]. A family of subgraphs $H_1, H_2, \cdots, H_h$ of G is said to be mixed cycle-decomposition of G if every subgraph H_i is isomorphic to some cycle C_k, for $k \geq 3$. Suppose G is mixed cycle-decomposable. A total labeling $f : V(G) \cup E(G) \to \{1, 2, \cdots, p+q\}$ is called an mixed cycle-magic labeling of G if there exists a positive integer k (called magic constant) such that for every copy H in the decomposition, $\sum_{v \in V(H)} f(v) + \sum_{e \in E(H)} f(e) = k$. A graph G that admits such a labeling is called a mixed cycle-magic decomposable graph. A mixed cycle-magic labeling f is called a mixed cycle-V-super magic labeling if $f(V(G)) = \{1, 2, \cdots, p\}$. A graph that admits a mixed cycle-V-super magic labeling is called a mixed cycle-V-super magic decomposable graph. A mixed cycle-magic labeling f is called a mixed cycle-E-super magic labeling if $f(E(G)) = \{1, 2, \cdots, q\}$. A graph that admits a mixed cycle-E-super magic labeling is called a mixed cycle-E-super magic decomposable graph.

In [14], Marimuthu and Stalin Kumar studied the Mixed cycle-V-super magic decomposition of complete bipartite graphs $G \cong K_{2m,2n}$ with $m \geq 2$ and $n \geq 3$. Here we studied the mixed cycle-E-super magicness of complete bipartite graphs and also prove that the graph $K_{4,4}$ is not mixed cycle-E-super magic decomposable under the decomposotion $(1,2,0) \in D(K_{4,4})$.

In this section, we consider the graph $G \cong K_{2m,2n}$ with $m \geq 2$ and $n \geq 3$. Clearly $p = 2(m+n)$ and $q = 4mn$.

Theorem 4.1. *If a non-trivial complete bipartite graph $G \cong K_{2m,2n}$ with $m \geq 2$ and $n \geq 3$ is mixed cycle-E-super magic decomposable, then the magic constant k is given by $k = \frac{2mn(3m+n+12mn+2)}{p_1+q_1+r_1}$.*

Proof. By Theorem 1.3, $D(K_{2m,2n}) = S_{4mn} = \{(p_1, q_1, r_1)/p_1, q_1, r_1 \in N \cup \{0\}$ and $4p_1 + 6q_1 + 8r_1 = 4mn\}$. Let $U = \{u_1, u_2, \cdots, u_{2m}\}$ and $W = \{w_1, w_2, \cdots, w_{2n}\}$ be two stable sets of G such that $V(G) = U \cup W$ and let $\{H_i/i = 1, 2, \cdots, p_1 + q_1 + r_1\}$ be a mixed-cycle decomposition of G. Let f be a mixed cycle-E-super magic labeling of G with magic constant k. Then $f(V(G)) = \{4mn + 1, 4mn + 2, \cdots, (4mn + 2(m + n))\}$, $f(E(G)) = \{1, 2, \cdots, 4mn\}$ and $k = \sum_{v \in V(H_i)} f(v) + \sum_{e \in E(H_i)} f(e)$ for every H_i in the decomposition of G. Let $f(U) = \{4mn + 1, 4mn + 2, \cdots, (4mn + 2m)\}$ and $f(W) = \{(4mn+2m+1), (4mn+2m+2), \cdots, (4mn+2(m+n))\}$. When taking the sum of vertex labels of all subgraphs H_i, $(i = 1, 2, \cdots, p_1 + q_1 + r_1)$ in the decomposition (p_1, q_1, r_1), the vertex labels $4mn + 1, 4mn + 2, \cdots, (4mn + 2m)$ occurs $\frac{2n}{2}$ times and the vertex labels $(4mn + 2m + 1), (4mn + 2m + 2), \cdots, (4mn + 2(m + n))$ occurs $\frac{2m}{2}$ times.
Hence,

$$\begin{aligned}
\sum_{i=1}^{p_1+q_1+r_1} f(V(H_i)) &= \frac{2n}{2}\{(4mn + 1) + (4mn + 2) + \cdots + (4mn + 2m)\} \\
&\quad + \frac{2m}{2}\{(4mn + (2m + 1)) + (4mn + (2m + 2)) \\
&\quad + \cdots + (4mn + (2m + 2n))\} \\
&= n\{8m^2n + \frac{2m(2m + 1)}{2}\} + m\{2n(4mn + 2m) + \frac{2n(2n + 1)}{2}\} \\
&= 8m^2n^2 + mn(2m + 1) + 8m^2n^2 + 4m^2n + mn(2n + 1) \\
&= 16m^2n^2 + 4m^2n + mn(2m + 2n + 2) \\
&= mn(16mn + 4m + 2(m + n + 1)).
\end{aligned}$$

