Journal of the Earth & Space Physics. Vol. 32, No. 1, 2006, P. 7-23

A hybrid precise gravimetric geoid model for Iran based on recent GRACE and SRTM data and the least squares modification of Stokes's formula

Kiamehr, R*.

*Geodesy Group, Royal Institute of Technology, SE -100 44 Stockholm, Sweden and Department of Geomatics, Zanjan University, 313, Zanjan, Iran

(Received: 18 Sep 2005 , Accepted:28 Feb 2006)

Abstract

Since 1986, several gravimetric geoid models have been published in the Iran region. It was found that the standard deviation of fitting between these models versus GPS/levelling data in most cases was worse than the currently available global geopotential models. A new hybrid gravimetric geoid model computed (IRG04) by using the least squares modification of Stokes formula based on the recent published GRACE based global geopotential model, the high-resolution Shuttle SRTM global digital terrain model and a new Iranian gravity anomaly database. The absolute and relative accuracies of the new geoid model tested versus the GPS/levelling points and they are estimated about 0.27 m and 3.8 ppm, respectively. Additional comparison between the IRG04 and the recent published gravimetric geoid models shows that the relative accuracy of the IRG04 is almost 4 times better than the most recent published models in this area.

Keywords: Gravity database, Least-squares modification of Stokes function, Geoid determination, SRTM, GRACE, GPS/levelling, Iran

1 INTRODUCTION

Many different methods for regional geoid determination have been proposed during recent years, each preferring its own set of techniques and philosophy, which makes it more difficult to judge what is the best method in a certain situation. Until now, there are three different geoid models computed for Iran based on the Remove-Compute-Restore (RCR) approach (see e.g., Forsberg 1990; Sansoʹ 1994), Helmert's scheme (Vaníček et al. 1995) and the ellipsoidal Bruns's formula (Ardalan and Grafarend 2004).

Based on the primary investigation that was carried out on these gravimetric geoid models (Kiamehr 1997, 2001, 2003a, 2003b, 2004 and Kiamehr and Sjöberg 2005a), it was found that standard deviation (SD) of fitting between the geoid models mentioned and GPS/levelling data both in the relative and absolute sense were almost same, or in some cases worse than the currently available GGMs.

Also, since the publication of the recent gravimetric geoid models, the quantity and quality of terrestrial gravity data have increased and several new GGMs from the recent satellite gravimetric missions (e.g., the Gravity Recovery and Climate Experiment Mission GRACE) have been released. On the other hand, a new high resolution Shuttle Radar Topography Mission (SRTM) global DEM has been released. The main purpose of this research is to test the potential of the Royal Institute of Technology (KTH) combined approach based on Least-Squares Modification of Stokes (LSMS) formula (Sjöberg, 1984, 1991, 2003c and 2003d) and these new data for the determination of a new geoid model for Iran.

The paper starts with a review of the KTH computational scheme to determine the geoid based on the LSMS formula with its additive correction terms. Then we continue with a brief review of the previous geoid models. In section 4 we explain briefly the procedure for the creation of the new Iranian gravity database and choosing the best GGM and DEM models, and parameters for the construction of the IRG04 model. Finally we evaluate and compare the accuracy of the new models and different geoid models versus the GPS/levelling approach in the absolute and relative senses.

2 GEOID DETERMINATION USING THE KTH APPROACH

In the computational scheme of the KTH method for the geoid determination (Sjöberg 2003b), the surface gravity anomalies and GGMs are used for determination of approximate geoidal heights ($\tilde{N}^0$) and all necessary corrections are added directly to $\tilde{N}^0$ (see Equation 1). In other approaches, these corrections are usually computed one by one in separate corrections in steps, so that in the first step the surface gravity anomalies are corrected by removing the effects of topographic and atmospheric external masses (or reducing them inside the geoid) as direct effects, and then, after applying Stokes's integral, their effects are restored (indirect effects). In addition, the gravity anomalies in Stokes's formula must refer to the geoid, so that a reduction of the observed gravity from the Earth's surface to the geoid is necessary; that is called downward continuation (DWC). In the KTH approach, all these separate effects are replaced by a total topographic effect (Sjöberg 2001).

The computational procedure for estimation of geoid height $\hat{N}$ in the KTH approach can be summarized by the following formula:

$$\hat{N} = \tilde{N}^0 + \delta N^{Topo}_{comb} + \delta N_{DWC} + \delta N^{a}_{comb} + \delta N_e, \tag{1}$$

where δN^{Topo}_{comb} is the combined topographic correction and it includes the sum of direct and indirect topographical effects on the geoidal heights, δN_{DWC} is the correction for the downward continuation effect (Sjöberg 2003b), δN^{a}_{comb} is the combined atmospheric correction (Sjöberg 2001) and it includes the sum of direct and indirect atmospherical effects, and δN_e is the ellipsoidal correction for the spherical approximation of the geoid in Stokes's formula to ellipsoidal surface of reference. (For more details about these correction terms see references). The approximate geoid height ($\tilde{N}^0$) can be computed by Sjöberg (2003c):

$$\tilde{N}^0 = \frac{c}{2\pi}\iint_{\sigma_0} S_L(\psi)\Delta\tilde{g}\,d\sigma + c\sum_{n=2}^{M}(Q^L_n + S_n)\Delta\tilde{g}^{EGM}_n, \tag{2}$$

where c = R / (2 γ), R is the mean Earth radius, ψ is the geocentric angle, $\Delta\tilde{g}$ is gravity anomaly, σ_0 is the unit sphere, γ is normal gravity on the reference ellipsoid and $\Delta\tilde{g}^{EGM}_n$ is the sea level gravity anomaly Laplace harmonic determined from EGM. The modified Stokes function is expressed as:

$$S_L(\psi) = S(\psi) - \sum_{n=2}^{L}\frac{2n+1}{2}s_n P_k(\cos\psi), \tag{3}$$

the modification parameters s^*_n and the truncation coefficients Q^L_n in Equation (3) can be calculated by

$$s^*_n = \begin{cases} s_n & \text{if} \quad 2 \le n \le L \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

and

$$Q^L_n = Q_n - \sum_{n=2}^{\infty}\frac{2k+1}{2}s_n e_{nk}, \tag{5}$$

where e_{nk} are the functions of a limited radius of the integration cap and Q_n are the Molodenskii's truncation coefficients that can be presented by

$$Q_n = \int_{\psi_0}^{\pi} S(\psi)P_n(\cos\psi)\sin\psi\, d\psi, \tag{6}$$

The LS choice of the parameters S_n is given by the solution to the system of equations (Sjöberg 1991)

$$\sum_{r=2}^{L} a_{kr}.s_r = h_k, \qquad k = 2,3,...,L, \tag{7}$$

Modification coefficients, which can be expressed via Q_n, e_{nk}, c_n, dc_n and σ^2_n by:

$$a_{kr} = (a^2_k + dc^*_k)\delta_{kr} - \frac{2r+1}{2}(\sigma^2_k + dc^*_n)e_{kr} - \frac{2k+1}{2}(\sigma^2_r + dc^*_r)e_{rk} + \frac{2k+1}{2}\quad\frac{2r+1}{2}\sum_{n=2}^{\infty}e_{nk}e_{nr}(\sigma^2_n + dc^*_n), \tag{8}$$

and

$$h_k = \frac{2\sigma^2_k}{k-1} - Q_k(\sigma^*_k + dc^*_n) + \frac{2k+1}{2}\sum_{n=2}^{\infty}Q_n e_{nk}(\sigma^2_n + dc^*_n) - \frac{2}{n-1}e_{nk}\sigma^2_n, \tag{9}$$

where

$$dc^*_n = \begin{cases} dc_n & \text{if} \quad 2 \le n \le M \\ c_n & \text{otherwise} \quad n > M, \end{cases} \tag{10}$$

For all data, errors are assumed to be random with zero mean so the norm of the total error can be obtained by adding their partial contributions. So, utilizing the "pure satellite" GGMs [instead of combined GGMs e.g., EGM96 (Lemoine et al. 1998) which used terrestrial gravity anomalies in their construction] is recommended in this approach.

