Advances in Industrial Engineering, Spring 2020, 54(2): 165-184
DOI: 10.22059/jieng.2021.323213.1762

RESEARCH PAPER

Optimizing a Reverse Logistics System by Considering Quality of Returned Products

Fariborz Jolai [a], Parisa Hashemi [a], Jafar Heydari [a,*], Alireza Bakhshi [a], Abbas Keramati [b]

a. *School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran*
b. *Ted Rogers School of Information Technology Management, Ryerson University, Toronto, Canada*

Received: 04 May 2021, Revised: 15 May 2021, Accepted: 15 May 2021

Abstract

Coordination is one of the critical issues in remanufacturing systems that can persuade supply chain parties to make optimal centralized decisions leading to higher profits. Accordingly, this paper aims to examine a reverse logistics system, including one manufacturer along with a collector who collects used products based on the consumers' willingness to return such products. Consumers' willingness is dependent on the take-back price, which is adjusted based on various quality levels affecting the processing cost of the collected items. This study developed mathematical models under both decentralized and centralized scenarios. Besides, to align the interests of both members and better profit-sharing, a cost-sharing contract is implemented. According to the results, in the coordination model, the take-back price of the high-quality level is increased compared to the decentralized model while the take-back price of the low-quality level is decreased. Hence, it suggests collecting and repairing higher-quality products to achieve higher profits for the whole system. Besides, the paper provides valuable suggestions for managers to resolve the conflicts of interest among participants of reverse logistics systems in an efficient manner.

Keywords:
Reverse Logistics;
Quality of Returned Product;
Coordination;
Recycling Strategy;
Cost-Sharing Contract

Introduction

In today's world, due to the growing consumption of various resources along with environmental concerns and social responsibilities, special attention is paid to closed-loop supply chains (CLSC) and reverse logistics [1,2,3]. Indeed, companies are working to develop a cycle for the return of the used products in order to save raw materials and prevent waste [4]. This issue plays a significant role in the competitive environment of developing countries, and it has become one of the basic demands of consumers [5]. Therefore, some famous high-tech companies adopted different remanufacturing plans (e.g., Kodak, FujiFilm, Hewlett–Packard, IBM Europe, and Xerox) [6].

Consumed goods can be classified into two main types, i.e., white and brown goods. The first group consists of commodities designed for a long lifespan, including appliances (e.g., trash compactors) that can be finally buried [7,8,9]. The latter group, called End-Of-Use (EOU) products, contains electronics (e.g., TVs and laptop computers) that become outdated with the development of new technologies [10,11,12]. However, because of environmental concerns,

[*] Corresponding author: (J. Heydari)
Email: J.Heydari@ut.ac.ir

they cannot be disposed of burying. Hence, it is preferred to repair or recover such goods collected through traditional or online channels [13]. Thus, the structure of reverse logistics and making appropriate decisions about the return policy can play a remarkable role in the profitability of supply chains [14].

Note that returned products vary in quality depending on their lifespan and how they are used. It is a big deal for decision-makers to manage the uncertainty of the returned products in terms of quality and quantity [15]. The quality of products is usually evaluated according to their integrity, lifespan, and maintenance strategy [16]. In fact, based on the quality levels of the collected products, different actions can be adopted to recover them [17]. In general, these different recovery actions include repairing higher quality products, remanufacturing or recycling low-quality products, and sending very low-quality products to be scrapped. Note that the processing cost and time allocated to returned products in checking the quality level, separating, sorting, and even the recycling cost will vary based on the product's quality.

Two approaches are applicable for product return policy. In the first one, called the active return approach, contrary to the second one called the passive return approach, the system members make an effort to motivate consumers to return products by paying incentives per each returned product [18]. Consumers will often resist returning end-of-life products; hence, preparations can be considered to motivate them to bring back such products. By offering incentives, companies can encourage consumers to return products, which is one of the critical factors in the success of reverse logistics [19]. There are various approaches to propose a framework for offering incentives. In some research, the incentive is considered a constant value [20]; in return, in others, it depends on several related factors, e.g., consumer's willingness [21]. Note that given the latter method, the effects of the returned product's quality on the incentive offered to consumers and recovery strategies are among topics that have not sufficiently been discussed in coordination problems. Therefore, by making a major change, the current study has addressed the coordination of a reverse logistics system through an incentive mechanism, i.e., a cost-sharing contract.

This paper aims to address the following research questions:

1. Can the cost-sharing contract be beneficial for both members?
2. What are the optimal terms of the cost-sharing contract?
3. How much is the proposed take-back price according to the quality levels of the returned products?

Motivated by this matter, we assume a two-level reverse channel, including a manufacturer and a collector who would like to reinforce consumers' willingness in returning consumed products and decides on the take-back price offered to consumers given the quality levels of the returned products. Two quality levels are considered based on the quality ratio of the returned products. Besides, the effect of a cost-sharing contract offered by the manufacturer is analyzed on decision-making.

The main contributions of this paper are as follow:

- Considering two quality levels for optimizing the take-back price offered to consumers; the first level involves products with a maximum of two years and the second level includes products with two to four years.
- Analyzing the effects of each quality level on decisions based on characterizing appropriate recycling strategy and processing cost in the recycling process.
- Considering the effects of overcrowding on the cost of processing, repairing, and remanufacturing; while the processing cost is shown as the effects of one quality level to another, the exponential cost function is considered for repair/remanufacturing cost.
- Offering a cost-sharing contract so that the reverse system can achieve coordination.

The rest of this paper is organized as follows. Section 2 reviews the subject literature and indicates the research gap addressed in this paper. In Section 3, the mathematical model and solving approach are presented for three scenarios. Numerical results and sensitivity analyses are provided in Section 4. In Section 5, managerial implications are presented. Section 6 includes the conclusions of this paper and suggestions for future studies.

Literature review

Reverse logistics design with quality consideration

Reverse logistics and recovery strategies are fields of CLSCs discussed in several studies considering special aspects. We reviewed the related aspects of this field to our study.

Using the appropriate method to recover products based on their nature is a topic investigated in the literature. Some studies addressed a reverse logistics network design model considering carbon-constrained [22,23,24,25]. Besides, some researchers studied the role of incentives on the return rate. The first study that distinguished the role of incentive mechanism in collection and recovery problems is presented by Guide Jr and Van Wassenhove [8]. Kaya [20] considered a CLSC in which the manufacturer collects used products by offering an incentive to consumers and then sells the products. In another similar study, researchers analyzed the influence of a buy-back contract in returning old products with three recovery options, i.e., product or component remanufacturing and raw material recovery [26].

In recent studies, special attention has been addressed to the role of product quality in making decisions. Cai et al. [27] determined optimal acquisition and production planning by considering two levels of quality products in a hybrid manufacturing/remanufacturing system. Liu et al. [16] developed a competition model for recycling waste electrical and electronic equipment that product price is based on product quality. Similarly, in another study, the impact of product quality on pricing decisions has been analyzed [28]. In some studies, the effect of product quality on the return rate has been addressed. Giri and Sharma [29] studied a CLSC that the return rate depends on the acceptable quality level, and the manufacturing process is assumed to be imperfect as well. Taleizadeh et al. [30] investigated the collection, product quality, and pricing decisions in two types of CLSCs: only dual-channel for recycling and dual-channel for forward and recycling process.

Reverse logistics coordination

Developing coordination contracts is a remarkable area in both the manufacturing and service industries [31]. He et al. [32] investigated the impact of various contracts on carbon-capped problems. Xie et al. [33] merged two revenue-sharing and cost-sharing contracts and proposed a solution to improve the quality of the returned products by rating collected products. Zhang et al. [34] examined how to distinguish the quality and value of products in a CLSC with defective returned products and demonstrated the coordination model through a revenue-sharing contract. Wang et al. [35] proposed two models for a dual-collection channel. To achieve the optimal strategy, they applied two-part tariff and revenue-sharing contracts. Recently, Bakhshi and Heydari [36] analyzed a put option contract in a reverse channel under remanufacturing capacity volatility.

Game theoretical models are one of the approaches to solve CLSC problems. Yi et al. [37] provided a game-theoretic framework for a dual recycling CLSC to determine optimal collection decisions. Genc and De Giovanni [38] modeled a Stackelberg game to assess a CLSC where the return rate is a function of price and quality. Another study employed a Stackelberg game for a competitive CLSC to cooperate members and examined the role of two contracts as well [39]. Toktaş-Palut [40] addressed green manufacturing processes in a game model along with a two-tariff contract. Recently, attitudes attracted toward optimal pricing and service policies through the Stackelberg game model in a dual-channel reverse supply chain [41,42].