Also,

$$\begin{aligned}
\sum_{i=1}^{p_1+q_1+r_1} f(E(H_i)) &= \{1 + 2 + \cdots + 4mn\} \\
&= \frac{4mn(4mn + 1)}{2} \\
&= 2mn(4mn + 1).
\end{aligned}$$

Since there are p_1 copies of C_4, q_1 copies of C_6 and r_1 copies of C_8, we have

$$\begin{aligned}(p_1+q_1+r_1)k &= \sum_{i=1}^{p_1+q_1+r_1} f(V(H_i)) + \sum_{i=1}^{p_1+q_1+r_1} f(E(H_i)) \\ &= mn(16mn+4m+2(m+n+1)) + 2mn(4mn+1) \\ &= mn(16mn+4m+2(m+n+1)+2(4mn+1)) \\ &= 2mn(8mn+2m+(m+n+1)+(4mn+1)) \\ &= 2mn(3m+n+12mn+2).\end{aligned}$$

Thus, $k = \frac{2mn(3m+n+12mn+2)}{p_1+q_1+r_1}$. □

Corollary 4.2. *Suppose $G \cong K_{2m,2n}$ with $m \geq 2$ and $n \geq 3$ is a mixed cycle-E-super magic decomposable graph under the decomposition $(p_1, q_1, r_1) \in D(K_{2m,2n})$ with magic constant $k = \frac{2mn(3m+n+12mn+2)}{p_1+q_1+r_1}$, then $(p_1+q_1+r_1)$ divides $2mn$ or $(3m+n+12mn+2)$.*

Theorem 4.3. *The graph $G \cong K_{4,4}$ is not a mixed cycle-E-super magic decomposable under the decomposition $(1, 2, 0) \in D(K_{4,4})$.*

Proof. Suppose G is mixed-cycle-E-super magic decomposable under the decomposition $(1, 2, 0) \in D(K_{4,4})$, then by Theorem 4.1,

$$\begin{aligned} k &= \frac{2mn(3m+n+12mn+2)}{p_1+q_1+r_1} \\ &= \frac{2(2)(2)(3(2)+2+12(2)(2)+2)}{1+2+0} \\ &= \frac{8(58)}{3}, \end{aligned}$$

which is not an integer. Hence $G \cong K_{4,4}$ is not a mixed cycle-E-super magic decomposable graph under the decomposition $(1, 2, 0) \in D(K_{4,4})$. □

Remark 4.4. *Suppose for some $m \geq 2$ and $n \geq 3$, $D(K_{2m,2n}) = \{(a, b, c)\}$ where $a, b, c \in N \cup \{0\}$ and $K_{2m,2n} = aC_4 + bC_6 + cC_8$. If $k = \frac{2mn(3m+n+12mn+2)}{a+b+c}$ is an integer. Then $K_{2m,2n}$ is not neccessary to be a mixed cycle-E-super magic decomposable graph. This fact is illustrated in the following examples.*

Example 4.1. *Consider the decomposition $(2, 0, 2) \in D(K_{4,6})$.*
Let $U = \{u_1, u_2, u_3, u_4\}$ and $W = \{w_1, w_2, w_3, w_4, w_5, w_6\}$ be two stable sets of $K_{4,6}$ such that $V(G) = U \cup W$. Let $\{H_1 = (u_1w_1u_2w_2), H_2 = (u_3w_1u_4w_6), H_3 = (u_1w_3u_2w_4u_3w_2u_4w_5)$, $H_4 = (u_1w_4u_4w_3u_3w_5u_2w_6)\}$ be a mixed cycle-decomposition of $K_{4,6}$, where H_1, H_2 are isomorphic to C_4 and H_3,H_4 are isomorphic to C_8. Define a total labeling $f : V(G) \cup E(G) \to \{1, 2, \cdots, 34\}$ by $f(u_i) = 24 + i$ and $f(w_j) = 28 + j$, for all $i = 1, 2, 3, 4$ and