The system of equations in Equation (7) is so ill-conditioned that it often cannot be solved by standard methods like Gaussian elimination. Based on Ågren (2004a), the instability of the optimum choice of parameters for the unbiased LSM method is completely harmless and truncated SVD (e.g., Press et al.1992) can be used to obtain a useful solution. The gravity anomaly degree variances (c_n) and GGM derived anomalies error degree variances (dc_n) (in Equations 8, 9 and 10) can be computed by

$$c_n = \frac{(GM)^2}{a^4}(n-1)^2 \sum_{m=0}^{n} (\overline{C}_{nm}^2 + \overline{S}_{nm}^2), \tag{11}$$

$$dc_n = \frac{(GM)^2}{a^4}(n-1)^2 \sum_{m=0}^{n} (d_{C_{nm}}^2 + d_{S_{nm}}^2), \tag{12}$$

where G is the universal gravitational constant, M is the mass of the Earth, a is the major semi-axis of the reference ellipsoid, $\overline{C}_{nm}$ and $\overline{S}_{nm}$ are the potential coefficients and $d_{C_{nm}}$ and $d_{S_{nm}}$ are the standard deviations of the potential coefficients coming from GGMs. The higher degree over the GGM harmonics generated synthetically using the Tscherning and Rapp (1974) model. For estimation of the error anomaly degree variances for the terrestrial gravity anomalies (σ_n^2), we use an isotropic error degree covariance function $C(\psi)$ (see Sjöberg 1986):

$$C(\psi) = c_1[\frac{1-\mu}{\sqrt{1-2\mu\cos\psi+\mu^2}} - (1-\mu)-(1-\mu)\mu\cos\psi], \tag{13}$$

and σ_n^2 is expressed by a reciprocal distance type function as:

$$\sigma_n^2 = c_1(1-\mu)\mu^n \quad , \quad 0<\mu<1, \tag{14}$$

the parameters c_1 and μ are computed using a covariance function $C(0)$ where ($\psi = 0$) and the correlation length τ [the value of the argument for which $C(\psi^0)$ has decreased to half of its correlation at $\psi = 0$ (Moritz, 1980)]. Based on studies by Nahavandchi (1998), Ellmann (2004) and Ågren (2004a), this study uses the correlation length $\tau = 0.1^\circ$. In Equation (13) when $\psi = 0$, we get

$$C(0) = c_1\mu^2 \Rightarrow C(\psi^0) = 0.5\,c_1\mu^2, \tag{15}$$

The solution for μ = 0.99899012912 can be found iteratively (Ellmann 2004).

2.1 THE ADDITIVE CORRECTIONS

As mentioned before, 4 different additive correction terms must be applied to approximate geoidal height ($\tilde{N}^0$) for determination of the final geoid height $\hat{N}$ (see Equation 1). The combined topographic correction term (Sjöberg 2001) can be computed by:

$$\delta N_{comb}^{Topo} = \delta N_{dir} + \delta N_{indir} \simeq -\frac{2\pi G\rho}{\gamma}H^2, \tag{16}$$

where ρ is the topographic mass density and H is the orthometric height. The DWC term in Eq. (1) can be computed for any point of interest P based on the LSM parameters (For more details about these correction terms, see Sjöberg (2003c) and Ågren 2004b) by:

$$\delta N_{DWC}(P) = \delta N_{DWC}^{(1)}(P) + \delta N_{DWC}^{L1,Far} + \delta N_{DWC}^{L2}(P), \tag{17}$$

where

$$\delta N_{DWC}^{(1)} = H_P\left[\frac{\Delta g(P)}{\gamma} + 3\frac{N_P^0}{r_P} - \frac{1}{2\gamma} - \frac{\partial \Delta g}{\partial r}\bigg|_P H_P,\right] \tag{18}$$

and

$$\delta N_{DWC}^{L1,Far} = c\sum_{n=2}^{M}(S_n^* + Q_n^L)\left[(\frac{R}{r_P})^{n+2} - 1\right]\Delta g_n(P), \tag{19}$$

and

$$\delta N_{DWC}^{L2} = \frac{c}{2\pi}\int_\sigma\int_0 S_L(\psi)\left[\frac{\partial \Delta g}{\partial r}\bigg|_P\right]\times(H_P - H_Q)d\sigma_Q, \tag{20}$$

where $r_p = R + H_p$, $\Delta g(P)$ is the gravity anomaly at the surface computation point P, σ_0 is a spherical cap with radius ψ_1 centred around P, H is the orthometric height of point P and gravity gradient $\frac{\partial \Delta g}{\partial r}$ in point P can be computed based on Heiskanen and Moritz (1967):

$$\left.\frac{\partial \Delta g}{\partial r}\right|_P = \frac{R^2}{2\pi}\iint_{\sigma_0} \frac{\Delta g_Q - \Delta g_P}{\ell_0^3} d\sigma_Q - \frac{2}{R}\Delta g(P), \tag{21}$$

where $\ell_0 = 2R\sin\frac{\psi_{PQ}}{2}$. Also for the modified Stokes formula, the approximate ellipsoidal correction (δN_e) for the geoid can be determined by a simple formula (for more details, see Sjöberg 2004):

$$\delta N_e \approx \psi_0^{\circ}[(0.12 - 0.38\cos^2\theta)\Delta g + 0.17N^0 \sin^2\theta]_{mm}, \tag{22}$$

where ψ_0° is the cap size (in units of degree of arc), θ is geocentric co-latitude, Δg is given in mGal and N^0 in m. In addition, the atmospheric correction δN_{comb}^a expressed in spherical harmonic terms of global height model (Sjöberg and Nahavandchi 2000) as a direct combined correction for the approximate geoidal height N^0 is given by:

$$\delta_{comb}^a = -\frac{2\pi\rho^0 R}{\gamma}\left[\sum_{n=2}^{\infty}\frac{2}{n-1}H_n - \sum_{n=2}^{\infty}(s_n + Q_n^L)H_n - \sum_{n=2}^{\infty}Q_n^L\frac{n+2}{2n+1}H_n\right], \tag{23}$$

where ρ^0 is the density of atmosphere at sea level. The elevation H of the arbitrary power v can be presented to any surface point with latitude and longitude (φ,λ) as

$$H^v(\varphi,\lambda) = \sum_{m=0}^{\infty}\sum_{m=-n}^{n} H_{nm}^v Y_{nm}(\varphi,\lambda), \tag{24}$$

where H_{nm}^v is the normalized spherical harmonic coefficient of degree n and order m that can be determined by the spherical harmonic analysis

$$H_{nm}^v = \frac{1}{4\pi}\iint_{\sigma} H^v(\varphi,\lambda)Y_{nm}(\varphi,\lambda)d\sigma, \tag{25}$$

here Y_{nm} is the fully-normalized spherical harmonic degree and order m. However, using just any high resolution DEM does not have any practical effect on the final results of solution (Sjöberg and Nahavandchi 2004, personal communication) so in this research, a $30' \times 30'$ digital elevation model is generated by averaging the Geophysical Exploration Technology (GETECH) $5' \times 5'$ DEM (GETECH 1995), using area weighting. The spherical harmonic coefficients are computed to degree and order 360. Figure 1 shows the result of additive correction terms to the geoid model on Iran.