Gap analysis

Several studies that are most similar to our research, which provides incentives for returned products by offering incentives as a linear distribution of consumers' willingness, are discussed below. Bai [43] studied reverse logistics and considered a willingness function for consumers to return used products. Afterward, Govindan and Popiuc [21] considered a CLSC and analyzed a revenue-sharing contract. Similarly, Heydari et al. [44] worked on this issue with debate on the government role, and Heydari et al. [18] developed this model by considering stochastic remanufacturing capacity. Another research examined uncertainty in the quality of the returned products along with remanufacturing capacity volatility [19].

In the current research, the role of the returned products' quality in the collection/recycling strategies is discussed. The most relevant studies to this paper are based on the development of the first model proposed by Bai [43] that introduced the consumers' willingness function in taking back used products. Among previous studies in this field, only Heydari and Ghasemi [19] addressed the quality of the used products as a random variable and determined a minimum acceptable quality level. In the current paper, two quality levels are considered in which different consumers' willingness functions are defined for each quality level. Furthermore, we present a processing cost function related to the amount of another quality level for each quality so that different quality levels lead to different actions for recycling. In Table 1**Table 1**, the most similar studies according to the main features have been compared with this study.

Table 1. Summarized literature review

Study	Coordination program	Quality level		Efficient factor on Rate of return			Quality role			Decision variables		
		Single	Multi	Advertising	Incentive	Constant/ Distribution	Demand	Rate of return	Recovery	Take-back price	Wholesale price	Production quantity/orde
Kaya [20]	Linear contract with transfer payments				□					□		□
Govindan and Popiuc [21]	Revenue-sharing				□					□		
Giri and Sharma [26]	-	□				□			□			
Heydari et al. [44]	Quantity discounts and Increasing fee			□	□					□		
Xie et al. [33]	Revenue-sharing and Cost-sharing	□				□			□			□
Modak et al. [28]	-	□				□	□				□	
Heydari et al. [18]	Revenue-sharing				□					□		
Heydari and Ghasemi [19]	Revenue-sharing	□			□			□		□		
Bakhshi and Heydari [36]	Put option				□							□
This study	Cost-sharing		□		□			□	□	□		

Problem description

Contrary to previous studies that have broadly concentrated on forwarding logistics dealing with inventory management, routing, location, and pickup/delivery [45,46,47,48], this research has looked at logistics issues from a different angle, i.e., the EOU products recycling/remanufacturing process management. In this study, a reverse logistics system is considered containing one manufacturer and one collector. The collector accumulates used products from consumers by offering incentives proportional to the quality levels of the returned products. The quality levels of the returned products are categorized into two levels, called quality level 1 (i.e., high quality) and quality level 2 (i.e., low quality). The collector tries to make a framework to motivate consumers, which is intended to provide a linear distribution for the consumers' willingness to return products, as presented below [21,43]:

$$W = f(d) = \begin{cases} \dfrac{d}{d_{max}}, & 0 < d < d_{max} \\ 1, & d \geq d_{max} \end{cases} \tag{1}$$

Where d is the take-back price offered to consumers for returned products and d_{max} is the maximum take-back price that motivates consumers to return all consumed products.

In this problem, each level of quality has a distinct consumers' willingness function. Since two quality levels are considered in this paper, we have two consumers' willingness functions. The maximum take-back price for quality level 1 is higher than that for quality level 2 ($d_{max1} > d_{max2}$). Besides, two constraints, specified in Eqs. 2 and 3 guarantee that the quantity of the returned products does not exceed products sold to consumers:

$$\frac{d_1^D}{d_{max1}} + \frac{d_2^D}{d_{max2}} \leq 1 \tag{2}$$

$$\frac{d_1^C}{d_{max1}} + \frac{d_2^C}{d_{max2}} \leq 1 \tag{3}$$

Let d_1^D and d_2^D, as well as d_1^C and d_2^C, denote the take-back price offered by the collector to return products in quality levels 1 and 2 in the decentralized and centralized models, respectively. Similarly, d_{max1} and d_{max2} are the maximum take-back price offered by the collector for this purpose.

After inspection and classification operations, the collector sells collected products to the manufacturer at an agreed price. The manufacturer performs two types of actions on the returned products based on quality levels. In this way, the manufacturer repairs products with a higher quality level and remanufactures the lower quality products. The purpose of this study is to determine optimal incentives for two quality levels to optimize the reverse logistics profit in three scenarios, i.e., decentralized, centralized, and coordinated by a cost-sharing contract. Fig. 1 depicts the structure of the assumed reverse system.

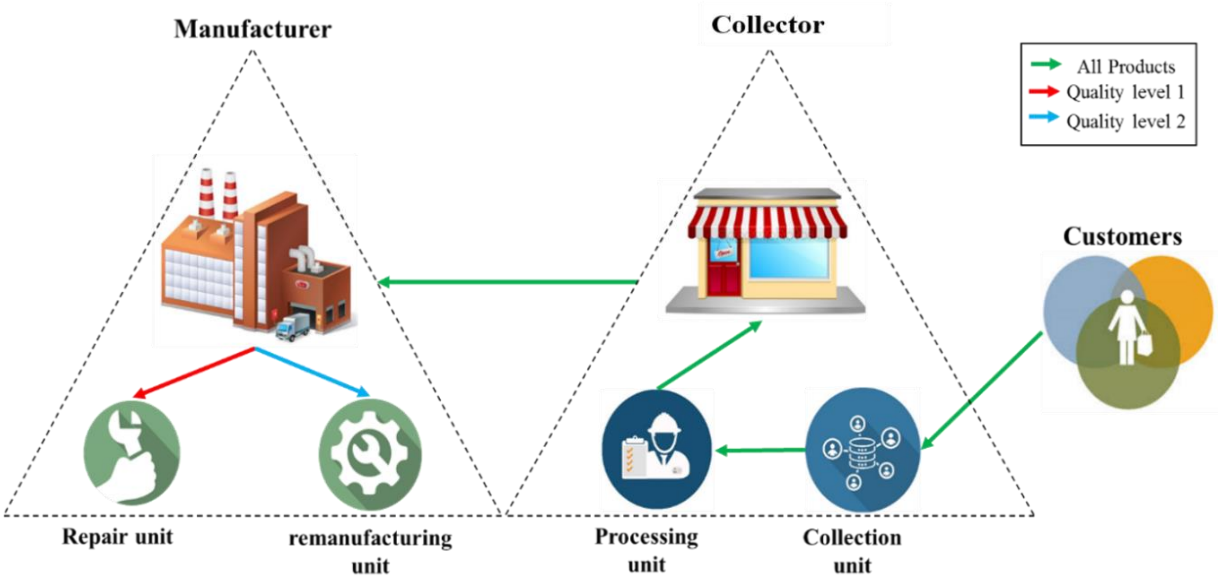

Fig 1. Structure of the investigated reverse system

The following assumptions are considered in the proposed models:

- The sales price of returned products to the manufacturer by the collector (P) is the same for all quality levels [27].
- The maximum take-back price offered for the returned products in quality level 1 (d_{max1}) is greater than quality level 2 (d_{max2}).
- The fixed processing cost of the collector for quality level 1 (α_0) is less than quality level 2 (β_0).
- The remanufacturing cost (C_{rem}) is greater than the repair cost (C_{rep}).
- The remanufactured and repaired products are identical in terms of quality and sales price (P_m).

Model formulation

The parameters and variables of models are listed as follow:

Parameters

D	Quantity of new product sold to consumers
P	Sales price of returned products sold to the manufacturer by the collector
W_1	Consumers' willingness for returning products in quality level 1
W_2	Consumers' willingness for returning products in quality level 2
C_1	Processing cost of returned products in quality level 1 for the collector
C_2	Processing cost of returned products in quality level 2 for the collector
C_{rep}	Repair cost ratio for the manufacturer
C_{rem}	Remanufacturing cost ratio for the manufacturer
d_{max1}	Maximum take-back price offered by the collector for quality level 1
d_{max2}	Maximum take-back price offered by the collector for quality level 2
P_m	Sales price of repaired and remanufactured products to consumers by the manufacturer

Decision variables in the decentralized model

d_1^D	Offered take-back price by the collector to return products in quality level 1
d_2^D	Offered take-back price by the collector to return products in quality level 2

Decision variables in the centralized model

d_1^C	Offered take-back price by the collector to return products in quality level 1
d_2^C	Offered take-back price by the collector to return products in quality level 2

Decision variables in the contract model

d_1^{Cont}	Offered take-back price by the collector to return products in quality level 1

d_2^{Cont}	Offered take-back price by the collector to return products in quality level 2

Decentralized model

The collector's profit function, including the profit of selling returned products and the processing cost for each quality level, is as follow:

$$\Pi_C^D = (P - d_1^D)W_1 D - C_1 W_1 D + (P - d_2^D)W_2 D - C_2 W_2 D \tag{4}$$

The first two terms are related to quality level 1; the former represents the profit from sales of the returned items to the manufacturer, and the latter indicates the processing cost of products. The second two terms show the same for quality level 2.