$j = 1, 2, 3, 4, 5, 6$. *Here* $m = 2$, $n = 3$, $p_1 = 2$, $q_1 = 0$ *and* $r_1 = 2$.
Suppose $K_{4,6}$ *is a mixed cycle-E-super magic decomposable graph under the decomposition* $(2, 0, 2) \in D(K_{4,6})$, *then by Theorem* 4.1,

$$\begin{aligned} k &= \frac{2mn(3m + n + 12mn + 2)}{p_1 + q_1 + r_1} \\ &= \frac{2(2)(3)(3(2) + 3 + 12(2)(3) + 2)}{2 + 0 + 2} \\ &= \frac{12(83)}{4} \\ &= 249. \end{aligned}$$

Hence $\sum f(H_1) = \sum f(H_2) = \sum f(H_3) = \sum f(H_4) = 249$.
But consider the labeling of H_1, *if we assign maximum edge labels to* H_1, $\sum f(E(H_1)) = 21 + 22 + 23 + 24 = 90$ *and by definition of* f, *we have*

$$\begin{aligned} \sum f(V(H_1)) &= f(u_1) + f(w_1) + f(u_2) + f(w_2) \\ &= (24 + 1) + (28 + 1) + (24 + 2) + (28 + 2) \\ &= 110. \end{aligned}$$

Thus, $\sum f(H_1) = 90 + 110 = 200 \neq 249$. *Hence, under the decomposition* $(2, 0, 2) \in D(K_{4,6})$, *the graph* $K_{4,6}$ *is not a mixed cycle-E-super magic decomposable.*

Example 4.2. *The graph* $K_{4,6}$ *is not mixed cycle-E-super magic decomposable under the decomposition* $(1, 2, 1) \in D(K_{4,6})$.
Let $U = \{u_1, u_2, u_3, u_4\}$ *and* $W = \{w_1, w_2, w_3, w_4, w_5, w_6\}$ *be two stable sets of* $K_{4,6}$ *such that* $V(G) = U \cup W$. *Let* $\{H_1 = (u_3w_1u_4w_2), H_2 = (u_1w_4u_2w_5u_3w_6), H_3 = (u_1w_1u_2w_6u_4w_3), H_4 = (u_1w_1u_2w_3u_3w_4u_4w_5)\}$ *be a Mixed Cycle- decomposition of* $K_{4,6}$, *where* H_1, *is isomorphic to* C_4, H_2,H_3, *are isomorphic to* C_6 *and* H_4 *is isomorphic to* C_8. *Define a total labeling* $f : V(G) \cup E(G) \to \{1, 2, \cdots, 34\}$ *by* $f(u_i) = 24 + i$ *and* $f(w_j) = 28 + j$, *for all* $i = 1, 2, 3, 4$ *and* $j = 1, 2, 3, 4, 5, 6$. *Here* $m = 2$, $n = 3$, $p_1 = 1$, $q_1 = 2$ *and* $r_1 = 1$.
Suppose $K_{4,6}$ *is a mixed cycle-E-super magic decomposable graph under the decomposition*

$(1,2,1) \in D(K_{4,6})$, *then by Theorem* 4.1,

$$\begin{aligned} k &= \frac{2mn(3m+n+12mn+2)}{p_1+q_1+r_1} \\ &= \frac{2(2)(3)(3(2)+3+12(2)(3)+2)}{1+2+1} \\ &= \frac{12(83)}{4} \\ &= 249. \end{aligned}$$

Hence $\sum f(H_1) = \sum f(H_2) = \sum f(H_3) = \sum f(H_4) = 249$.
But consider the labeling of H_1, *if we assign maximum edge labels to* H_1, $\sum f(E(H_1)) = 21+22+23+24 = 90$ *and by definition of* f, *we have*

$$\begin{aligned} \sum f(V(H_1)) &= f(u_3)+f(w_1)+f(u_4)+f(w_2) \\ &= (24+3)+(28+1)+(24+4)+(28+2) \\ &= 114. \end{aligned}$$