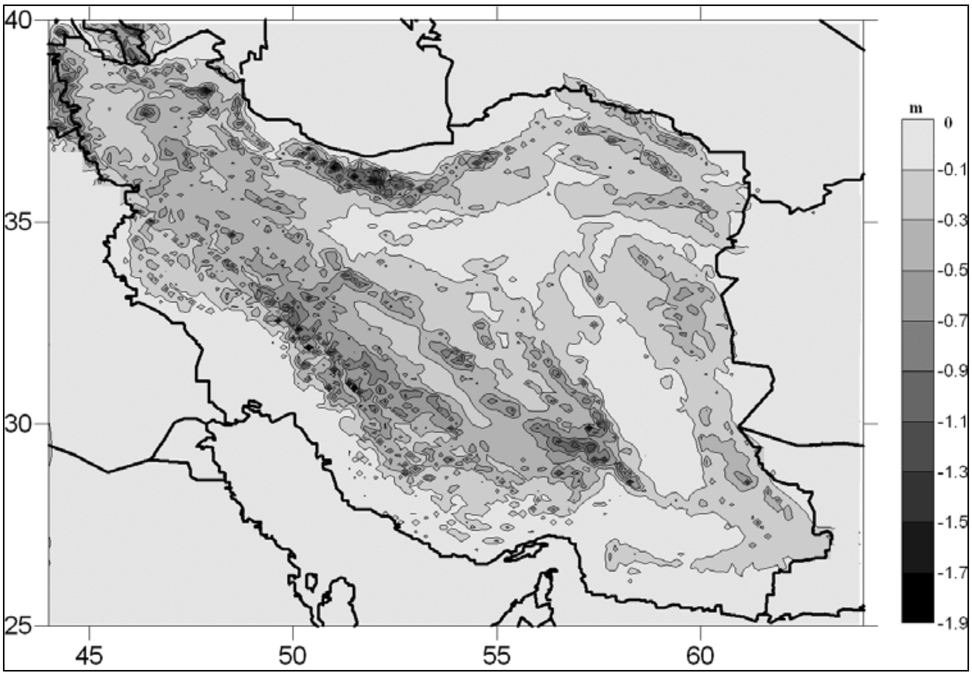
3 OVERVIEW OF CURRENT IRANIAN GRAVIMETRIC GEOID MODELS

Since 1986, different local gravimetric geoid models have been computed for Iran using various methods. The computation of the first geoid model was conducted by using a regional geopotential model improvement approach (Weber and Zomorrodian 1988). The method was based on the tailored GPM2 (degree and order 180) global geopotential model of Wenzel (1985).

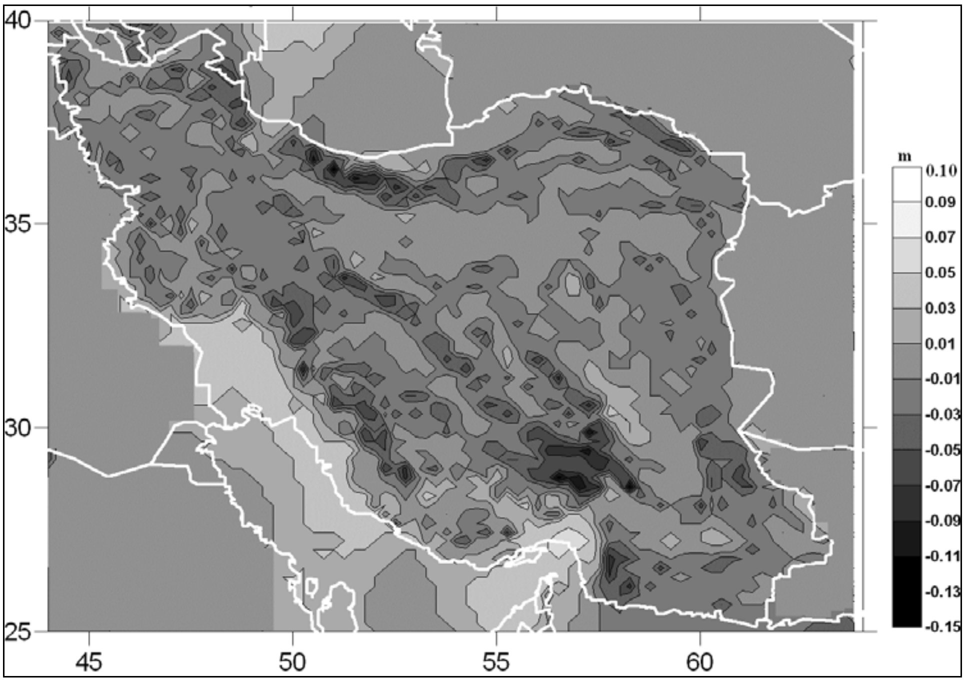
In the second stage (Hamesh and Zomorrodian 1992), a 1 by 1 km gridded DEM (extracted from scanned 1/250000 maps), together with the OSU89B (Rapp and Pavlis 1990) Global Geopotential Model (GGM) and the BGI gravity database was used for determination of the geoid by using the method of remove-compute-restore (RCR). Further results showed that fitting the geoid model at the GPS/levelling points improved by eliminating the terrestrial gravity data (Δg) from the solution of the geoid model, implying that the final presented geoid model (RCR) was computed based on the OSU89B GGM and DEM data only (Hamesh and Zomorrodian ibid).

Another research project by Ardalan and Grafarend (2004) resulted in the Tehran University Geoid model (TUG), which was computed in a new manner based on the ellipsoidal Bruns's formula and without applying Stokes's formula. The combination of the BGI gravity database with the recent observed gravity data from the National Cartographic Centre of Iran (NCC) together with the National Imagery and Mapping Agency (NIMA) GLOBE (Hastings 1996) 1×1 km global DEM were used in determination of this model.

(a)


40
35
30
25
45
50
55
60
m
0
-0.1
-0.3
-0.5
-0.7
-0.9
-1.1
-1.3
-1.5
-1.7
-1.9

(b)


40
35
30
25
45
50
55
60
m
0.10
0.09
0.07
0.05
0.03
0.01
-0.01
-0.03
-0.05
-0.07
-0.09
-0.11
-0.13
-0.15

