Advances in Industrial Engineering, Winter 2020, 54(1): 15-23
DOI: 10.22059/jieng.2021.317846.1749
RESEARCH PAPER

Timetabling of Metro Trains in a Dynamic Demand Situation Considering the Capacity of Trains and Stations on Peak and Off-Peak Times

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Received: 27 February 2021, Revised: 12 April 2021, Accepted: 12 April 2021

Abstract

This paper aims to propose a mathematical model to minimize the total waiting time of passengers in metro systems. The main contribution of this paper is considering the capacity of trains and stations, as well as the assumption of a constant interval for travelling between two successive stations. To reach this aim, the sum of dwell time and travel time is assumed constant. The dwell time is considered a function of the number of passengers who can board the train. To show the effectiveness of the proposed model, a numerical example is studied. The parameters of the metro system are considered according to Tehran Urban and Suburban Railway Operation Co. The results show the effect of an increase in the capacity of trains and the number of trains in reduction of the total waiting time. Furthermore, in this study, the best amount of Headway in order to minimize the waiting time is six minutes.

Keywords:
Dynamic Demand;
Mathematical Model;
Metro Timetabling;
Scheduling;
Waiting Time

Introduction

Global warming is an increasingly important topic these days because of the pressure from governmental and non-governmental organizations (NGOs) [1]. One of the causes of this phenomenon is the increasing amount of carbon dioxide (CO_2) emissions, which comes from a large part of transportation. One of the most effective ways to reduce the amount of CO_2 emissions is Implementation systems that use energy sources without CO_2 emissions such as electricity.

In the economy of each country, an important role is assigned to rail transportation which moves a significant amount of passengers and freight [2]. In particular, the metro system is a safe, fast, and convenient electric railway that has a significant role in reducing air pollution in urban areas. The major fields studied on metro systems are staffing, rolling stock, timetabling, line design, and network design. A metro system can be evaluated by metrics like energy consumption and travel time. Subway system operating companies are satisfied with minimizing the energy consumption, and the passengers enjoy minimizing the travel time [3].

The train timetabling problem (TTP) tries to obtain an optimal schedule for a set of trains that satisfies the constraints of the train capacities and their maximum velocity and does not violate the operational constraints such as dwell time, headway, speed profile and uncertainties emerged in peak-hours passenger demand.

In this paper, a mixed-integer linear programming (MILP) model for timetable optimization is presented to minimize total passenger waiting times. The challenging point that distinguishes

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between this study and the previous works is considering the limitation in the train capacity as well as the constraint of the capacity of the stations. In practice, the timetable of metros is predetermined and fixed, and it is declared to the public, therefore, the sum of traveling time in each section between stations and dwell time at each station is supposed to be a fixed value. In other words, an optimal train timetabling is designed that in peak or non-peak hours of a day, if the trains arrive station n at time T, they arrive station $n+1$ at time $T+k$, where k is a constant. In practice, the issue is handled by the train driver by controlling the dwell time and by adjusting the speed of the train.

The rest of the paper is organized as follows. In Section 2, the related works in the literature are reviewed. In Section 3, the mathematical model is proposed. The experimental results are shown in Section 4 and finally, the conclusions are presented in the last section.

Literature Review

Train timetabling models generate a timetable based on infrastructure constraints. The main objectives of the train timetabling problems are minimizing total travel time, unnecessary delays, and passenger waiting time.

Timetable design and its dependence on passenger demands are studied by several authors. Higgins et al. presented an approach for solving the train timetabling problem over a single-track line [4]. The objective function aims to minimize the total train delay time and also minimize the operating costs. Brännlund et al. presented a mathematical optimization model for the train timetabling problem [5]. Minimizing unnecessary delays along the track was considered as the objective function. Caprara et al. [6] and Caprara et al. [7] developed MILP models for the train timetabling problem. The objective is to achieve an optimal schedule that does not violate infrastructure capacities and some operating constraints. Kroon et al. [8] assumed that the demand is regular and tried to improve periodic timetables and minimize random disturbances. A single-track train timetabling model with a single objective is presented by Zhou and Zhong [9]. They considered a set of operational and safety requirement constraints and minimized the total travel time.