The manufacturer's profit function consists of the profit of selling repaired and remanufactured products to consumers and the repair/remanufacturing costs:

$$\Pi_M^D = (P_m - P)W_1 D - \frac{1}{2}C_{rep}(W_1 D)^2 + (P_m - P)W_2 D - \frac{1}{2}C_{rem}(W_2 D)^2 \tag{5}$$

The first two terms are related to quality level 1. The former represents the profit from sales of the repaired products to consumers, while the latter expresses the cost of repairing products. The second two terms show the same as previous for quality level 2. The cost of repair and remanufacturing due to the production line's over-capacity is considered in the literature [49,50].

It is also assumed that the collector considers one processing facility for both quality levels; therefore, due to capacity limitations, input amounts of each quality level can influence another level's processing cost. Note that the increase in the input amount of a quality level results in overcrowding and requiring more resources for its processing, which means the possibility of facing a shortage of capacity for another, and it can cause delays, extra storage, etc. This issue results in increasing the processing cost of another quality level. The quantity of each quality level depends on the consumers' willingness to return products. Therefore, the processing cost of each quality level is presented as a function of consumers' willingness of another quality level:

$$\begin{aligned} C_1 = F(W_2) = \alpha_0 + \alpha W_2 = \alpha_0 + \alpha \frac{d_2^D}{d_{max2}} \\ C_2 = F(W_1) = \beta_0 + \beta W_1 = \beta_0 + \beta \frac{d_1^D}{d_{max1}} \end{aligned} \tag{6}$$

Where α_0 is the fixed processing cost of quality level 1 per unit and α is the impact factor of consumers' willingness to return products with quality level 2 on the processing cost of quality level 1. Besides, β_0 demonstrates the fixed processing cost of quality level 2 per unit, and β is identified as the impact factor of consumers' willingness to return products with quality level 1 on the processing cost of quality level 2 for the collector.

Under the decentralized scenario, the collector decides on the optimal d_1^D and d_2^D, then attempts to maximize its profit. Now, by substituting Eqs. 1 and 6 into Eq. 4, we will have:

$$\Pi_C^D = (P - d_1^D)\frac{d_1^D}{d_{max1}}D - \left(\alpha_0 + \alpha\frac{d_2^D}{d_{max2}}\right)\frac{d_1^D}{d_{max1}}D + (P - d_2^D)\frac{d_2^D}{d_{max2}}D - (\beta_0 + \beta\frac{d_1^D}{d_{max1}})\frac{d_2^D}{d_{max2}D} \tag{7}$$

Theorem 1. *In the decentralized scenario, the collector's profit function* Π_C^D *is concave in* d_1^D *and* d_2^D *that guarantees the optimal* d_1^D *and* d_2^D *which are given as follow*:

$$d_1^D = \frac{2d_{max1}d_{max2}(P-\alpha_0) - d_{max1}(\alpha+\beta)(P-\beta_0)}{4d_{max1}d_{max2} - (\alpha+\beta)^2} \tag{8}$$

$$d_2^D = \frac{2d_{max1}d_{max2}(P-\beta_0) - d_{max2}(\alpha+\beta)(P-\alpha_0)}{4d_{max1}d_{max2} - (\alpha+\beta)^2} \tag{9}$$

Note that the proof of all presented theorems is given in Appendix.

Centralized model

In this case, the entire system profit is considered, and in the systemic view, the optimal amount of variables is calculated. The supply chain profit is obtained by the sum of profit functions of the retailer and the manufacturer and is equal to:

$$\Pi_{SC}^C = (P_m - d_1^C - \alpha_0)\frac{d_1^C}{d_{max1}}D + (P_m - d_2^C - \beta_0)\frac{d_2^C}{d_{max2}}D - \frac{1}{2}C_{rep}\left(\frac{d_1^C}{d_{max1}}D\right)^2 - \frac{1}{2}C_{rem}\left(\frac{d_2^C}{d_{max2}}D\right)^2 - (\alpha+\beta)\frac{d_1^C}{d_{max1}}\frac{d_2^C}{d_{max2}}D \tag{10}$$

Theorem 2. *In the centralized* scenario, *the supply chain profit function* Π_{SC}^C *is concave in* d_1^C *and* d_2^C, *and the optimal* d_1^C *and* d_2^C *are given by*:

$$d_1^C = \frac{d_{max1}(P_m(C_{rem}D + 2d_{max2} - \alpha - \beta) - \alpha_0(C_{rem}D + 2d_{max2}) + \beta_0(\alpha+\beta))}{C_{rem}C_{rep}D^2 + 2D(C_{rem}d_{max1} + C_{rep}d_{max2}) + 4d_{max1}d_{max2} - (\alpha+\beta)^2} \tag{11}$$

$$d_2^C = \frac{d_{max2}(P_m(C_{rep}D + 2d_{max1} - \alpha - \beta) - \beta_0(C_{rep}D + 2d_{max1}) + \alpha_0(\alpha+\beta))}{C_{rem}C_{rep}D^2 + 2D(C_{rem}d_{max1} + C_{rep}d_{max2}) + 4d_{max1}d_{max2} - (\alpha+\beta)^2} \tag{12}$$

Coordination by a cost-sharing contract

Results indicate that making centralized decisions leads to higher system profit than the decentralized model, but centralized decisions often result in a reduction in one of the members' profit. In order to satisfy members to the coordinated decision-making, it is necessary to provide incentives and motivations by defined contracts that compensate for losses incurred by members. In this model, the collector faces profit reduction under centralized decisions and requires a stimulus to join this coordination. Most studies proposed a revenue-sharing contract in tackling similar problems; however, to achieve this aim, a cost-sharing contract is proposed and analyzed in this study. In fact, cost-sharing is a process in which SC members can work together to secure savings in business operations. Indeed, cost-sharing enables SC members to cope with the competitive challenges of the rising cost. The manufacturer is inclined to increase the return rate of high-quality products. Thus, according to the cost-sharing contract, the manufacturer will be responsible for supplying a fraction of the purchasing cost of the returned products with quality level 1 as well as a fraction of the fixed cost of processing them. Also, the sales price of the returned products sold to the manufacturer by the collector is considered as a contract parameter to determine under the contract scenario. Hence, there are three contract parameters including P_c for the sales price of the returned products sold to the manufacturer by the collector under contract, λ $(0 < \lambda < 1)$ for a fraction of the purchasing cost of the returned products with quality level 1 $\left((d_1^{Cont})\left(\frac{d_1^{Cont}}{d_{max1}}\right)D\right)$, and γ $(0 < \gamma < 1)$ which is shown a fraction of the fixed processing cost of the returned products with quality level 1 $(\alpha_0\left(\frac{d_1^{Cont}}{d_{max1}}\right)D)$.

The profit functions of the collector and the remanufacturer under the cost-sharing contract are presented as below:

$$\Pi_C^{Cont} = (P_c - (1-\lambda)d_1^{Cont})\left(\frac{d_1^{Cont}}{d_{max1}}\right)D - \left((1-\gamma)\alpha_0 + \alpha\left(\frac{d_2^{Cont}}{d_{max2}}\right)\right)\left(\frac{d_1^{Cont}}{d_{max1}}\right)D + (P_c - d_2^{Cont})\left(\frac{d_2^{Cont}}{d_{max2}}\right)D - (\beta_0 + \beta\left(\frac{d_1^{Cont}}{d_{max1}}\right))\left(\frac{d_2^{Cont}}{d_{max2}}\right)D \tag{13}$$

$$\Pi_M^{Cont} = (P_m - P_c - \lambda d_1^{Cont})\left(\frac{d_1^{Cont}}{d_{max1}}\right)D - \left(\frac{1}{2}\right)C_{rep}\left(\left(\frac{d_1^{Cont}}{d_{max1}}\right)D\right)^2 + (P_m - P_c)\left(\frac{d_2^{Cont}}{d_{max2}}\right)D - \left(\frac{1}{2}\right)C_{rem}\left(\left(\frac{d_2^{Cont}}{d_{max2}}\right)D\right)^2 - \gamma\alpha_0\left(\frac{d_1^{Cont}}{d_{max1}}\right) \tag{14}$$

In Eq. 13, the first two terms are the profit and cost trade-offs for quality level 1 that the former displays the profit obtained from selling returned products to the manufacturer which under the cost-sharing contract, $(1-\lambda)$ fraction of d_1^{Cont} is paid by the collector. The latter indicates the processing cost of the collected products that $(1-\gamma)$ of fixed processing cost (α_0) is paid by the collector. The second two terms, like the previous one, illustrate the profit and cost phrases for quality level 2. In Eq. 14, the first two terms are related to quality level 1, which the first one represents the profit from sales of repaired products to consumers as well as the fraction of sales price of the returned products with quality level 1 incurred by the contract to the manufacturer. The second one shows the repair cost paid by the manufacturer for returned products. The second two terms, same as the previous one, show the profit and the cost of remanufacturing process for quality level 2. The last term expresses the fraction of fixed processing cost of quality level 1 provided by the manufacturer under the cost-sharing contract.

