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RESEARCH PAPER

Multivariate Process Incapability Index Considering Measurement Error in Fuzzy Environment

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Abstract

Process Capability Indices (PCI) show that the process conforms to the specification limits; when the product quality depends on more than one characteristic, Multivariate Process Capability Indices (MCPI) are used. By modifying the process capability indices, the process incapability indices are created; these indices then provide information about the accuracy and precision of the process separately. In the real world, in most cases, the parameters cannot be specified precisely; therefore, the use of fuzzy sets can solve this problem in statistical quality control. The purpose of this paper is to present, for the first time, a Multivariate Process Incapability Index by considering the measurement error in a fuzzy environment. The presented index is shown for practical examples solved by considering Triangular Fuzzy Numbers; then the capability of the model is compared to the time when fuzzy logic is not used. The obtained results emphasize that ignoring the measurement error also leads to the incorrect calculation of process capability, causing a lot of damage to manufacturing industries, especially high-tech ones.

Keywords:
Fuzzy Multivariate Process Incapability Index;
Fuzzy Measurement Error;
Multivariate Normal Distribution;
Fuzzy Logic;
High Technology Manufacturing Processes

Introduction

One of the main factors in determining the customer's satisfaction is the quality of products. Of course, the quality of products is also of special importance for manufacturers. One aspect of process quality control is the process capability analysis. One method of process capability analysis is using process capability indices. By using these indices, the process capability is reported as a number indicating the degree to which the manufactured products conform to the specification limits.

Capability indices are divided into univariate and multivariate categories. If the quality of the manufactured products depends on one characteristic, the univariate capability indices are applied; if the quality of products depends on more than one characteristic and these characteristics are dependent on each other, multivariate capability indices are used. To check the controllability of processes in which the production sensitivity is high and also, their deviations are very low, the process capability indices are not appropriate and the process incapability is not well defined. In other words, conventional methods are not appropriate to tackle this problem. Therefore, it is necessary to use process incapability indices, because these deviations are well detected and the process capability or incapability is determined.

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Electronic products, for example, have a strict range of tolerance and specification than other types of products. Therefore, process performance sensitivity will have a greater impact on product quality management. Therefore, it is necessary to use a high-accuracy index.

The structure of this paper is as follows: In the second part, the research that has been done on the process capability and incapability indices in a fuzzy environment is examined. In the third part, the concepts of fuzzy logic and how to perform mathematical operations on fuzzy numbers will be presented; then mathematical relations will be presented for the first time to calculate the multivariate process incapability index by considering the measurement error in a fuzzy environment. Then, in Section 4, by solving a real example and applying analyses, the performance of the presented index is examined. Finally, some conclusion and suggestions for future research are presented in the fifth section.

Literature review

Fuzzy theory was introduced by Prof. Zadeh in 1965 for data with non-statistical uncertainty.

Univariate and multivariate fuzzy process capability indices

There are several articles in the literature on fuzzy univariate and multivariate process capability indices; these are such as Yongting [1], Lee et al. [2], Lee [3], Sadeghpour Gildeh [4], Parchami et al. [5,6], Parchami and Machinchi [7,8], Hsu and Shu [9], Kahrman and Kaya [10,11], Kaya and Kahrman [12-20], Ramezani et al. [21], Sadeghpour Gildeh and Moradi [22], Sadeghpour Gildeh and Angoshtari [23], Abbasi Ganji and Sadeghpour Gildeh [24-26], and Hashemian and Akbari [27].

Multivariate process incapability index

Abbasi Ganji [28] first introduced a multivariate process incapability vector including two components for processes in which the quality characteristic follows the multivariate normal distribution. This index is calculated due to the ratio of the volume of a tolerance region to the volume of the process region.

Process incapability index with measurement error

Sadeghpour Gildeh and Abbasi Ganji [29] for the first time, presented the process incapability index by taking into account measurement errors and examined its statistical properties and obtained its Maximum Likelihood Estimation (MLE).

Fuzzy process incapability indices

However, very few studies have addressed the process incapability index. Kahrman and Kaya [30] calculated C_{pp} index by using fuzzy set theory and applied it in a piston manufacturer firm. Kaya and Baracli [31] calculated C_{pp} index with asymmetric tolerances by using fuzzy set theory and applied it in a piston manufacturer firm. Kaya [32] calculated C_{pp} index by using fuzzy set theory and used it in a decision-making process to select the most appropriate supplier.