A more general problem is obtained when demand is considered to be dynamic. Mu and Dessouky [10] proposed two single-objective models for scheduling freight trains. Barrena et al. [11] take the dynamic passenger demand for rapid transit lines into consideration. The main objective of the presented model is to provide a timetable focusing on passenger convenience. They presented a mathematical model for the problem by introducing the flow variables. The Madrid subway system is taken into account as a case study. Extensive computational experiments show the effectiveness of their proposed algorithm.

Xu et al. [12] minimized the delay-ratio considering the train velocity and obtained an optimally balanced train schedule. An exact optimization model is used to provide the schedule and a hybrid algorithm is developed. Sun et al. [13] considered the problem of designing a demand-driven timetable and presented three mathematical model formulations. Yin et al. [14] studied a subway system with an online adjusting timetable to improve the efficiency in an environment that in each station the passenger demand is not certainly known. Jamili and Aghaee [15] proposed a stop-skipping method to obtain a solution that is robust against the uncertainties including variable passengers number boarded and alighted and special conditions in an urban railway line.

Yaghini et al. [16] investigated the locomotive assignment and train scheduling problems, simultaneously. Hassannayebi et al. [17] tried to obtain an optimal schedule for public transit terminals that minimizes passenger waiting times. To find an upper bound, heuristic rules were developed and inserted in the enhanced non-linear formulation to mitigate the required computational effort. Qi et al. [18] took women-only passenger cars into account. They proposed a heuristic algorithm as well as a simulation-based model and applied their approach to the case of the Beijing metro Yizhuang line. Kamandanipour et al. [19] addressed multi-class capacity allocation and dynamic pricing problems in passenger railroad transportation using a stochastic data-driven optimization method. Yang et al. [20] made an effort to enhance the efficacy of an urban rail line considering the passenger demand to be unbalanced. Gong et al. [21] studies a stochastic train timetabling problem and assumed the passenger demand to be random and dynamic.

Reviewing the literature we came to the conclusion that there are few works investigating the problem of train timetabling in a dynamic environment. However, none of these researches considered the limitations on the train and the station capacities, which is practically a challenging point. Based on this research gap, in this paper, an MILP model is presented for train timetabling problems considering capacity constraints and satisfy optimal driving policy with a constant running time between neighbor stations.

Problem Description

In this section, an optimal timetabling approach is described which aims to minimize the sum of the passenger waiting times. Also, we develop peak/off-peak timetable which leads to uncertainty on dwell time and considers structural restriction.

Notations

Indices

$S = \{1,2,\dots,n\}$	Set of station
$T = \{0,1,\dots,p\}$	Set of time instants defining the planning horizon. The time horizon is discretized into time intervals with the length of δ. It means that $t \in T$ corresponds to a time instance that δt time units has passed since the beginning of the time horizon.
$M = \{1,\dots m\}$	Set of train services which can be considered either finite or infinite.

Parameters

δ	Discretization constant by witch the planning horizon is divided into time intervals. Is the shortest valid time in the model and time intervals smaller than δ are not taken into account.
d_{ij}^t	Passenger demand between stations i and j at the end of time interval $[t-1,t]$
TD	Total passenger demand which is equal to $\sum_{t\in T}\sum_{i\in s}\sum_{j\in s:j>i} d_{ij}^t$ (TD is used as a big number to formulate some constraints)
C_k	The capacity of train k
C_i	The capacity of station i
h_{min}	Minimum value of headway
$\tau_{i,i+1}$	Time constant between station i and $i+1$
$TT_{i,i+1}$	Total travel time between station i and $i+1$
w_i	Dwell time on station i