Theorem 3. *Under the cost-sharing contract, after determining the concavity of the collector's profit function Π_C^{Cont} in d_1^{Cont} and d_2^{Cont}, the optimal amounts of take-back price (d_1^{Cont} and d_2^{Cont}) are obtained as follow:*

$$d_1^{Cont} = \frac{d_{max1}(2d_{max2}(P_c - (1-\gamma)\alpha_0) - (\alpha+\beta)(P_c - \beta_0))}{4(1-\lambda)d_{max1}d_{max2} - (\alpha+\beta)^2} \tag{15}$$

$$d_2^{Cont} = \frac{d_{max2}(((1-\gamma)\alpha_0 - P_c)(\alpha+\beta) - 2d_{max1}(1-\lambda)(\beta_0 - P_c))}{4(1-\lambda)d_{max1}d_{max2} - (\alpha+\beta)^2} \tag{16}$$

Theorem 4. *In the cost-sharing contract, the sales price of the returned product to the manufacturer (P_c), and the fraction of the processing cost of products with quality level 1 (γ) procured by the manufacturer, are obtained as:*

$$P_c = (-(\alpha+\beta)(C_{rem}\,D(-P_m + \alpha_0) + P_m(\alpha+\beta) + (C_{rep}\,D + 2\mathrm{d}_{\max1})(2\mathrm{d}_{\max2}\,P_m + C_{rem}\,D\beta_0))\;/\;(C_{rem}D(C_{rep}D + 2d\mathrm{d}_{\max1}) + 2C_{rep}D\mathrm{d}_{\max2} + 4\mathrm{d}_{\max1}\mathrm{d}_{\max2} - (\alpha+\beta)^2) \tag{17}$$

$$\gamma = -\frac{1}{d_1^C d_{max2}\alpha_0}(d_{max1}((d_2^D)^2 + 3(d_2^C)^2) + d_2^D(d_1^D(\alpha+\beta) + d_{max1}(-P+\beta_0)) + d_2^C(2d_1^C(d_{max2}+\alpha+\beta) + d_{max1}(-P+\beta_0)) + d_{max2}((d_1^D)^2 + d_1^D(-P+\alpha_0) + d_1^C(d_1^C(1 + (\alpha+\beta)/d_{max1}) + (-P+\beta_0)))) \quad (18)$$

Theorem 5. *In the coordination scenario, the feasible interval of λ is as follows:*

$$\lambda_{min} = \frac{1}{2(d_1^C)^2 d_{max1} d_{max2}^2}(C_{rem} D d_{max1}^2((d_2^C)^2 - (d_2^D)^2) + d_{max2}(4d_1^C d_2^C d_{max1}(\alpha+\beta) + 2d_1^C d_{max1}^2(\beta_0 + 2d_1^C) + C_{rep} D d_{max2}((d_1^C)^2 - (d_1^D)^2) + 2d_{max1}d_{max2}(\alpha_0\, d_1^C + 2(d_1^C)^2 - 2d_1^D P + 2d_1^D P_m - 2d_1^C P_m) - 2d_{max1}^2(d_2^D P - d_2^D P_m + d_2^C P_m)) \quad (19)$$

$$\lambda_{max} = \frac{1}{(d_1^C)^2 d_{max2}}((d_1^D d_2^D + d_1^C d_2^C)(\alpha+\beta) + d_{max1}(\beta_0 d_2^D + (d_2^D)^2 + (d_2^C)^2 - d_2^D P) + d_{max2}(\alpha_0 d_1^D + (d_1^D)^2 + (d_1^C)^2 - d_1^D P)) \quad (20)$$

To simplify calculations, we considered the middle of the interval of λ as given in Eq. 21:

$$\lambda_{mid} = \frac{1}{4(d_1^C)^2 d_{max1} d_{max2}^2}(C_{rem} D d_{max1}^2((d_2^C)^2 - (d_2^D)^2) + d_{max2}(2d_{max1}(\alpha + \beta)(d_1^D d_2^D + 3d_1^C d_2^C) + 2d_{max1}^2(\beta_0 d_2^D + (d_2^D)^2 + \beta_0 d_2^C + 3(d_2^C)^2 - 2d_2^D P + d_2^D P_m - d_2^C P_m) + C_{rep} D d_{max2}((d_1^C)^2 - (d_1^D)^2) + 2d_{max1}d_{max2}(\alpha_0\, d_1^D + (d_1^D)^2 + \alpha_0\, d_1^C + 3(d_1^C)^2 - 2d_1^D P + d_1^D P_m - d_1^C P_m)) \quad (21)$$

To portray the efficient performance of the proposed cost-sharing contract, note that $\lambda_{min} < \lambda_{max}$. Due to the complexity of calculations, although it is no possible to prove this analytically, our numerical instances demonstrate that this always happens in numerical terms. Given the results acquired from numerical examples, $[\lambda_{min}, \lambda_{max}]$ is a non-empty interval, and it can be concluded that the proposed cost-sharing contract acts properly.

Debate on the model constraints

For each scenario, there is a constraint that ensures the quantity of returned products does not exceed the total amount of sold products (Eqs. 2 and 3). With respect to using a non-binding method to solve the model and meet such constraints, a heuristic approach is presented as follows:

Calculate the value of $\frac{d_1}{d_{max1}} + \frac{d_2}{d_{max2}}$ for each scenario.

In this step, we face two conditions:

Condition 1: If $\frac{d_1}{d_{max1}} + \frac{d_2}{d_{max2}} \le 1$, the values are optimal and algorithm is finished.

Condition 2: If $\frac{d_1}{d_{max1}} + \frac{d_2}{d_{max2}} = m > 1$, then replace these values instead of the main value of d_1 and d_2, respectively: $d'_1 = \frac{d_1}{m}$ and $d'_2 = \frac{d_2}{m}$.

This algorithm reduces the values of two variables to an equal ratio and guarantees that the mentioned constraints are satisfied, but this approach does not guarantee optimal values, and a complete search is needed to find the optimal solution. However, in this research, we used this heuristic approach because we were looking for an applicable model that can be solved by analytical methods.

Numerical results

Ten examples are investigated, and the results obtained from three scenarios are compared. Datasets used in the experimental instances encompass all the assumptions and requirements of the proposed models. Note that we used a set of datasets pertinent to previous studies in this field [21,51,52,53,54], chiefly developed based on real cases. Specifically, by scaling and rectifications, these values can be applied in the remanufacturing industry. Accordingly, in order to reinforce consumers' willingness to return end-of-life products, we have endeavored to prove and demonstrate the performance and efficiency of our proposed models by using real data sets in previous research and solving numerous experimental instances. The considered dataset is presented in Table 2. Meanwhile, equations obtained for all decision variables are closed-form relations; therefore, they can be solved through any mathematics software. The assessment of the experiments indicates the results mentioned as follows:

Table 2. The considered numerical instances

Instance	Parameters										
	P	D	d_{max1}	d_{max2}	α_0	β_0	α	β	C_{rep}	C_{rem}	P_m
1	1200	5000	900	500	500	800	50	20	0.4	1	2500
2	1200	5500	900	500	500	800	50	20	0.4	1	2500
3	1200	5000	900	500	300	700	50	20	0.4	1	3500
4	1200	5000	900	500	500	800	100	50	0.4	1	2500
5	1200	5000	900	500	500	800	50	20	1	2	4500
6	1200	5000	1000	400	500	800	50	20	0.4	1	2500
7	1100	5000	500	300	500	800	50	20	0.4	1	2700
8	1300	5000	900	500	500	800	50	20	0.4	1	3500
9	1200	5000	900	500	500	800	100	20	0.4	1	2500
10	1000	3500	750	400	200	600	30	10	0.07	0.6	2000

Regarding numerical results acquired in Table 3, the collector's profit decreases in the centralized scenario; however, by increasing the manufacturer's profit, the whole system's profit increases. Therefore, a coordination mechanism is implemented in which the manufacturer incurred a fraction of the collector's costs. In line with previous studies wherein they proved the efficiency of cost-sharing contracts on their proposed system, in the proposed reverse system in this study, the results illustrate that this mechanism works well and increases the profit of both members and the entire system as well. Comparison of the model variables, which are take-back prices offered to consumers in three types of the decision-making process, indicates that the take-back price of quality level 1 has been increased in the centralized model compared to the decentralized one while the take-back price of quality level 2 has been decreased in the centralized case. This highlights that an increase in the returned products amount at quality level 1 will increase the whole system's profit. Applying a cost-sharing contract in which the manufacturer takes over part of the take-back price and processing cost of products with quality level 1 triggers an incentive for the collector to increase the take-back price of quality level 1.