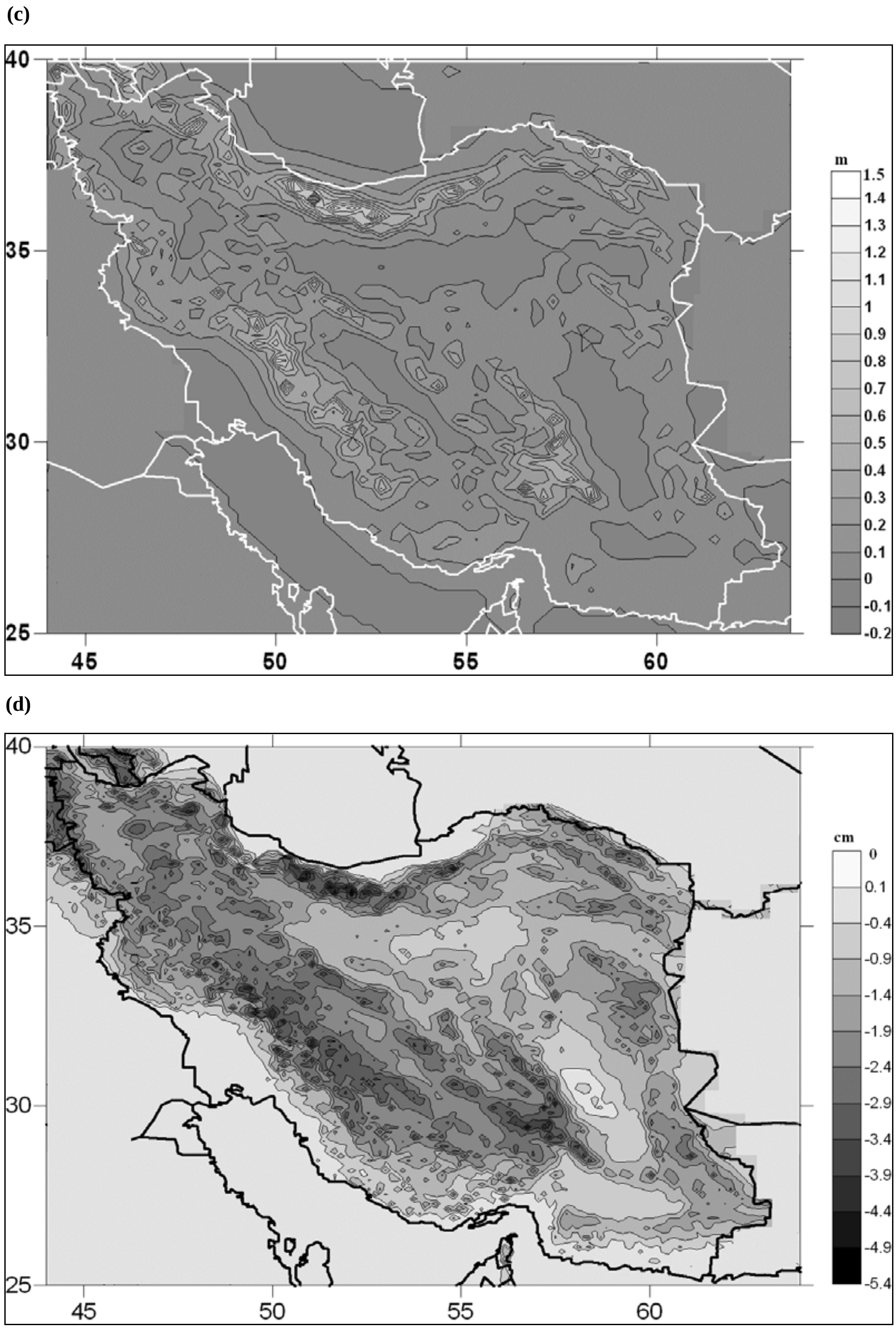


Figure 1. The combined topographic effect (a), DWC effect (b), total atmospheric effect (c) and the ellipsoidal correction (d) on the geoid model in Iran.

In the other effort, Najafi (2004) used the Stokes-Helmert scheme (Vaníček et al., 1995) for the computation of a new geoid model (KNTUG) for the central part of Iran ($30^\circ \leq \varphi \leq 34^\circ$ and $50^\circ \leq \lambda \leq 54^\circ$). The long-wavelength part of the KNTUG model was determined using the EIGEN-01S CHAMP satellite-only model (http://op.gfz-potsdam.de/champ/index_CHAMP.html), and the short wavelength contributions were determined by the total BGI and NCC terrestrial gravity data and the GLOBE 1 km global DTM model (Najafi 2004).

4 THE NEW IRANIAN GRAVIMETRIC GEOID MODEL (IRG04)

4.1 TERRESTRIAL GRAVITY ANOMALIES DATABASE

Regardless of the effect of choosing a proper computational method in the determination of a geoid model, the quantity and quality of the gravity anomaly database plays a major role in its final result. Kiamehr (2005) created a more complete and also refined new gravity database for Iran and all possible outliers were detected and removed from the database by using the Least Squares Collocation (LSC) approach. For the generation of the gravity database, a total number of 26125 point and mean gravity data were collected from different data sources. The original datasets used, bounded between $23° < \varphi < 42°$ and $41° < \lambda < 67°$, are: a) 9566 data from Bureau Gravimetriqe International (BGI 1992) gravity database; b) 8949 gravity data from the NCC and c) 7610 marine free-air gravity anomalies. The available observations have been observed during a long time span, using different equipment, methods and reference frames. It should be mentioned that the data type in question is usually very heterogeneous: very dense high-quality observations might have been collected in some geophysically interesting areas (e.g., the oil areas in the south-west), while the data are sparse and of diverse quality in other places. Also, it can be added that the lack of accurate heights is often a source of crucial errors.

The total area of Iran is estimated to be 1,648,195 km square, so it is simple to show that we have about one gravity point per 65 km square. The largest gaps are mostly located in the Zagros and Alborz mountains areas, Lut and Kavir central desert areas, Sistan & Balochestan provinces (in the south-east of country) and marine areas such as the Persian Gulf, Oman and Caspian Seas. In order to fill the gaps in the areas mentioned (both inside and outside Iranian territory), the original $0.5^\circ \times 0.5^\circ$ surface free-air gravity anomaly data (that was used in the modelling of the EGM96) and also those free-air gravity anomalies that were derived from satellite altimetry (Sandwell and Smith, 1997) were used in our database.

A special method was used for the interpolation of free-air gravity anomalies (for more details, see Kiamehr 2005) in order to take into account the effect of topography. The overall accuracy for the current database is estimated to be near 10 mGal. The minimum, maximum, mean and standard deviation of data are -182, 352, 3 and 51 mGal. The predicted $80'' \times 90''$ grid of free-air gravity anomalies (without outliers) is presented in figure 2.

4.2 DIGITAL ELEVATION MODEL (DEM)

Kiamehr and Sjöberg (accepted) evaluated the absolute accuracy of different DEMs in the Iranian region and explained their procedure for creating a new $15''$ precise DEM model for Iran (IRD04) based of SRTM 100 m high resolution data. They found very large differences between the GLOBE and SRTM models, with a range of -750 to 550 metre. They showed that this difference can cause a large error in the range of -160 to 140 mGal in free air correction and -60 to 60 mGal in simple Bouguer anomaly corrections. They also found that the geoid height difference in the range of -1.1 to 1 m due to the use of these two DEMs (Kiamehr and Sjöberg, ibid). The minimum, maximum, mean and standard division heights in the IRD04 DEM are-84.5, 5033.1, 758.1 and 760.6 m respectively. The overall absolute vertical accuracy of the SRTM in Iran was estimated to be approximately 6.2 m. This new DEM is used in the interpolation of free-air anomalies and also in computation of the topographic corrections of a new geoid model of Iran.

4.3 GLOBAL GEOPOTENTIAL MODEL

For the choice of the best GGM in the combined solution of the LSMS formula, several GGMs have been tested [e.g., EGM96, GGM01(S and C), GGM02 (S and C), EIGEN-GRACE 02S and CG01C] (Kiamehr and Sjöberg 2005a). The study showed that the combination of the newly released GRACE model (GGM02C) with EGM96 (for degrees and orders 200-360) performs the best with respect to GPS/levelling compared to the other recent satellite gravimetry missions.

However, in practice because of the interaction between terrestrial data and GGM in computation of the gravimetric geoid model using the least squares modification of Stokes's (LSMS) formula, we find that GGM02S (satellite-only) model gives more or less the same results compared with the combined GGM02C and EGM96 models. We know that in the construction of the GGM02C and EGM96 models, some surface gravity data are used that are not fitted with conditions of LSMS approach. Thus, we chose the GGM02S model in determination of new gravimetric geoid of Iran. The GGM02S gravity model (see, http://www.csr.utexas.edu/grace/gravity/) was estimated with 363 days GRACE data and its field was estimated to degree and order 160, and the solution appears to retain the correct signal power spectrum up to about degree 120. The GGM02S model was used with maximum degree and order of 110 in determination of the least square modification parameters in this research.

4.4 GPS/LEVELLING DATA

For the evaluation of the GGMs, DEMs and also comparison of the newly released geoid model versus current available gravimetric geoid models, 260 GPS/levelling data was used in this research. From these points, 35 points belong to the precise 1st order Iranian GPS and levelling network and others belong to 2nd order networks.

The measurements of the GPS network started in August 1988 and continued through 1990, using single frequency GPS receivers. The observations and computation was renewed and completed recently using dual frequency GPS receivers. From the result of latest network adjustment (Nilforoushan 2003, Personal communication), the mean standard deviation of the geodetic heights (σ_h) was estimated at approximately 0.2 m.

The orthometric heights (H) are believed to be accurate to 0.7 m in the absolute level of accuracy, because of neglecting the effect of the sea surface topography, presence of different systematic errors in observations and uncertainty about definition and establishment of the height reference system used in the adjustment of the network (Hamesh, 1991). The relative accuracy of the orthometric heights for the 1st order-leveling network is quite good and estimated near the 3 ppm (Hamesh 1991).

For evaluation of the geoid models (especially in the relative sense), we used the 35 most precise GPS/levelling points through the five selected traverses (in the north, west, east, centre and south-east of the country). The minimum, maximum and average distances between these points are 52, 115 and 80 km, respectively. Figure 3 shows the location of the selected GPS/levelling points and selected traverses on the topographic map of Iran.

4.5 THE NEW GEOID MODEL PARAMETERS

Choosing the proper GGM and modification parameter is an essential step in the determination of the geoid model using the LSMS formula. The main objective of the modification procedure is to minimize the effects of errors in the estimation of the geoid. The modification methods proposed by Sjöberg (1984, 1991, 2003a and 2003b) allow for minimization of the truncation errors, the influence of erroneous gravity data, geopotential coefficients and combination of different data sources in the least-squares (LS) sense at the same time in an optimal form. Terrestrial gravity observations distributed in Iran are non-homogeneous and often affected by different systematic errors (see section 4.1 for more details).