Abbasi Ganji and Sadeghpour Gildeh [33] used the Buckley's approach to calculate the fuzzy univariate process incapability index, C''_{pp} , and made decisions using the fuzzy critical value to decide on that index.

Table 1 summarizes the research conducted on incapability indices.

Table 1. Summary of the research on incapability indices

Author(s)	Year	Assumptions				
		Process Incapability Index	Multivariate	Univariate	Measurement Error	Fuzzy Logic
Kahrman and Kaya [30]	2011	✓	×	✓	×	✓
Kaya and Baracli [31]	2012	✓	×	✓	×	✓
Kaya [32]	2014	✓	×	✓	×	✓
Abbasi Ganji and Sadeghpour Gildeh [33]	2016	✓	×	✓	×	✓
Sadeghpour Gildeh and Abbasi Ganji [29]	2019	✓	×	✓	✓	×
Abbasi Ganji [28]	2019	✓	✓	×	×	×
Current research		✓	✓	×	✓	✓

According to Table 1, it is clear that so far no article has been presented on the fuzzy multivariate incapability index in which measurement error is considered, and this paper complements previous research in this field. The innovation of this paper is the combination of multivariate parameters, measurement error and fuzzy logic with the process incapability index that present for the first time and for high technology manufacturing processes is very useful; because In the practical environment, there are several situations that we cannot cluster the parameters exactly; so using fuzzy sets is important and in reality, the products depend on several quality characteristics and also the measurement error can not be ignored.

Methodology

In this section, first, the assumptions are stated and then some basic definitions regarding the concepts of the fuzzy theory are presented; then mathematical relations are provided to calculate the multivariate process capability index considering the measurement error in the fuzzy environment.

Assumptions

It is assumed that the process under study has a multivariate normal distribution and the process is under control. Assume that the quality characteristic X is as a vector $p \times 1$ and has a normal multivariate distribution with the mean vector, μ, and the variance-covariance matrix, Σ, is as $N_p(\mu, \Sigma)$. The specification limits for the quality characteristic X_i for i is from 1 to p, from the upper and lower limits LSL_i and USL_i. Also, T is the target value that for each X_i is within this range.

Assuming that the measurement error is normal (E), in the multivariate mode, the error is a vector $p \times 1$ with a normal multivariate distribution and a zero mean vector $\mu_E = 0$, and the variance-covariance matrix Σ_E is as $N_p(\mu_E, \Sigma_E)$. It should be noted that X and E are independent of each other. Considering the measurement error, the quality characteristic is defined as $G = X + E$ in the multivariate mode, with a normal multivariate distribution as $N_p(\mu_G, \Sigma_G)$, which is $\mu_G = \mu$ and $\Sigma_G = \Sigma + \Sigma_E$.

Definitions and concepts

A. Triangular fuzzy number: A fuzzy number is a normal and convex set of R whose membership function is Piecewise Continuous.

B. Left-Right (LR) Fuzzy Numbers: LR fuzzy numbers mean that by using the α cut, any fuzzy number can be decomposed into right and left parts. The α level set of the fuzzy number $\tilde{A}$ is defined as $[A^L(\alpha), A^R(\alpha)]$. The α cut for a triangular fuzzy number is shown in Fig.1. Fuzzy number $\tilde{A} = (a, m, b)$ is an LR fuzzy number [34] if:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{m-a} & a \le x \le m \\ \dfrac{b-x}{b-m} & m \le x \le b \end{cases} \tag{1}$$

$$\begin{cases} x^L = a + \alpha(m-a) \\ x^R = b - \alpha(b-m) \end{cases} \tag{2}$$