Table 3. Numerical results of variables and profit functions obtained for three scenarios

Instance	Variables and Profit functions																	
	d_1^D	d_2^D	$\prod_C^D$	$\prod_M^D$	$\prod_{SC}^D$	d_1^C	d_2^C	$\prod_C^C$	$\prod_M^C$	$\prod_{SC}^C$	d_1^{cont}	d_2^{cont}	γ	λ_{mid}	P_c	$\prod_C^{cont}$	$\prod_M^{cont}$	$\prod_{SC}^{cont}$
1	336.92	186.89	1028912.04	2415724.79	3444636.83	469.08	138.6	913522.82	2870843.11	3784365.93	469.08	138.6	0.2787	0.2179	1113.7	1198776.5	2585589.3	3784365.9
2	336.91	186.89	1131803.2	2388101.3	3519904.6	445.96	128.1	1026591.6	2896493.8	3923085.5	445.96	128.1	0.4745	0.0917	1090.9	1333393.6	2589691.8	3923085.5
3	433.68	233.13	1667038.04	7025002.2	8692040.2	581.35	177.02	1520835.7	7846862.7	9367698.4	581.35	177.02	0.7446	0.1418	1099.3	2004867.1	7362831.3	9367698.4
4	324.05	172.99	976090.01	2444731.2	3420821.2	464.07	135.22	861707.5	2865874.5	3727582.04	464.07	135.22	0.0919	0.2962	1147.8	1129470.3	2598111.6	3727582.04
5	336.91	186.89	1028912.03	7099617.3	8128529.3	526.33	166.32	828391.6	8096658.2	8925049.9	526.33	166.32	0.1802	0.2967	1173.6	1427172.2	7497877.6	8925049.9
6	333.52	188.32	1054479.3	1901159.5	2955638.9	494.97	114.85	867040.5	2828136.4	3695176.9	494.97	114.85	0.8665	0.0126	1064.4	1424248.3	2270928.6	3695176.9
7	283.91	129.64	1179621.7	4053256.2	5232877.9	343.62	93.82	1128146.8	4415779.9	5543926.7	343.62	93.82	0.2681	0.0572	1035.8	1335146.1	4208780.6	5543926.7
8	383.54	235.08	1440031.1	6188322.01	7628353.2	575.35	180.35	1213855.5	7330120.8	8543976.3	575.35	180.35	0.5065	0.1888	1205.5	1897842.7	6646133.5	8543976.3
9	328.63	178.09	995183.69	2436144.37	3431328.06	465.92	136.49	880767.86	2867871.63	3748639.49	465.92	136.49	0.1594	0.2681	1135.1	1153839.4	2594800.08	3748639.49
10	390.52	189.58	1060747.6	2539501.2	3600248.9	514	125.86	957737.3	2934746.4	3892483.8	514	125.86	0.9948	0.1581	879.1	1206865.1	2685618.7	3892483.8

Let us express sensitivity analyses. For sensitivity analyses, the effect of important parameters on the model behavior is discussed. Sensitivity analysis is applied to the model parameters. The efficient parameters are recognized, and their changing trend on the variables and the profit functions has been investigated. First, the effect of the sales price of returned products (P) is examined.

As shown in Fig. 2**Fig 2**, the sales price of the returned products (P) does not affect the centralized model. However, in the decentralized model, by increasing P, the take-back price value of both qualities (d_1^D, d_2^D) is initially increased, then fixed at the same value for both. The decentralized profit function also behaves like the take-back prices, and after an initial increase, it faces a decrease, then follows a constant trend. This decreasing and then fixed trend is due to the constraints in Eqs. 2 and 3. Unifying the take-back price value of two quality levels demonstrates that in the high sales price of the returned products, the collector does not discriminate between the products' qualities in offering the take-back prices. This result is totally new in the literature.

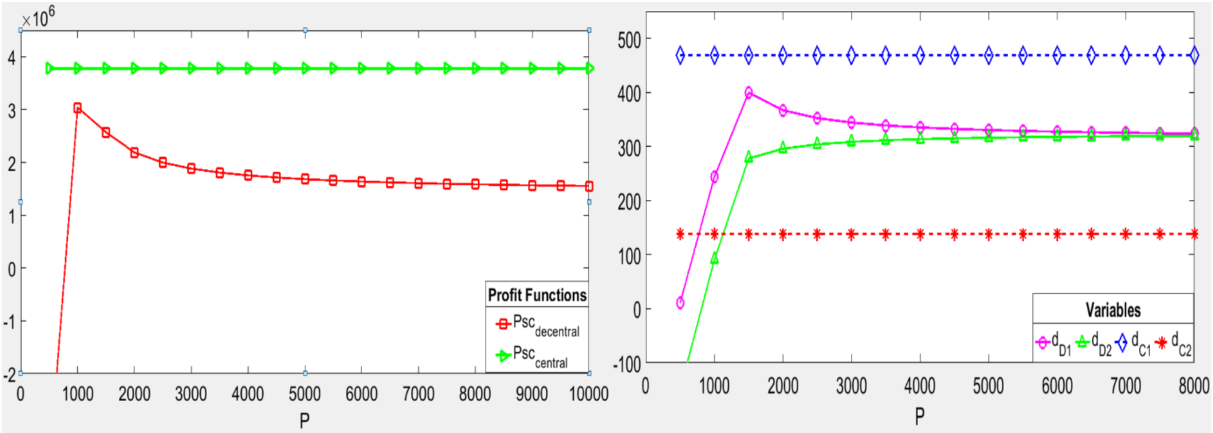

Fig 2. The sensitivity analysis of the sales price of returned products (P)

In the following, the effect of the sales price of the repaired and remanufactured products (P_m) is analyzed. The increasing P_m results in increasing the take-back price values in the centralized model (d_1^C and d_2^C) at first and after imposing the constraint in Eqs. 2 and 3, they become constant. This parameter does not affect the decentralized model variables specified in Eqs. 8 and 9, and it happened because, in the decentralized model, the collector decides on the take-back price while P_m is related to the manufacturer. Moreover, the profit functions of these models are incremental with increasing P_m. Note that increasing the sales price of the remanufactured products has a significant impact on profitability, corresponding to previous literature [19]. Therefore, it seems to be an efficient and practical idea to invest in advertising areas to raise consumer environmental awareness and encourage them to buy recycled products.

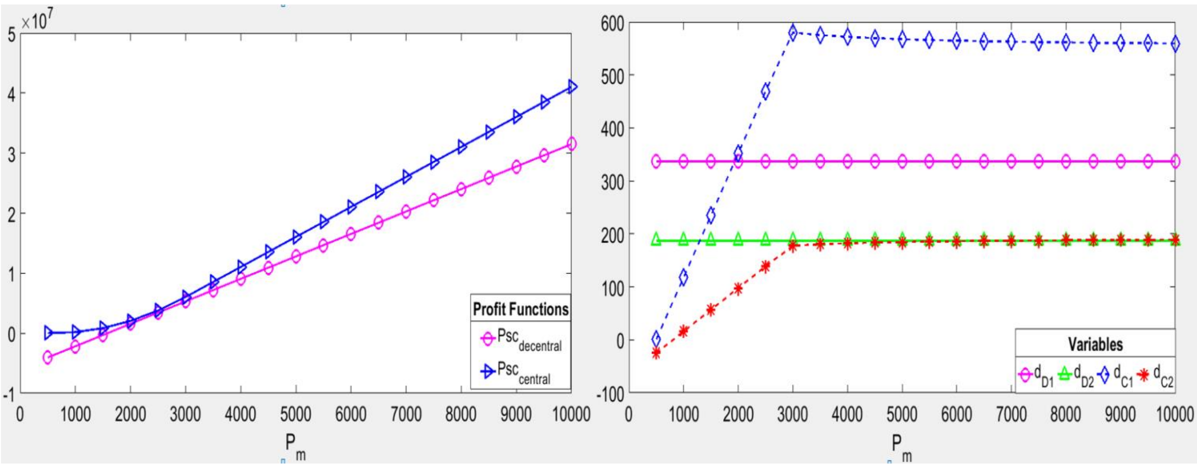

Fig. 3. The sensitivity analysis of the sales price of repaired and remanufactured products (P_m)

In what follows, the effect of the remanufacturing and repair costs (C_{rem} and C_{rep}) is investigated. By simultaneously increasing the repair and remanufacturing costs, we can see a reduction of the take-back prices in the centralized case (d_1^C *and* d_2^C). This increase leads to a reduction in profit in both centralized and decentralized models as well. Since the collector determines the take-back price in the decentralized model, these variable diagrams are flat. Note that the negative impact of increasing these costs on the decentralized model profit is intense. Hence, there is a greater sensitivity to these parameters in the decentralized case than the centralized case in which, after a period of decline, the profit function continues to take a steady trend. These results are new in the reverse logistics literature.