Decision variable

x_{kij}^t	Binary decision variable whose value is equal to one if and only if train k leaves station i to station j at time t.
y_{kij}^t	Binary decision variable whose value is equal to one if and only if at least one passenger board train k to leave station i to station j at time t.
u_{kij}^t	Integer decision variable indicating the number of passengers boarded train k from station i to station j.
q_k^t	Integer decision variable indicating the number of passengers in train k at time t.
f_{ij}^t	Number of passengers waiting in station i to go station j at the end of time interval $[t-1,t]$

Mathematical model

In this section, the following mixed integer programming model can be constructed.

$$\text{Minimize Z} = \delta \sum_{t\in T}\sum_{i\in S}\sum_{j\in S} f_{ij}^{t}$$

$$f_{ij}^{t} = f_{ij}^{t-1} + d_{ij}^{t} + \sum_{k\in M} u_{kij}^{t} \qquad \forall i\in S, \forall j\in S: j>i \quad \forall t\in T\backslash\{0\} \qquad (1)$$

$$q_k^t \le q_k^{t-1} + \sum_{j'\in S: j'>j} u_{kjj'}^{t} - \sum_{t'<t}\sum_{i\in S: j>i} u_{kij}^{t'} + TD(1-x_{kj}^{t}) \qquad \forall k\in M, \ \forall t\in T\backslash\{0\}, \forall j\in S \qquad (2)$$

$$q_k^t \ge q_k^{t-1} + \sum_{j'\in S: j'>j} u_{kjj'}^{t} - \sum_{t'<t}\sum_{i\in S: j>i} u_{kij}^{t'} - TD\left(1-x_{kj}^{t}\right) \qquad \forall k\in M, \ \forall t\in T\backslash\{0\}, \forall j\in S \qquad (3)$$

$$q_k^t \le q_k^{t-1} + TD \times \textstyle\sum_{j\in S} x_{kj}^{t} \qquad \forall k\in M, \forall t\in T\backslash\{0\} \qquad (4)$$

$$q_k^t \ge q_k^{t-1} - TD \times \sum_{j\in S} x_{kj}^{t} \qquad \forall k\in M, \forall t\in T\backslash\{0\} \qquad (5)$$

$$q_k^t \le C_k \qquad \forall k\in M, \forall t\in T \qquad (6)$$

$$\sum_{j\in S} f_{ij}^{t} \le C_i \qquad \forall i\in S, \forall t\in T \qquad (7)$$

$$u_{kij}^{t} \le \left(\sum_{t'=0}^{t}\sum_{j'\in S; j'>i} d_{ij'}^{t'}\right) x_{ki}^{t} \qquad \forall i\in S\backslash\{n\}, \ j\in S: j>i \qquad (8)$$

$$x_{ki}^{t} \le \sum_{t'=0}^{t} x_{k-1,i}^{t'} \qquad \forall t\in T \qquad (9)$$

$$\sum_{t\in T}\sum_{k\in M} x_{k1}^{t} \le m \qquad (10)$$

$$\sum_{t\in T} x_{kj}^{t} \le 1 \qquad \forall j\in S, \ \forall k\in M \qquad (11)$$

$$\sum_{t\in T} t x_{k(i+1)}^{t} \ge \sum_{t\in T} t x_{ki}^{t} + h_{min}\sum_{t\in T} x_{ki}^{t} \qquad \forall i\in S, k\in M\backslash\{m\} \qquad (12)$$

$$TT_{i,i+1} + w_i = \tau_i \qquad \forall i\in S\backslash\{n\}, k\in M \qquad (13)$$

$$\sum_{t\in T} t y_{ij}^{t} = w_i \qquad \forall i,j\in S, \forall t\in T \qquad (14)$$

$$w_i = \begin{cases} w_i^{min} & ; \ f_{ij}^{t} \le C_k - q_k^t \\ w_i^{min} + 0.05(C_k - q_k^t) & ; \ f_{ij}^{t} > C_k - q_k^t \end{cases} \qquad (15)$$