Thus, $\sum f(H_1) = 90+114 = 204 \neq 249$. *Hence, under the decomposition* $(1,2,1) \in D(K_{4,6})$, *the graph* $K_{4,6}$ *is not a mixed cycle-E-super magic decomposable.*

Example 4.3. *The graph* $K_{4,6}$ *is not mixed cycle-E-super magic decomposable under the decomposition* $(4,0,1) \in D(K_{4,6})$.
Suppose $K_{4,6}$ *is mixed-cycle-E-super magic decomposable under the decomposition* $(4,0,1) \in D(K_{4,6})$, *then by Theorem* 4.1,

$$\begin{aligned} k &= \frac{2mn(3m+n+12mn+2)}{p_1+q_1+r_1} \\ &= \frac{2(2)(3)(3(2)+3+12(2)(3)+2)}{4+0+1} \\ &= \frac{12(83)}{5}, \end{aligned}$$

which is not an integer. Thus, under the decomposition $(4,0,1) \in D(K_{4,6})$, *the graph* $K_{4,6}$ *is not a mixed cycle-E-super magic decomposable.*

Example 4.4. *The graph* $K_{4,6}$ *is not mixed cycle-E-super magic decomposable under the decomposition* $(3,2,0) \in D(K_{4,6})$.
Suppose $K_{4,6}$ *is mixed-cycle-E-super magic decomposable under the decomposition* $(3,2,0) \in$