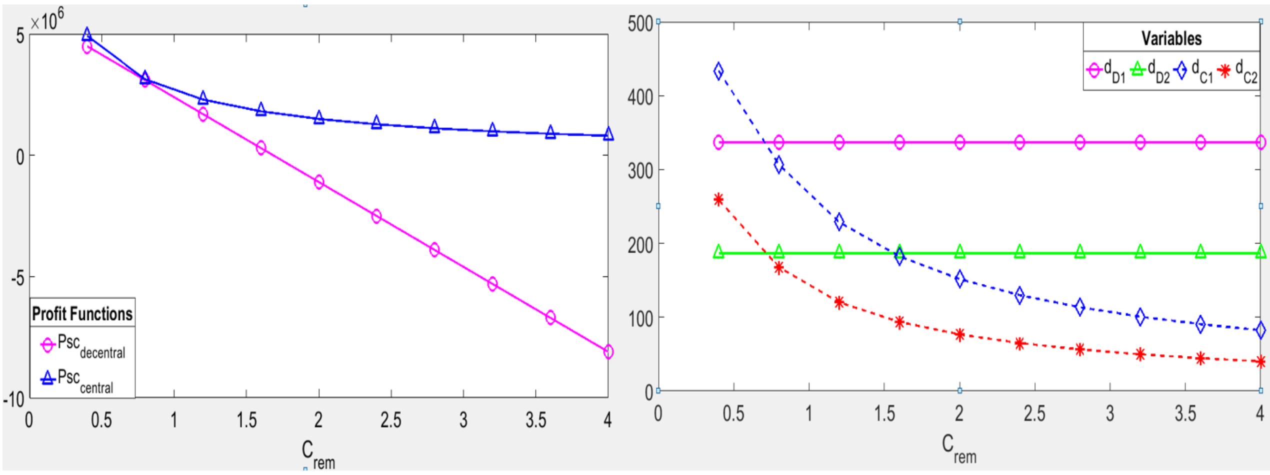

Fig. 4. The sensitivity analysis of the remanufacturing and repair costs (C_{rem}, C_{rep})

Finally, the effect of the impact factor of consumers' willingness to return products for a quality level on the processing cost of another quality level will be surveyed. Simultaneous increase of these two parameters (α, β) has no significant effect at first; however, in a higher value of α, increasing these parameters augments the take-back price of quality level 1 and a decreasing trend of the take-back price of quality level 2. This decline in the take-back price of level 2 demonstrates that the increase in the cost of capacity shortages from products of level 2 is not cost-effective from one point and consequently reduces its amount. Such results are quite novel in the related literature.

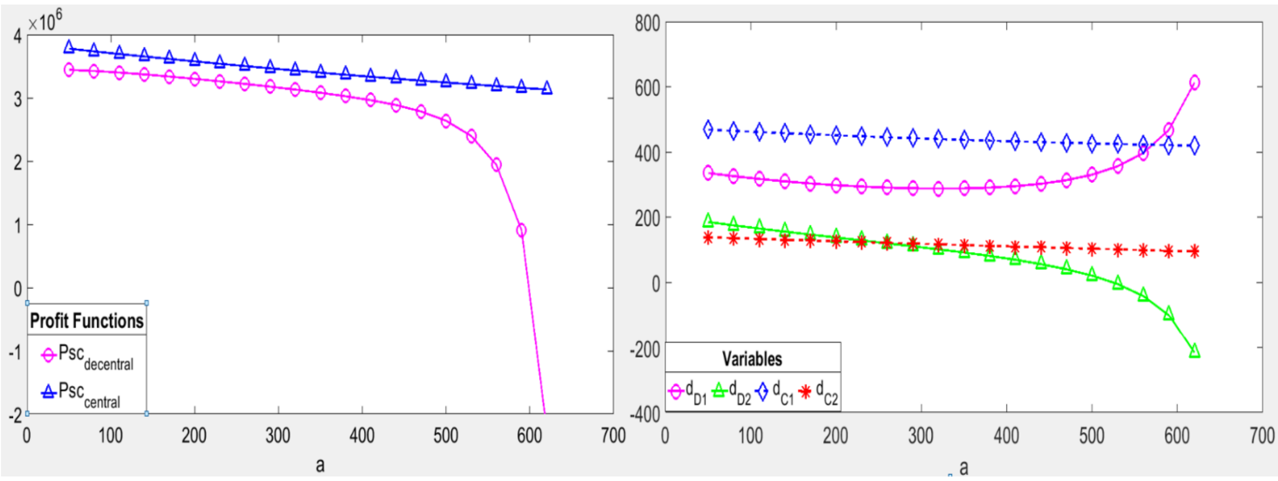

Fig. 5. The sensitivity analysis of the impact factor of consumers' willingness to return products for a quality level on the processing cost of other quality levels (α, β)

Managerial Implication

The findings of this study can help decision-makers to manage reverse logistics systems from various perspectives better. At first, our findings can help decision-makers to resolve the conflicts of interest among participants of reverse logistics systems in an efficient manner. Besides, some valuable managerial insights can be extracted from our findings. Three pieces of advice that can help decision-makers are as follows:

- Applying a cost-sharing contract initiated by the manufacturer can convince the retailer to make better decisions. Additionally, if the contract parameters are appropriately adjusted, it can achieve the best possible performance of the whole system, i.e., channel coordination.
- The take-back price offered for quality level 1 is always higher than the quality level 2. By removing the conflicts of interest through a cost-sharing contract, the manufacturer expects to receive more high-quality items and fewer low-quality items. Therefore, it is recommended that the manufacturer considers sufficient capacity to repair high-quality items as well as planning for the idle capacity of the remanufacturing line.
- Since the higher sales price of the remanufactured products directly affects profitability, upstream managers can focus on suitable advertising procedures to enhance consumer environmental awareness as well as stimulating them to buy such products.

Conclusion

In this study, we provide an analytical solution for the problem of collecting used items in a reverse logistics system, including a remanufacturer and a collector. In the investigated problem, the rate of returning used products by consumers depends on the monetary incentive paid to them by the collector as take-back price. Consumers have this choice to return used products that last two years or less over their lifetime (i.e., high quality or quality level 1) and products that have been used between two and four years (i.e., low quality or quality level 2). The monetary incentive paid to consumers for a returned product depends on the quality level of the returned item. Moreover, in accordance with the actual practice, the consumers' willingness to return used products is a function of the take-back price. On the other hand, the cost of repairing high-quality products is less than the cost of remanufacturing low-quality products. While the manufacturer prefers high-quality products, the retailer likes the low take-back price of low-quality items. The conflicts of interest between two members of the reverse logistics system can result in a deficiency of the whole system. This study at first optimizes the system and determines the optimal take-back prices for both quality levels, then proposes a cost-sharing mechanism that aligns both parties' interests and better profit-sharing.

While some parameters are stochastic in real-world cases, this study is developed under a deterministic environment. As an interesting further study, it is possible to assume a stochastic return rate for used items. Besides, optimizing the forward logistics system and integrating it with the proposed reverse logistics system (i.e., optimizing the CLSC model) is another opportunity for future research. Finally, consumers who have low income, usually willing to buy used products instead of new products; therefore, future studies can consider such an assumption.

Acknowledgements

The authors appreciate two reviewers for their prompt and constructive comments which have helped to improve the presentation of the paper, substantially.