When the recently published GRACE models with very high accuracy in the low to medium degrees are used, it becomes important to use a kernel modification that effectively filters out the long wavelength errors from the gravity anomalies (Featherstone 2003). For this reason we need a proper weighting scheme for data as a priori or empirical stochastic model. However, usually the true errors for the gravity data are not known (e.g. Featherstone 2003) and we can determine just a general estimation for their accuracies in model. But, in Table 1(c) we can see that our pre-estimated accuracies for gravity data (10 mGal) gives the best results through the LSMS formula models (Kiamehr 2005).

In the Section 4.3 we explained that the combination of the GGM02C with EGM96 models gives the best fit versus GPS/levelling data. In order to test the effect of LSMS approach on the final results of the geoid model, we tested the potential of 5 different GGMs with their full effective degree and orders with the same cap size (ψ) and gravity data. Table 1(a) shows the results of this comparison versus 260 GPS/levelling data. We can see that the GGM02S has the same fitting level compared to the combined GGM02C with EGM96 models. It can be interrelated as a result of the interaction of the gravity data through the LSMS formula model.

On the other hand, we can see that the GGM02S has superior fitting compared to the other selected models. Therefore, we selected the GGM02S satellite-only model with maximum degree and order of $M = L = 110$ [see Equation (2 and 3)] in determination of the least square modification parameters.

In addition, from Table 1(b and c) we can see that the best fitting results comes by choosing the cap size $\psi = 3^{\circ}$ and $\sigma_{\Delta g} = 10$ mGal. These comparisons give very good information about properties of different sources of data and their effects on geoid models.

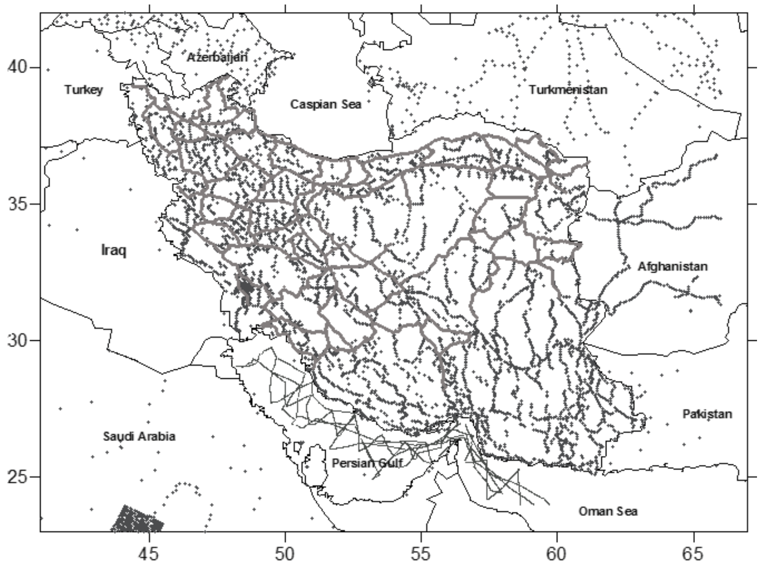


Figure 2. Gravity data distribution in Iran.

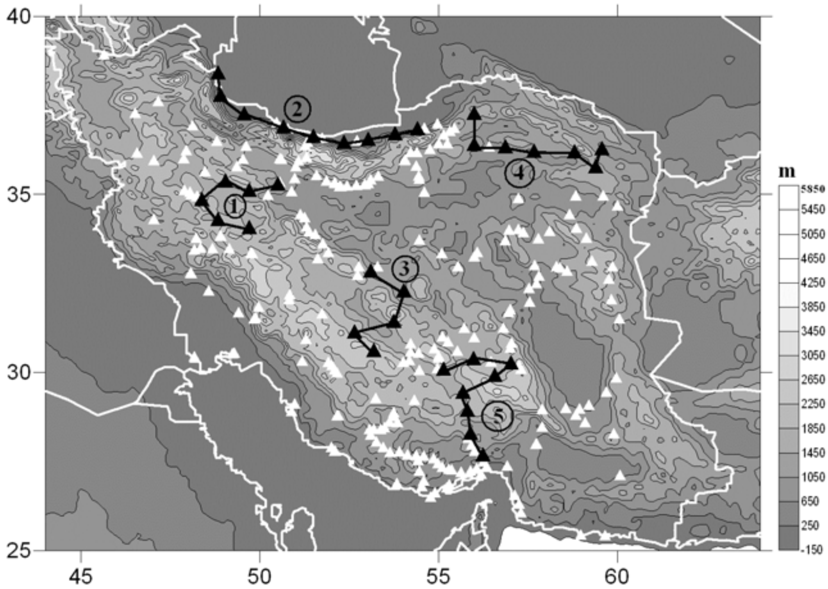


Figure 3. Distribution of the 260 GPS/levelling points (including the 35 precise points in the 5 traverses) on Iran DEM.

The final geoid model was computed based on the free-air gravity anomalies in the $80'' \times 90''$ grid size, GGM02S global geopotential model and 100 m SRTM DEM with cap size $(\psi = 3^\circ)$ and $\sigma_{\Delta g} = 10$ mGal. We also found that cap size $(\psi = 3^\circ)$ gives the best results concerning the quality and distribution of the gravity anomaly data. We think it proves the presence of different systematic errors in gravity data and very large local correlations between them. This comparison also proves the pre-estimated accuracy for gravity data ($\sigma_{\Delta g} = 10$ mGal) (Kiamehr 2005). Figure 11 shows contour map of the IRG04 geoid model.

5 EVALUATION OF THE NEW GEOID MODEL

A reasonable indication about the accuracy of geoid models can be obtained from the comparison with the GPS/levelling data. It can be done by the determination of the Root-Mean-Square (RMS) of fitting between gravimetric and geometric geoid models. The various wavelength errors in the gravity solution may be approximated by different kinds of functions in order to fit the quasi-geoid to a set of GPS levelling points through an integrated least squares (LS) adjustment. Several models can be used, ranging from a simple linear regression to a seven parameter similarity transformation model (Kotsakis and Sideris 1999). The seven parameter model gives the best fitting with minimum standard division in all selected GGM and gravimetric geoid models in Iran. Based on the 35 most precise GPS/levelling points, we obtain a RMS of 7 parameter fitting for IRG04 geoid model near 0.27 m.

In addition to testing in the absolute sense, the best way for evaluating the real potential of geoid models is the testing of their fitting versus GPS/levelling data in the relative view. For this purpose we computed the difference between two orthometric heights difference ($\delta\Delta H$) derived from direct levelling (ΔH_{Level}) and GPS with geoid models ($\Delta H_{(GPS\,with\,G\,GMs\,or\,Geoid)}$). This difference can be presented in the relative form in parts per million (ppm):

$$\text{ppm} = \text{mean}\left|\frac{(\delta\Delta H_{GGM-Level})_{mm}}{D_{ij_{(km)}}}\right|, \qquad (26)$$

where D_{ij} is the length of the baseline. Table 2 shows the results of fitting in the absolute and relative senses for the IRG04 and the current gravimetric geoid models. As the KNTUG model is available only in a limited area, we find just two precise GPS/levelling points there, so it is not possible to make any comparison for this model. As we mentioned before, the evaluation of geoid models in the relative sense gives a more realistic view about potential of geoid models. For example, the difference between the IRG04 and RCR geoid models in the absolute accuracy view is just 0.21 m but in the relative view there is a large difference, nearly 4 times between these two models. Also, we can see a large improvement (almost double) between IRG04 and TUG in the absolute view. Again in the relative view the improvement is almost 4 times. However, in order to get the best results, more and well distributed GPS/levelling data are needed.