In this mathematical model, the objective function tries to minimize the total waiting time of all passengers. Constraint (1) calculates the value of f_{ij}^{t}. This constraint implies that the number of passengers waiting at station *i* going to station *j* at the end of time interval $[t-1, t]$ is equal to the number of passengers waiting at the end of time interval $[t-2, t-1]$ plus the number of passengers entering station *i* at time instant *t* to go station *j* minus the number of passengers who board the train at station *i* to go station *j* at time instant *t*. Constraints (2) and (3) imply that if train *k* is sent out at time *t* then the number of passengers boarded in train *k* at time *t*, q_k^t, is equal to the number of passengers boarded in train *k* at time *t*-1, q_k^{t-1}, plus the number of passengers who alight from the train at time *t*. Constraints (4) and (5) state that if train *k* is not sent out at time *t* then q_k^t is equal to q_k^{t-1} because no change has been made to the number of passengers of the train *k* at time *t*. Constraints (6) and (7) ensure that the capacity constraint of trains and stations is not exceeded at any time. Constraint (8) is aimed to forces an upper bound on the variable u_{kij}^t. None of the trains is allowed to be departed from a station before the departure time of its previous train that station. This fact is guaranteed using constraint (9). Constraint (10) is used to limit the number of trains used in the timetable. To ensure that each train is sent out once from the first station constraint (11) is added to the model. Minimum headway time constraint is demonstrated via constraint (12). Constraint (13) ensures that the sum of travel time and dwell time between two successive stations is fixed for each train. Allowable time to boarding and alighting for passengers is showed in constraint (14). The permissible amount of dwell time is shown in constraint (15).

Numerical Example

Line 4 of Tehran metro is investigated in a normal day and the mathematical model is optimized based on data from Tehran urban and suburban railway company. The structural and operational properties of this line are shown in Tables 1 and 2, respectively.

Table 1. Structural properties of line 4 of Tehran metro

Property	**Value**
Total path length	21 km
Number of stations	18
Number of active trains	13
Number of active cars	91
Capacity of each train	1290 people
Interval between two successive trains	7 min
Utilization time	5:30 – 22:30

Table 2. Current timetable of line 4 of Tehran metro

Station	Ereme Sabz	Ekbatan	Azadi	Ostad Moein	Doctor Habibollah	Shademan
Depart Time (min)	0	2	6	9	11	13
Station	Towhid	Enghelab	Te'atre Shahr	Ferdowsi	Darvaze Dowlat	Darvaze Shemiran
Depart Time (min)	15	18	20	23	25	27
Station	Shohada	Sheikholreeis	Piroozi	Nabard	Niroo Havaei	Kolahdooz
Depart Time (min)	30	32	34	36	38	40

Numbers which is assigned to depart times start in time 0 at the first station (Erame Sabz) and end in time 40 at the last station (Kolahdooz). According to the objective function, the total waiting times of passengers are to be minimized. As shown in constraint 13, we implemented this model using the current timetable without change it.

Test problems are almost the same test problems presented by Barrena et al. (2014). The problem has many aspects. The examples are referred to as: $TT - n - p - \delta - m - C_i - C_k$ where *n* is the number of stations, *p* is the planning horizon, δ is the discretization constant, *m* is the number of trains, and finally C_i and C_k represent the capacity of trains and stations, respectively. The parameters used in these examples are listed in Table 3.

Table 3. Parameters index

Parameter	n	p	δ	m	Ci	Ck	$hmin$
Value	18	200	1(s)	(4,18)	(25000,39000)	(1500,3000)	420(s)
Parameter	wi min	f_{ij}^{t}					
Value	(30,60)	(0,3000)					

In this section, the relation between total waiting time of passengers and capacity of trains, number of trains, and headway are illustrated in Figs. 1 to 3, respectively.