References

[1] Agrawal, S., and Singh, R. K. (2020). Outsourcing and reverse supply chain performance: a triple bottom line approach. *Benchmarking: An International Journal*.

[2] Mathiyazhagan, K., Rajak, S., Panigrahi, S. S., Agarwal, V., and Manani, D. (2020). Reverse supply chain management in manufacturing industry: a systematic review. *International Journal of Productivity and Performance Management*.

[3] Dev, N. K., Shankar, R., and Qaiser, F. H. (2020). Industry 4.0 and circular economy: Operational excellence for sustainable reverse supply chain performance. *Resources, Conservation and Recycling*, *153*, 104583.

[4] Lampón, J. F., Pérez-Elizundia, G., and Delgado- Guzmán, J. A. (2021). Relevance of the cooperation in financing the automobile industry's supply chain: the case of reverse factoring. Journal of Manufacturing Technology Management.

[5] Govindan, K., Soleimani, H., and Kannan, D. (2015). Reverse logistics and closed-loop supply chain: A comprehensive review to explore the future. *European Journal of Operational Research*, *240*(3), 603-626.

[6] Qiang, Q., Ke, K., Anderson, T., and Dong, J. (2013). The closed-loop supply chain network with competition, distribution channel investment, and uncertainties. *Omega, 41*(2), 186-194.

[7] Blumberg, D. F. (2004). *Introduction to management of reverse logistics and closed loop supply chain processes*: CRC Press.

[8] Guide Jr, V. D. R., and Van Wassenhove, L. N. (2001). Managing product returns for remanufacturing. *Production and Operations Management, 10*(2), 142-155.

[9] Lehnig, A. C., Dewal, R. S., Baer, L. A., Kitching, K. M., Munoz, V. R., Arts, P. J., ... and Stanford, K. I. (2019). Exercise training induces depot-specific adaptations to white and brown adipose tissue. *Iscience*, *11*, 425-439.

[10] Lizarraga-Mollinedo, E., Carreras-Badosa, G., Xargay-Torrent, S., Remesar, X., Mas-Pares, B., Prats-Puig, A., ... and Bassols, J. (2021). Catch-up growth in juvenile rats, fat expansion, and dysregulation of visceral adipose tissue. *Pediatric Research*, 1-9.

[11] Stevels, A. (2001, December). Experiences with the take-back of white and brown goods in the Netherlands. In *Proceedings Second International Symposium on Environmentally Conscious Design and Inverse Manufacturing* (pp. 489-493). IEEE.

[12] Lechner, G., and Reimann, M. (2015). Reprocessing and repairing white and brown goods-the RUSZ case: an independent and non-profit business. *Journal of Remanufacturing*, *5*(1), 1-29.

[13] Giri, B., Chakraborty, A., and Maiti, T. (2017). Pricing and return product collection decisions in a closed-loop supply chain with dual-channel in both forward and reverse logistics. *Journal of manufacturing systems*, *42*, 104-123.

[14] Singh, R. K., and Agrawal, S. (2018). Analyzing disposition strategies in reverse supply chains: fuzzy TOPSIS approach. *Management of Environmental Quality: An International Journal*.

[15] Wang, M., Wang, B., and Chan, R. (2020). Reverse logistics uncertainty in a courier industry: a triadic model. *Modern Supply Chain Research and Applications*.

[16] Liu, H., Lei, M., Deng, H., Leong, G. K., and Huang, T. (2016). A dual channel, quality-based price competition model for the WEEE recycling market with government subsidy. *Omega, 59*, 290-302.

[17] Aćimović, S., Mijušković, V., and Rajić, V. (2020). The impact of reverse logistics onto green supply chain competitiveness evidence from Serbian consumers. *International Journal of Retail and Distribution Management*.

[18] Heydari, J., Govindan, K., and Sadeghi, R. (2018). Reverse supply chain coordination under stochastic remanufacturing capacity. *International Journal of Production Economics, 202*, 1-11.

[19] Heydari, J., and Ghasemi, M. (2018). A revenue sharing contract for reverse supply chain coordination under stochastic quality of returned products and uncertain remanufacturing capacity. *Journal of Cleaner Production, 197*, 607-615.

[20] Kaya, O. (2010). Incentive and production decisions for remanufacturing operations. *European Journal of Operational Research, 201*(2), 442-453.

[21] Govindan, K., and Popiuc, M. N. (2014). Reverse supply chain coordination by revenue sharing contract: A case for the personal computers industry. *European Journal of Operational Research, 233*(2), 326-336.
[22] Yu, H., and Solvang, W. D. (2017). A carbon-constrained stochastic optimization model with augmented multi-criteria scenario-based risk-averse solution for reverse logistics network design under uncertainty. *Journal of cleaner production, 164*, 1248-1267.
[23] Tseng, S. H., Wee, H. M., Song, P. S., and Jeng, S. (2019). Optimal green supply-chain model design considering full truckload. *Kybernetes*.
[24] Aldoukhi, M., and Gupta, S. M. (2020). Use of Maximal Covering Location Problem to Design a Closed Loop Supply Chain Network Under Product Substitution. In *Applications of Management Science*. Emerald Publishing Limited.
[25] Kushwaha, S., Ghosh, A., and Rao, A. K. (2020). Collection activity channels selection in a reverse supply chain under a carbon cap-and-trade regulation. *Journal of Cleaner Production, 260*, 121034.
[26] Dutta, P., Das, D., Schultmann, F., and Fröhling, M. (2016). Design and planning of a closed-loop supply chain with three way recovery and buy-back offer. *Journal of Cleaner Production, 135*, 604-619.
[27] Cai, X., Lai, M., Li, X., Li, Y., and Wu, X. (2014). Optimal acquisition and production policy in a hybrid manufacturing/remanufacturing system with core acquisition at different quality levels. *European Journal of Operational Research*, *233*(2), 374-382.
[28] Modak, N. M., Modak, N., Panda, S., and Sana, S. S. (2018). Analyzing structure of two-echelon closed-loop supply chain for pricing, quality and recycling management. *Journal of Cleaner Production, 171*, 512-528.
[29] Giri, B., and Sharma, S. (2015). Optimizing a closed-loop supply chain with manufacturing defects and quality dependent return rate. *Journal of manufacturing systems, 35*, 92-111.
[30] Taleizadeh, A. A., Moshtagh, M. S., and Moon, I. (2018). Pricing, product quality, and collection optimization in a decentralized closed-loop supply chain with different channel structures: Game theoretical approach. *Journal of Cleaner Production, 189*, 406-431.
[31] Heydari, J., Chaharsooghi, S. K., and Alipour, L. (2009). Animation supply chain modelling and diagnosis: a case study in animation industry of Iran. *International Journal of Business Performance and Supply Chain Modelling*, *1*(4), 319-332..
[32] He, L., Mao, J., Hu, C., and Xiao, Z. (2019). Carbon emission regulation and operations in the supply chain supernetwork under stringent carbon policy. *Journal of cleaner production, 238*, 117652.
[33] Xie, J., Zhang, W., Liang, L., Xia, Y., Yin, J., and Yang, G. (2018). The revenue and cost sharing contract of pricing and servicing policies in a dual-channel closed-loop supply chain. *Journal of Cleaner Production, 191*, 361-383.
[34] Zhang, Z., Liu, S., and Niu, B. (2020). Coordination mechanism of dual-channel closed-loop supply chains considering product quality and return. *Journal of cleaner production, 248*, 119273.
[35] Wang, N., Song, Y., He, Q., and Jia, T. (2020). Competitive Dual-Collecting Regarding Consumer Behavior and Coordination in Closed-Loop Supply Chain. *Computers and Industrial Engineering*, 106481.
[36] Bakhshi, A., and Heydari, J. (2021). An optimal put option contract for a reverse supply chain: case of remanufacturing capacity uncertainty. *Annals of Operations Research*, 1-24.
[37] Yi, P., Huang, M., Guo, L., and Shi, T. (2016). Dual recycling channel decision in retailer oriented closed-loop supply chain for construction machinery remanufacturing. *Journal of Cleaner Production, 137*, 1393-1405.
[38] Genc, T. S., and De Giovanni, P. (2017). Trade-in and save: A two-period closed-loop supply chain game with price and technology dependent returns. *International Journal of Production Economics, 183*, 514-527.
[39] Tang, S., Wang, W., and Zhou, G. (2020). Remanufacturing in a competitive market: A closed-loop supply chain in a Stackelberg game framework. *Expert Systems with Applications, 161*, 113655.

[40] Toktaş-Palut, P. (2021). An integrated contract for coordinating a three-stage green forward and reverse supply chain under fairness concerns. *Journal of Cleaner Production*, *279*, 123735.