In order to show the advantage of including the terrestrial gravity anomalies and the DEM on the accuracy of the new geoid model, we also compared the results of GPS/levelling fitting between GGM02 models and IRG04 geoid models (see Table 3). The RMS of fitting for GGM02S and GPS/levelling in the absolute and relative senses are 1.23 m and 19 ppm, respectively, which is not un-comparable with the IRG04. Figure 5 shows the discrepancy between GPS/levelling data and current local gravimetric geoid models in Iran.

Figure 6 shows the corresponding discrepancies between the IRG04 and TUG models. The largest differences between these two models are mostly located in the rough topographic areas in the Alborz and Zagros mountains (north and west). According to this comparison it seems that the IRG04 model also has a better fit with the GPS/levelling data in these areas. This improvement may be because of using the high resolution SRTM DEM in interpolation step and also terrain corrections and its accuracies in mountainous areas (Kiamehr 2005).

It is also important to mention here that the GGM02C has a better (or same) accuracy as the TUG and RCR gravimetric geoid models. We think with the current non-homogeneous and poorly distributed and also low density gravity data in Iran (1 data per 65 km), the current relative accuracy for the IRG04 geoid model could be reasonable.

As mentioned before, the mean distances of GPS/levelling data in this research is nearly 80 km, and some of the GPS data were collected with single frequency GPS receivers. It is clear therefore that in the short or medium baselines (say 5-10 km), we can achieve better relative accuracies for IRG04 the model because most of the errors in GPS and levelling observations were eliminated in short baselines (e.g., tropospheric error).

To sum up, we can observe significant improvement in accuracy between the IRG04 and recent TUG gravimetric geoid models. From the overall view of different local and global geoid models, the best results were obtained for the IRG04 geoid model (see Figure 7). More investigation is needed with denser and more precise GPS/levelling data for testing the potential of the IRG04 geoid (especially in rough areas).

Table 1. Effect of choosing the different GGM (**a**), cap size (ψ) (**b**) and standard deviation for Δg (**c**) in the LSMS solution of geoid models versus GPS/levelling points after 7 parameter fitting approach.

(a)

GGMs: $(\sigma_{\Delta g} = 10\,\text{mGal})$ $(\psi = 3^\circ)$	**EGM96**	**GGM01S**	**EIGEN-GRACE 02S**	**GGM02S**	**GGM02C +EGM96**
RMS (m)	**0.88**	**0.67**	**0.63**	**0.57**	**0.57**

(b)

(ψ°) **:** **GGM:GGM02S** $(\sigma_{\Delta g} = 10\,\text{mGal})$	1°	2°	3°	5°
RMS (m)	**0.66**	**0.69**	**0.57**	**0.63**

(c)

$\sigma_{\Delta g} = (\text{mGal})$ **:** **GGM:GGM02S** $(\psi = 3^\circ)$	**1**	**5**	**10**
RMS (m)	**0.78**	**0.69**	**0.57**

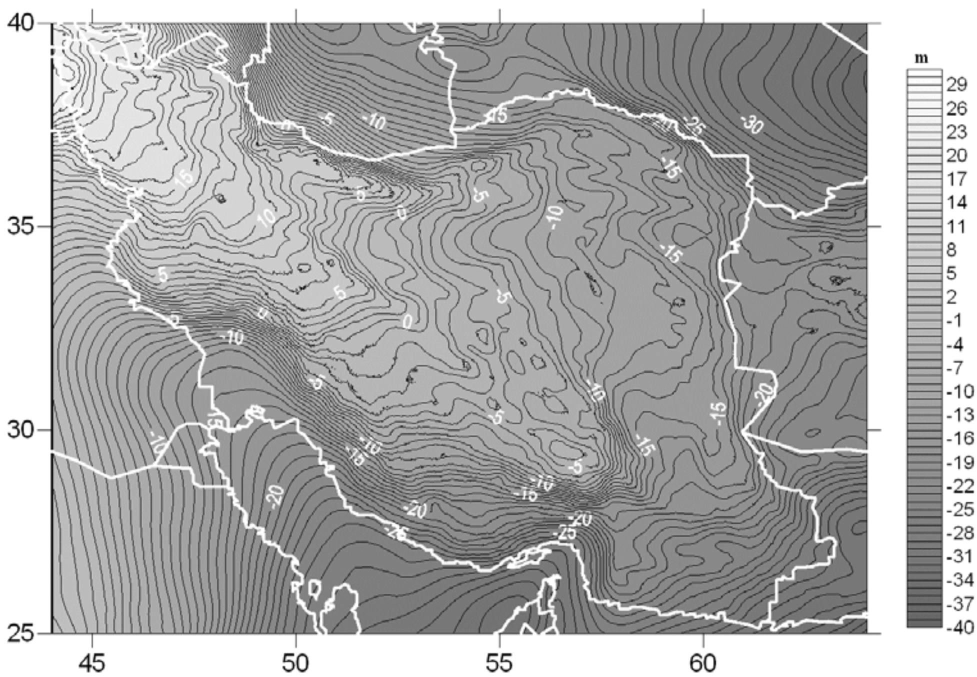


Figure 4. The new Iranian gravimetric geoid model (IRG04). Contour interval is 1 m.

Table 2. Statistical analysis of fitting between the local gravimetric geoid models and GPS/levelling points. (**a**) in the absolute view before and after the 7-parameter fitting. (**b**) in the relative view based of Equation (26). (Unit: metre)

(a)

Gravimetric Geoid Models	$N_{GPS-Lev} - N_{KNTUG}$ N=22 (Regional Model) **Before**	**After**	$N_{GPS-Lev} - N_{RCR}$ N=260 **Before**	**After**
Min.	-11.244	-1.631	-2.46	-1.934
Max.	-4.525	1.809	2.792	3.259
Mean	-9.559	0.000	-0.559	0.000
RMS	1.324	0.844	0.801	0.763
Gravimetric Geoid Models	$N_{GPS-Lev} - N_{TUG}$ N=258 **Before**	**After**	$N_{GPS-Lev} - N_{IRG04}$ N=260 **(New model)** **Before**	**After**
Min.	-5.14	-4.261	-2.257	-1.678
Max.	3.458	3.496	2.063	2.570
Mean	-0.517	0.000	-0.74	0.000
RMS	1.262	1.07	0.577	0.55

(b)

Traverse	**TUG RMS**	**RCR RMS**	**IRG04 RMS**
1 (West)	1.942	1.17	0.48
2 (North)	1.183	1.20	0.58
3 (Centre)	No DATA	1.33	0.48
4 (East)	1.93	0.49	0.18
5 (South)	0.407	2.00	0.14
Min	-2.686	-3.419	-0.82
Max	2.551	2.750	0.79
Mean	0.009	0.132	0.02
RMS ALL	1.239	1.310	0.40
ALL (ppm)	15.4	16	3.8

Table 3. Statistical analysis of fitting between GRACE GGM02 and GPS/levelling points. **(a)** in the absolute view before and after 7 parameter fitting. **(b)** in the relative view based of Eq. (26). (Unit: metre)

(a)

GGMs	**GGM02S GRACE (160)** **Before**	**After**	**GGM02C GRACE (200)** **Before**	**After**
Min.	-4.076	-3.671	-2.96	-2.558
Max.	2.714	3.013	1.87	2.018
Mean	-0.239	0.000	-0.243	0.000
RMS	1.230	1.218	0.854	0.837

(b)

Traverse	GGM-02S (120) RMS	GGM02C (200) RMS
1	0.451	0.500
2	2.461	1.530
3	1.860	1.477
4	0.778	0.733
5	1.348	1.223
Min	-3.221	-2.598
Max	3.569	1.943
Mean	-0.01	0.03
RMS ALL	1.540	1.11
ALL (ppm)	19	13.8