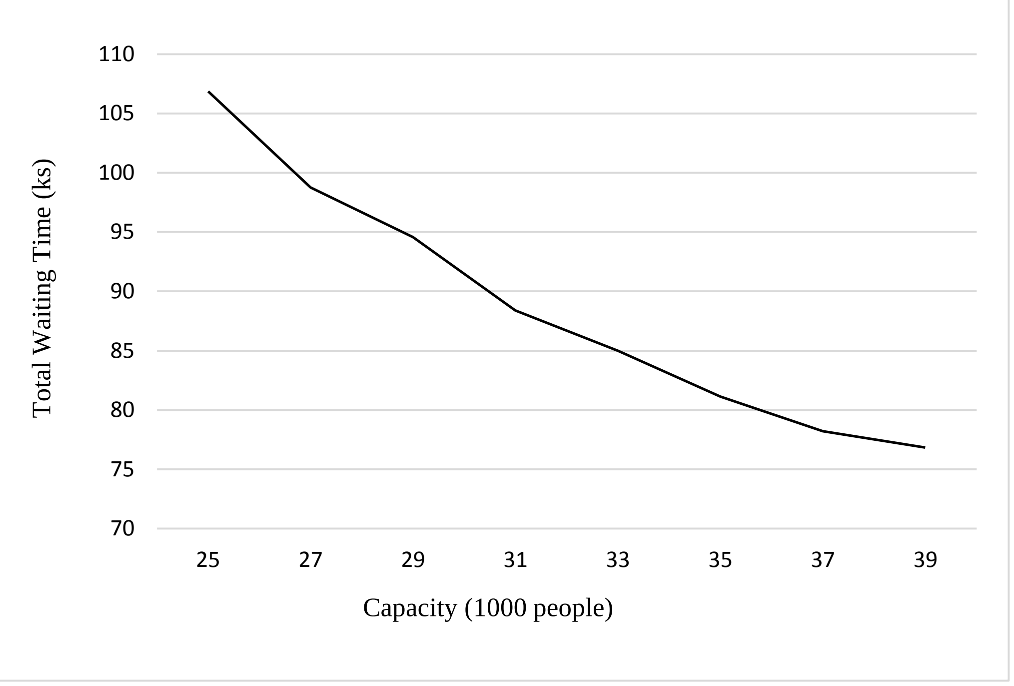

Fig. 1. Effect of Trains' Capacity on Total Waiting Time

It is worth mentioning that train capacity in normal situation is 1290 and multiplication of this number by 18 (number of station) lead to about 23000 people. If overload capacity of trains is considered, the number of people in each train is equal to 1600 and naturally total capacity will be increased. As shown in Fig. 1, increasing the capacity of trains leads to decreasing Total Waiting Time. However, the slope of changes decreased in greater capacities. So extreme increase in capacity is not economical. Since according to Tehran metro statistics, each car of metros price is 1 million dollars, a trade-off should be implemented between the waiting time and trains' capacity.

The next item studied in this section is the number of trains. As illustrated in Fig. 2, total waiting time and the number of trains are in an inverse relationship. After 18 trains, the waiting time is almost constant. It is noticeable that the number of active trains depends on the number of stations and the path length. In this example, the number of trains is variable between 4 and 18.