[41] Wu, D., Chen, J., Li, P., and Zhang, R. (2020). Contract coordination of dual channel reverse supply chain considering service level. *Journal of Cleaner Production*, *260*, 121071.

[42] Jin, L., Zheng, B., and Huang, S. (2021). Pricing and coordination in a reverse supply chain with online and offline recycling channels: A power perspective. *Journal of Cleaner Production*, *298*, 126786.

[43] Bai, H. (2008). *Reverse Supply Chain Coordination and Design for Profitable Returns--an Example of Ink Cartridge*. Worcester Polytechnic Institute.

[44] Heydari, J., Govindan, K., and Jafari, A. (2017). Reverse and closed loop supply chain coordination by considering government role. *Transportation Research Part D: Transport and Environment*, *52*, 379-398.

[45] Jafarkhan, F., Yaghoubi, S., Gilani Larimi, N., and Farhang Moghadam, B. F. M. (2019). The Inventory–Routing Problem for Distribution of Red Blood Cells considering Compatibility of Blood Group and Transshipment between Hospitals. *Advances in Industrial Engineering*, *53*(3), 31-44.

[46] Changizi, M., Rahmani, D., and Rmezanian, R. (2020). A Multi-Visit Heterogeneous Drone Routing Model Considering Recharging Decision in Disaster. *Advances in Industrial Engineering*, *54*(1), 53-73.

[47] Zafari, F., and Shishebori, D. (2019). Designing a Multi-Objective Three-Stage Location-Routing Model for Humanitarian Logistic Planning under Uncertainty. *Advances in Industrial Engineering*, *53*(4), 149-167.

[48] Bakhtiari, M., Ebrahimnejad, S., and Yavari-Moghaddam, M. (2019). A Mathematical Model for Solving Location-Routing Problem with Simultaneous Pickup and Delivery Using a Robust Optimization Approach. *Advances in Industrial Engineering*, *53*(4), 185-208.

[49] Chen, C. (2001). Design for the environment: A quality-based model for green product development. *Management Science*, *47*(2), 250-263.

[50] Gao, J., Han, H., Hou, L., and Wang, H. (2016). Pricing and effort decisions in a closed-loop supply chain under different channel power structures. *Journal of Cleaner Production*, *112*, 2043-2057.

[51] Modak, N. M., Kazemi, N., and Cárdenas-Barrón, L. E. (2019). Investigating structure of a two-echelon closed-loop supply chain using social work donation as a Corporate Social Responsibility practice. *International Journal of Production Economics*, *207*, 19-33.

[52] Mondal, C., and Giri, B. C. (2020). Pricing and used product collection strategies in a two-period closed-loop supply chain under greening level and effort dependent demand. *Journal of cleaner production*, 121335.

[53] Stekelorum, R., Laguir, I., Gupta, S., and Kumar, S. (2021). Green supply chain management practices and third-party logistics providers' performances: A fuzzy-set approach. *International Journal of Production Economics*, *235*, 108093.

[54] Santana, J. C. C., Guerhardt, F., Franzini, C. E., Ho, L. L., Júnior, S. E. R. R., Cânovas, G., ... and Berssaneti, F. T. (2021). Refurbishing and recycling of cell phones as a sustainable process of reverse logistics: A case study in Brazil. *Journal of Cleaner Production*, *283*, 124585.

Appendix: Proof of Theorems

Proof of Theorem 1. To determine that the collector profit function is strictly concave in d_1^D and d_2^D, we form Hessian matrix of Π_C^D:

$$H\left(\Pi_C^D(d_1^D, d_2^D)\right) = \begin{bmatrix} -\left(\frac{2D}{d_{max1}}\right) & -\left(\frac{(\alpha+\beta)D}{d_{max1}d_{max2}}\right) \\ -\left(\frac{(\alpha+\beta)D}{d_{max1}d_{max2}}\right) & -\left(\frac{2D}{d_{max2}}\right) \end{bmatrix} \quad (1A)$$

For concavity of the profit function Π_C^D in d_1^D and d_2^D, Hessian matrix must be negative definite for d_1^D and d_2^D and to attain this situation, the following conditions are required to establish:

1. Negative first minor: $-\left(\frac{2D}{d_{max1}}\right) < 0$
2. Positive second minor: $\left(\frac{(2D)^2}{d_{max1}d_{max2}}\right) - \left(\frac{(\alpha+\beta)D}{d_{max1}d_{max2}}\right)^2 > 0$

Since all parameters are positive, the first condition holds; therefore, the collector's profit function is concave, and the values obtained for d_1^D and d_2^D are optimal; a second condition for the parameters must be satisfied.
The optimal d_1^D and d_2^D are calculated by setting zero the term obtained by deriving the collector's profit function and solving two obtained equations.

Proof of Theorem 2. To examine the concavity of the logistics system's profit function Π_{SC}^C in d_1^C and d_2^C, Hessian matrix is obtained as follows:

$$H\left(\Pi_{SC}^C(d_1^C, d_2^C)\right) = \begin{bmatrix} -\left(\frac{2D}{d_{max1}} + \frac{C_{rep}D^2}{d_{max1}^2}\right) & -\left(\frac{(\alpha+\beta)D}{d_{max1}d_{max2}}\right) \\ -\left(\frac{(\alpha+\beta)D}{d_{max1}d_{max2}}\right) & -\left(\frac{2D}{d_{max1}} + \frac{C_{rem}D^2}{d_{max2}^2}\right) \end{bmatrix} \quad (2A)$$

Now the negative definite condition of Hessian matrix is investigated:

1. The negativity of the first principal minor: $-\left(\frac{2D}{d_{max1}} + \frac{C_{rep}D^2}{d_{max1}^2}\right) < 0$
2. The positivity of the second principal minor:

$$\left(\frac{2D}{d_{max1}} + \frac{C_{rep}D^2}{d_{max1}^2}\right)\left(\frac{2D}{d_{max1}} + \frac{C_{rem}D^2}{d_{max2}^2}\right) - \left(\frac{(\alpha+\beta)D}{d_{max1}d_{max2}}\right)^2 > 0 \quad (3A)$$

Based on the positive parameters, the first condition is established. Parameters must also be set up in such a way that the second condition is satisfied.

The optimal d_1^C *and* d_2^C are calculated by deriving from Π_{SC}^C and equal the obtained derivatives to zero.

Proof of Theorem 3. The procedure of proofing concavity of the collector's profit function Π_C^{Cont} in d_1^{Cont} and d_2^{Cont} is:

$$H\left(\Pi_C^{Cont}(d_1^{Cont}, d_2^{Cont})\right) = \begin{bmatrix} \left(\frac{2D(\lambda-1)}{d_{max1}}\right) & -\left(\frac{(\alpha+\beta)D}{d_{max1}d_{max2}}\right) \\ -\left(\frac{(\alpha+\beta)D}{d_{max1}d_{max2}}\right) & -\left(\frac{2D}{d_{max2}}\right) \end{bmatrix} \tag{4A}$$

Since $0 < \lambda < 1$, the phrase $(\lambda - 1)$ is negative; hence, the first minor of the matrix is negative. To establish the second condition of the concavity, it is necessary to satisfy the following statement: $-\left(\frac{2D(\lambda-1)}{d_{max1}}\right)\left(\frac{2D}{d_{max2}}\right) - \left(\frac{(\alpha+\beta)D}{d_{max1}d_{max2}}\right)^2 > 0$

The optimal d_1^{Cont} and d_2^{Cont} are calculated by deriving from Π_C^{Cont} and equal the obtained derivatives to zero and then solve the equations.

Proof of Theorem 4. In order to make coordination state in the reverse logistics, the variables in the coordinated model with the centralized model are equalized:

$$d_1^C = d_1^{Cont} \tag{5A}$$

$$d_2^C = d_2^{Cont} \tag{6A}$$

By substituting Eqs. 11 and 15 into Eq. 5A and also substituting Eqs. 12 and 16 into Eq. 6A, two equations are obtained, then by solving them, P_c and γ are calculated.

Proof of Theorem 5. If both members' profit in a coordination case is more than the decentralized one, members approve of making coordination decisions. Thus, the channel will be coordinated if λ is selected from the calculated range. Hence, two following conditions must be satisfied:

$$\Pi_M^D < \Pi_M^{Cont} \tag{7A}$$

$$\Pi_C^D < \Pi_C^{Cont} \tag{8A}$$

By substituting Eqs. 5 and 14 into Eq. 7A and also substituting Eqs. 4 and 13 in Eq. 8A, two inequalities are obtained, then by solving them, the upper and lower bounds of λ are calculated.