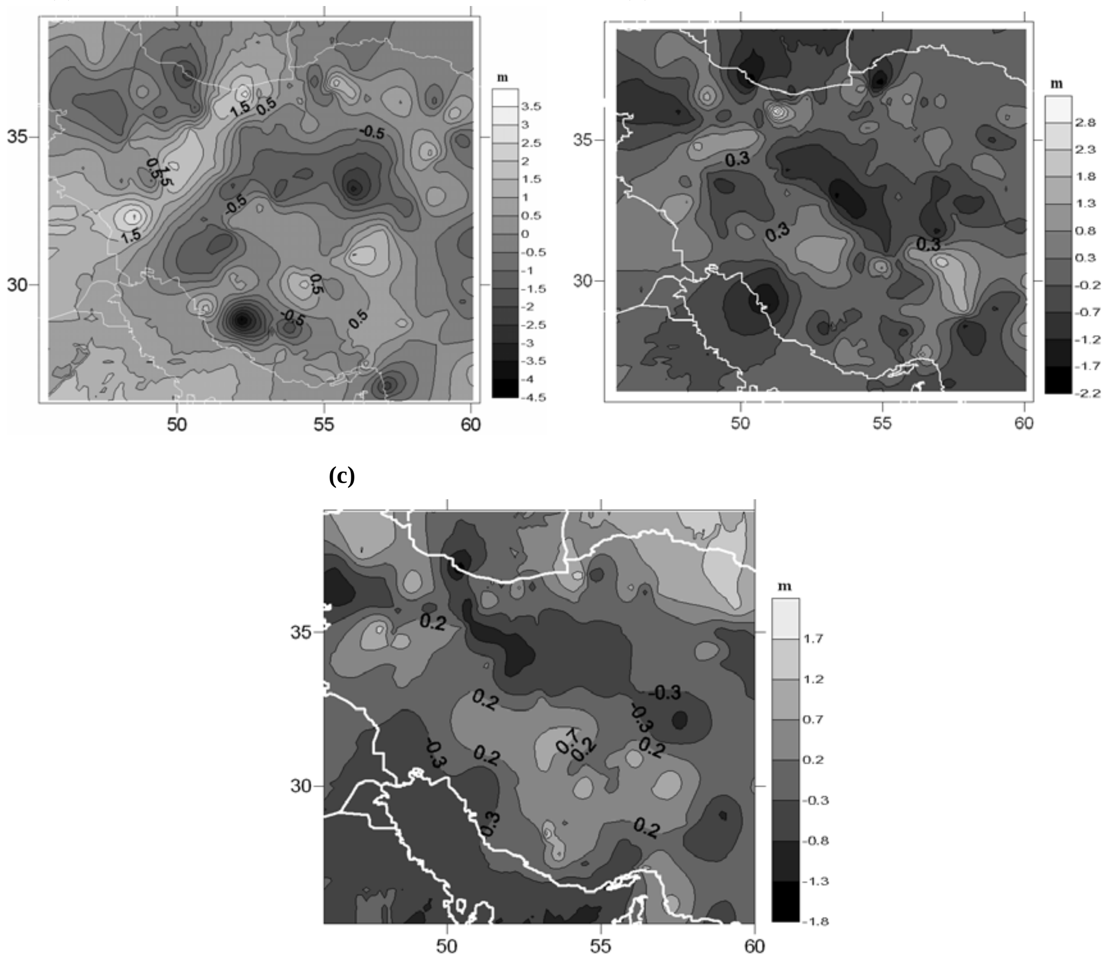


Figure 5. Discrepancy between GPS/levelling and local gravimetric geoids. (**a**): TUG, (**b**): RCR and (**c**): IRG04. (contour interval is 0.5 m).

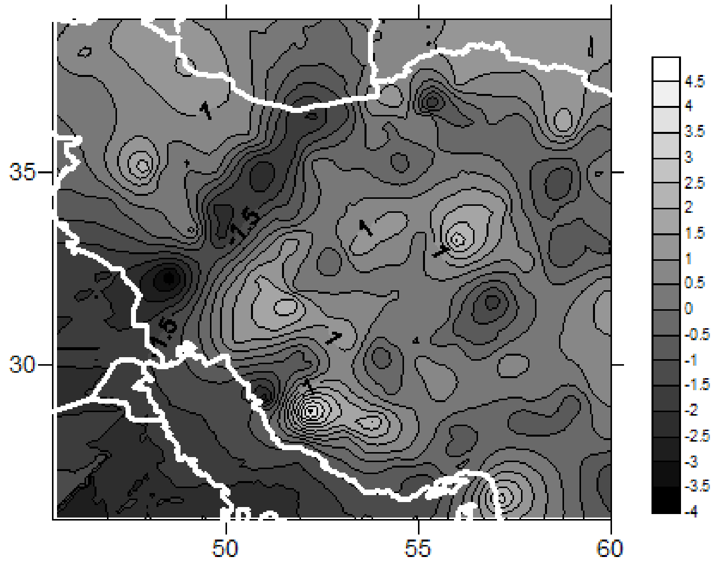
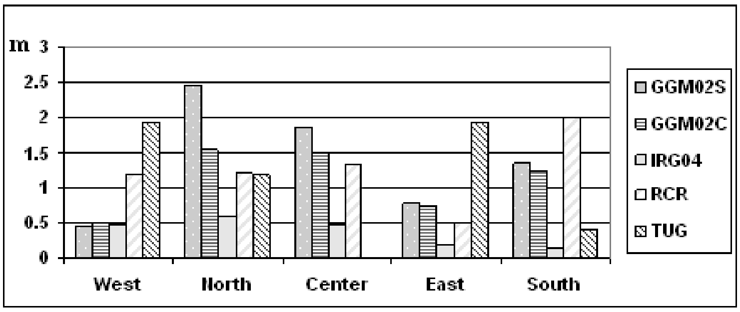


Figure 6. Discrepancies between the IRG04 and TUG geoid models. Contour maximum and minimum are +4.5 m (brightest region) and-4 m (darkest region), respectively, contour interval is 0.5 m.

(a)



(b)

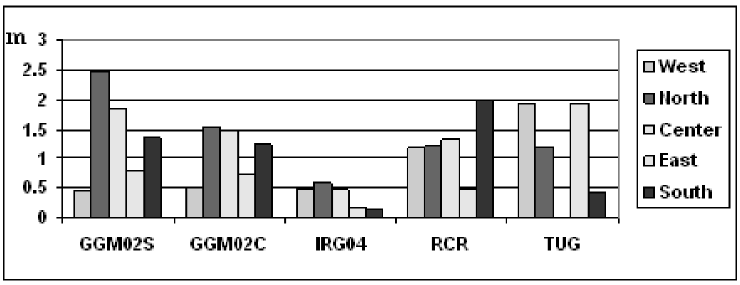


Figure 7. Comparıson of GGMs versus GPS/levelling points in the absolute (a) and relative view (b).

6 CONCLUSIONS

This research computes a new Iranian gravimetric geoid model based on the least square modification of Stokes's formula. During the research, a new Iranian gravity anomaly database was created and all possible outliers were detected and removed from the database (Kiamehr 2005). Also, the new Iranian DEM model (IRD04) was created with 15″ resolution based on the newly released 100 m SRTM DEM. In the computation of the new geoid model, we used the most recent data, including the new NCC gravity anomaly database, GGM02S GGM and SRTM DEM. The absolute and relative accuracies of the IRG04 gravimetric geoid model were tested versus GPS-levelling data. Based on 260 GPS/levelling points (a combination of the 1st and 2nd order data) the RMS of fitting for the IRG04 (after 7 parameter fitting model) was estimated near 0.55 m in the absolute view, the estimated accuracy by using the 35 accurate GPS/levelling points reach up to 0.27 m.

We know that the results of observation of GPS and levelling in the relative form have very high accuracy. This leads us to estimate the relative accuracy for the IRG04 geoid model based on ΔH (GPS/geoid) versus height differences from levelling. We found that the testing of the geoid models in the relative accuracy view gives realistic information about the potential of geoid models. According the results summarized in Figure 7 the IRG04 geoid model with 3.8 ppm is currently the best geoid model that fits the levelling data in Iran.

ACKNOWLEDGMENTS

The author is grateful to Prof. Lars Sjöberg for his constructive discussions and scientific review of this paper. Most of the gravity anomalies and GPS/levelling data that are used in this research were prepared by the National Cartographic Centre of Iran whose cooperation is kindly acknowledged.