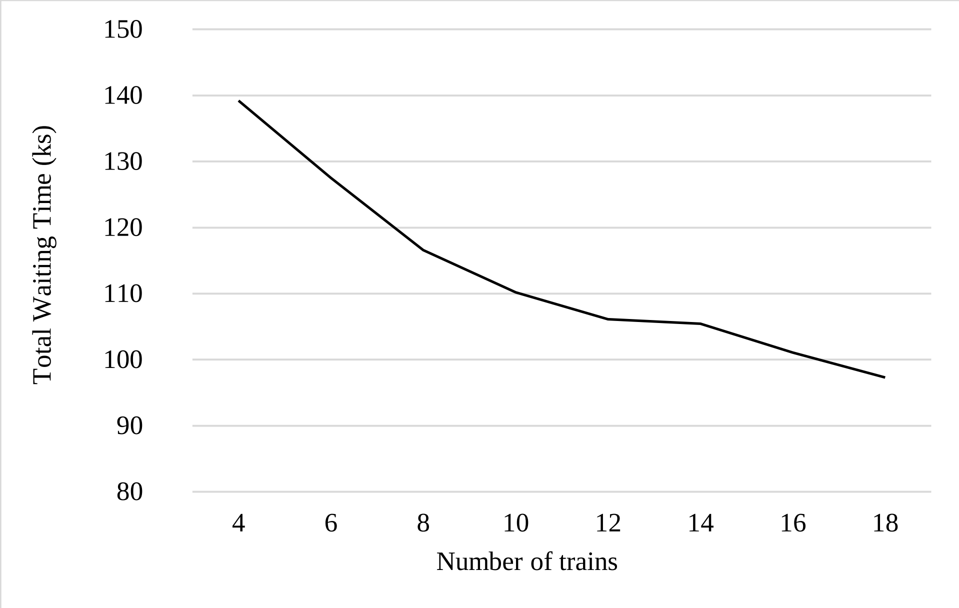

Fig. 2. Effect of trains' number on Total Waiting Time

The next item investigated in this research is Headway, which is defined as the gap between departure times of two successive trains from the first station. This amount varies between 4 and 12 in this case. Effect of headway on Total Waiting Time is shown in Fig. 3. As shown in this figure, the optimal value of headway is 6 minutes. We can describe this phenomenon as follows. If Headway has a very high amount, the time that trains dwell in stations will increase. So Total Waiting Time will grow. On the other hand, if Headway is less than 4 minutes, to avoid occurring accidents, trains must break and stop among the path. This fact leads to increasing the waiting time. Also, reduction of Headway needs to more trains and more expensive operation.

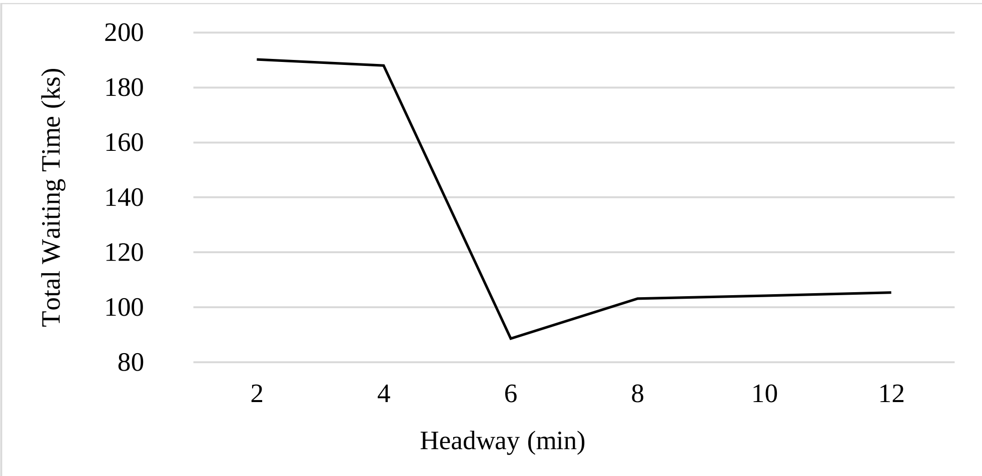

Fig. 3. Effect of Headway on Total Waiting Time

Conclusion

In this paper, an optimal timetabling approach is proposed for minimizing the total waiting time of passengers in metro stations. The main contribution of this model is considering structural constraints such as trains capacity and stations capacity. Also, dwell time of trains is considered as a function of the number of passengers waiting in stations. We developed this model in 2 situations namely peak and off-peak times. In peak times, if the number of passengers exceeds the empty capacity of trains, the value of dwell time increase per person. To simplifying the timetable optimization problem, we make some assumption which comes as follows. Furthermore, the capacity of all stations is assumed equal to 3000. In this case effect of some parameters including trains capacity, number of trains, and headway on total waiting time are investigated.

We implement a proposed model on Line 4 of the Tehran Metro system, to show its effectiveness. Some test problems were derived from literature using real data and solved by the GAMS-CPLEX solver. The obtained results show that total waiting time is reduced by increasing the capacity of trains and the number of active trains. Also, best value for Headway is equal to 6 minutes.

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