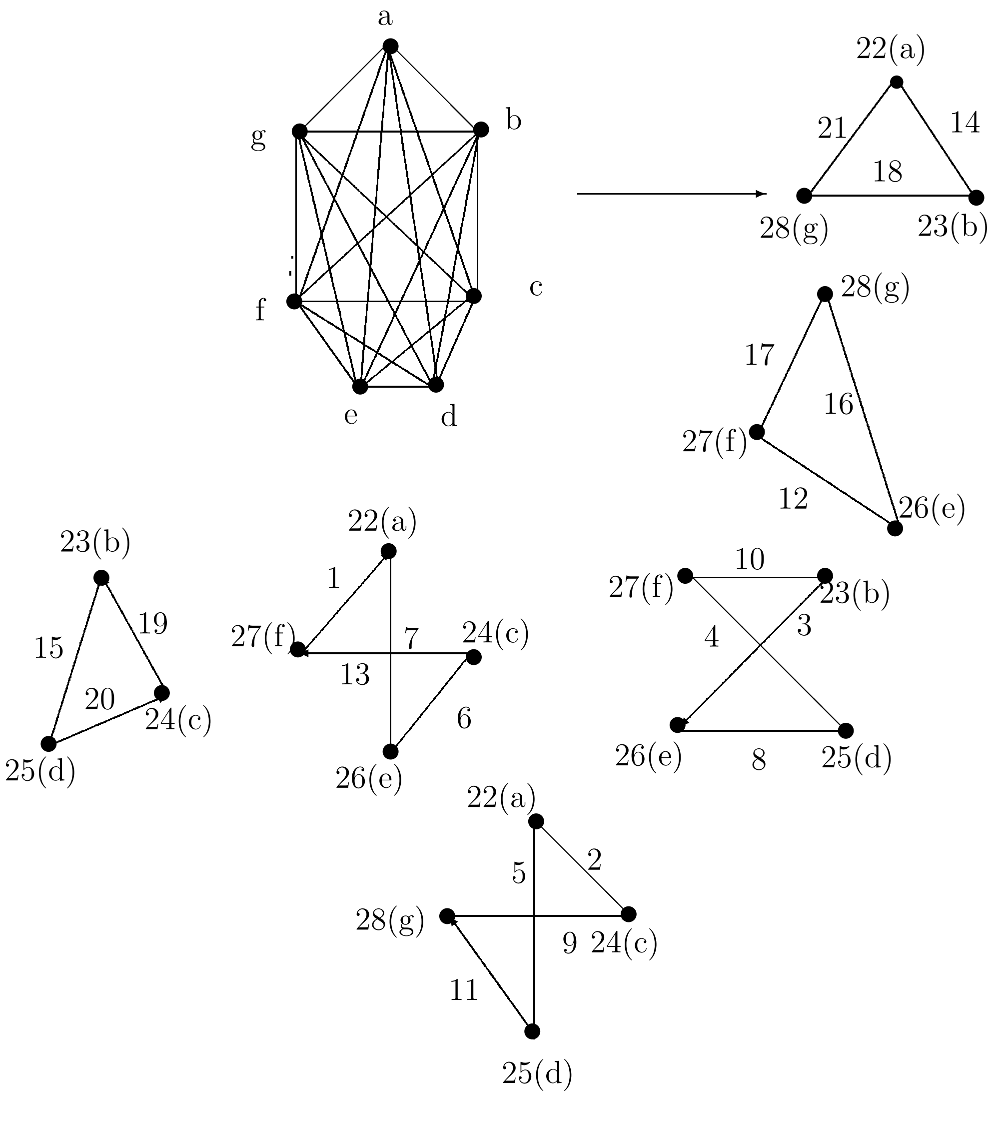

Figure 1: mixed cycle-E-super magic decompositon of K_7

$D(K_{4,6})$, then by Theorem 4.1,

$$\begin{aligned} k &= \frac{2mn(3m+n+12mn+2)}{p_1+q_1+r_1} \\ &= \frac{2(2)(3)(3(2)+3+12(2)(3)+2)}{3+2+0} \\ &= \frac{12(83)}{5}, \end{aligned}$$

which is not an integer. Thus, under the decomposition $(3,2,0) \in D(K_{4,6})$, the graph $K_{4,6}$ is not a mixed cycle-E-super magic decomposable.

Theorem 4.5. *Suppose the decomposition of $K_{2m,2n}$ has at least one C_4 and at least one C_8, then $K_{2m,2n}$ is not mixed cycle-E-super magic decomposable.*

Proof. Let $(p_1, q_1, r_1) \in D(K_{2m,2n})$ with $p_1 \geq 1$ and $r_1 \geq 1$. Suppose $K_{2m,2n}$ is mixed cycle-E-super magic decomposable, then by Theorem 4.1, there exists a mixed cycle-E-super magic labeling f such that $k = \frac{2mn(3m+n+12mn+2)}{p_1+q_1+r_1}$. Let us assume that $p_1 = 1$, $q_1 \neq 0$ and $r_1 = 1$. Let H_1 and H_2 be the subgraphs of $K_{2m,2n}$ isomorphic to C_4 and C_8 respectively. Let us label H_1, if we assign maximum edge labels to the edges of H_1, we have $\sum f(E(H_1)) = (4mn-3)+(4mn-2)+(4mn-1)+(4mn) = 16mn-6$ and if we assign maximum vertex labels to the vertices of H_1, we have $\sum f(V(H_1)) = \{((2m+2n)+4mn-3)+((2m+2n)+4mn-2)+((2m+2n)+4mn-1)+((2m+2n)+4mn)\}$ $=16mn+8m+8n-6$, thus $\sum f(H_1) = (16mn-6)+(16mn+8m+8n-6) = 32mn+8m+8n-12 \neq k$. Similarly if we assign minimum edge labels to the edges of H_2, we have $\sum f(E(H_1)) = (1+2+3+\cdots+8)$=36, and minimum vertex labels to the vertices of H_2, we have $\sum f(E(H_2)) = \{(4mn+1)+(4mn+2)+(4mn+3)+\cdots+(4mn+8)\}$ $=32mn+36$, thus $\sum f(H_2) = (36)+(32mn+36) = 32mn+72 \neq k$. In both cases we get a contradiction, hence the graph $K_{2m,2n}$ is not mixed cycle-E-super magic decomposable if its decomposition has at least one C_4 and at least one C_8. □

5 Conclusion

In this paper, we studied the mixed cycle-E-super magic decomposition of $K_{2m,2n}$. If its decomposition has at least one C_4 and at least one C_8, then $K_{2m,2n}$ is not mixed cycle-E-super magic decomposable. Figure 1 shows that the complete graph K_7 is mixed cycle-E-super magic decomposable with magic constant $k = 126$.
It is natural to have the following problem.

Open Problem 5.1. *Discuss the mixed cycle-E-super magic decomposition of complete graphs*

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