REFERENCES

Ardalan, R., and Grafarend, E., 2004, High resolution geoid computation without applying Stokes's formula case study: High resolution geoid of Iran, J. Geodesy, **78**, 138-156.

Ågren, J., 2004a, Regional geoid determination methods for the era of satellite gravimetry, Ph.D. thesis, Royal Institute of Technology, Stockholm, Sweden.

Ågren, J., 2004b, The analytical continuation bias in geoid determination using potential coefficients and terrestrial gravity data. J. Geodesy, **78**, 314-332.

BGI, 1992, BGI Bulletin d' Information 70.

Ellmann, A ., 2001, Least squares modification of Stokes' formula with applications to the Estonian geoid. Royal Institute of Technology, Division of Geodesy Report No. 1056 (Licentiate Thesis), Stockholm.

Ellmann, A., 2004, The geoid for the Baltic countries determined by the least squares modification of Stokes' formula. Doctoral dissertation in geodesy No. 1061, Royal Institute of Technology (KTH), Stockholm.

Featherstone, WE., 2003, Tests of two forms of Stokes's integral using a synthetic gravity field based on spherical harmonics. In EW Grafarend, FW Krumm and VS Schwarze (Eds.): Geodesy-The Challenge of the Third Millenium, Springer, 163-171.

Forsberg, R., 1990, A new high-resolution geoid of the Nordic area. In: Rapp RH, Sansó F (eds) Determination of the geoid. Springer, Berlin Heidelberg NewYork, 241-250.

GETECH, 1995, Global DTMS. Geophysical Exploration Technology (GETECH), University Leeds.

Hamesh, M., 1991, Technical report about adjustment of Iranian first order levelling network, NCC J. Surveying, **1**, 9-20.

Hamesh, M., and Zomorrodian, H., 1992, Iranian gravimetric geoid determination Second step. NCC J. Surveying, 2 (6), 17-24, 52-63.

Hastings, D., 1996, The Global Land One-km Base Elevation (GLOBE) Digital Elevation Model. IGBP Newsletter, 11-12. (http://www.ngdc.noaa.gov/seg/topo/globe.shtml).

Heiskanen, WA., and Moritz, H., 1967, Physical Geodesy. W H Freeman and Co., New York, London and San Francisco.

Kiamehr, R., 1997, Iranian geoid problem and recommendation, 1997, 4th International Conferences of civil engineering, Sharif University of Technology, Tehran, Iran.

Kiamehr, R., 2001., Potential of Iranian geoid for GPS/Leveling, Proc National Cartographic Center of Iran, Geomatics 80 conferences, Tehran, Iran.

Kiamehr, R., 2003a, Comparison relative accuracy of EGM96 and Iranian gravimetric geoid, 6th International conference of Civil Eng., Isfahan University of Technology, Iran.

Kiamehr, R., 2003b, Discussion about relative accuracy of IFAG geoid, Journal of Sepehr Iranian National Geography Organization, **42**, 13-18.

Kiamehr, R., 2004, The relative accuracies of recent satellite gravimetric models in Iran, IAG International Symposium Gravity, Geoid and Space Missions-GGSM2004, Porto, Portugal.

Kiamehr, R., 2005, Qualification and refinement of the Iranian gravity database, Proc national Cartographic Center of Iran Geomatics 84 Conferences, Tehran.

Kiamehr, R., Sjöberg, LE., 2005a, The qualities of Iranian gravimetric geoid models versus recent gravity field missions, J. Studia Geophysica et Geodaetica, **49**, 289-304.

Kiamehr, R., Sjöberg, LE., (accepted) Effect of the SRTM global DEM in the determination of a high-resolution geoid model of Iran, J. Geodesy.

Kotsakis, C., Sideris, MG., 1999, On the adjustment of combined GPS/levelling/geoid networks. J. Geodesy, **73(8)**, 412-421.

Lemoine, FG., Kenyon, SC., Factor, JK., Trimmer, RG., Pavlis, NK., Chinn, DS., Cox, CM., Klosko, SB., Luthcke, SB., Torrence, MH., Wang, YM., Williamson, RG., Pavlis, EC., Rapp, RH., Olson, TR., 1998, The Development of the Joint NASA GSFC and NIMA Geopotential Model EGM96. NASA Goddard Space Flight Center, Greenbelt, Maryland, 20771 USA.

Moritz, H., 1980, Advanced Physical Geodesy. Wichmann, Karlsruhe Omang OCD, Forsberg R (2003) How to handle the topography in geoid determination: three examples. J. Geodesy, **74**, 458-466.

Najafi, M., 2004, Technical report of the KNT University geoid model, Department of research, National Cartographic Centre (NCC), TOTAK project, Iran.

Nahavandchi, H., 1998, Precise gravimetric-GPS geoid determination with improved topohgraphic corrections applied over Sweden. Ph.D. thesis, Royal Institute of Technology, Stockholm, Sweden.

Press, WH., Teukolsky, SA., Vetterling, WT., Flannery, BP., 1992, Numerical recipes in Fortran, 2nd edn. Cambridge University Press, Cambridge.

Rapp, RH. And Pavlis, NK., 1990, The development and analysis of geopotential coefficients models to spherical harmonic degree 360, J. Geophys. Res., **95 B13**, 21885-21911.

Sandwell, DT., WHF Smith, 1997, Marine gravity anomaly from Geosat and ERS 1 satellite altimetry, J. Geophys. Res., **102**, 10039-10054.

Sansó, F., 1994, (ed) Lecture notes, international school for the determination and use of the geoid. International geoid service, DIIAR-politechnico di Milano.

Sjöberg, LE., 1984, Least squares modification of Stokes' and Vening Meinesz' formulas by accounting for truncation and potential coefficient errors. Manuscr Geod., **9**, 209-229.

Sjöberg, LE., 1986, Comparison of some methods of modifying Stokes' formula. Pro 10th general meeting of the NKG., 268-278. Also in Boll Geod Sci., **46(2)**, 229-248.

Sjöberg, LE., 1991, Refined least squares modification of Stokes' formula. Manuscrcript Geodesy, **16**, 367-375.

Sjöberg, LE., 2001, Topographic and atmospheric corrections of gravimetric geoid determination with special emphasis on the effects of degree zero and one. J. Geodesy, **75**, 283-290.

Sjöberg, LE., 2003a, Improving modified Stokes' formula by GOCE data. Boll. Geod. Sci. Aff., **61 (3)**, 215-225

Sjöberg, LE., 2003b, A computational scheme to model the geoid by the modified Stokes formula without gravity reductions. J. Geodesy, **74**, 255-268.

Sjöberg, LE., 2003c, A general model of modifying Stokes' formula and its least squares solution. J. Geodesy, **77**, 459-464.

Sjöberg, LE., 2003d, A solution to the downward continuation effect on the geoid determined by Stokes' formula. J. Geodesy, **77**, 94-100.

Sjöberg, LE., 2004, A spherical harmonic representation of the ellipsoidal correction to the modified Stokes formula. J. Geodesy, **78**(2004), 180-186.

Sjöberg, LE., and Nahavandchi, H., 2000, The atmospheric geoid effects in Stokes' formula. Geophys. J. Int., **140**, 95-100.

Tscherning, CC., and Rapp, R., 1974, Closed Co variance Expressions for Gravity Anomalies, Geoid Undulations, and Deflections of the Vertical Implied by Anomaly Degree-Variance Models. Rep of the Dept of Geodetic Sci and Surv No 208 The Ohio State Univ, Columbus, Ohio.

Vaníček, P., Najafi, M., Martinec, Z., Harrie, L., and Sjöberg, LE., 1995, Higher-degree reference field in the generalized Stokes-Helmert scheme for geoid computation. J. Geodesy, **70**, 176-182.

Weber, G., and Zommorrodian, H., 1988, Regional geopotential model improvement for the regional Iranian geoid determination. Bulletin Géodésique, **62**, 125-141.

Wenzel, G., 1985, Hochauflösende Kuglefunktionsmodelle für das Gravitations potential der Erde. Wissenschaftliche Arbeiten der Fachrichtung Vermessungswesen der Universidad Hannover Nr. 135.