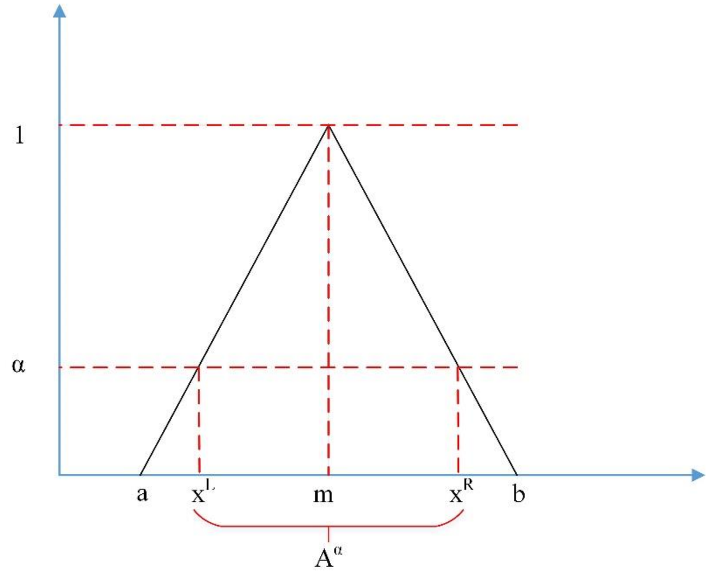

Fig.1. α cut for a triangular fuzzy number

C. Mathematical operations on LR fuzzy numbers: Consider two triangular fuzzy numbers $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$. α cut for these two fuzzy numbers is $\tilde{A}(\alpha) = [A^L(\alpha), A^R(\alpha)]$ and $\tilde{B}(\alpha) = [B^L(\alpha), B^R(\alpha)]$, and the mathematical operations for them are as follows [35].

$$\tilde{A}(\alpha) \oplus \tilde{B}(\alpha) = [A^L(\alpha) + B^L(\alpha), A^R(\alpha) + B^R(\alpha)] \tag{3}$$

$$\tilde{A}(\alpha) \ominus \tilde{B}(\alpha) = [A^L(\alpha) - B^R(\alpha), A^R(\alpha) - B^L(\alpha)] \tag{4}$$

$$\begin{gathered} \tilde{A}(\alpha) \otimes \tilde{B}(\alpha) = [\alpha, \beta] \\ \alpha = min\big(A^L(\alpha)B^L(\alpha), A^L(\alpha)B^R(\alpha), A^R(\alpha)B^L(\alpha), A^R(\alpha)B^R(\alpha)\big) \\ \beta = max\big(A^L(\alpha)B^L(\alpha), A^L(\alpha)B^R(\alpha), A^R(\alpha)B^L(\alpha), A^R(\alpha)B^R(\alpha)\big) \end{gathered} \tag{5}$$

$$\begin{gathered} \tilde{A}(\alpha) \oslash \tilde{B}(\alpha) = [\gamma, \delta] \\ \gamma = min\big(A^L(\alpha)/B^L(\alpha), A^L(\alpha)/B^R(\alpha), A^R(\alpha)/B^L(\alpha), A^R(\alpha)/B^R(\alpha)\big) \\ \delta = max\big(A^L(\alpha)/B^L(\alpha), A^L(\alpha)/B^R(\alpha), A^R(\alpha)/B^L(\alpha), A^R(\alpha)/B^R(\alpha)\big) \end{gathered} \tag{6}$$

$$\begin{cases} \lambda \odot \tilde{A}(\alpha) = [\lambda.A^L(\alpha), \lambda.A^R(\alpha)] & if \lambda > 0 \\ \lambda \odot \tilde{A}(\alpha) = [\lambda.A^R(\alpha), \lambda.A^L(\alpha)] & if \lambda < 0 \end{cases} \tag{7}$$

D. Fuzzy matrix: A triangular fuzzy matrix with the dimension $m \times n$ is a matrix with the dimension $m \times n$, where all its elements are triangular fuzzy numbers, that is, $\widetilde{M} = \left(\widetilde{m}_{ij}\right)_{m\times n}$, where $\widetilde{m}_{ij} = \left(a_{ij}, b_{ij}, c_{ij}\right)$ [34]. If $m = n$, it is called a square matrix. It should be noted that this paper is presented based on a situation where all the elements of the matrix are symmetric Triangular Fuzzy Number; so, $b_{ij} - a_{ij} = c_{ij} - b_{ij}$.

E. Fuzzy determinant: suppose $\tilde{A} = \left(\tilde{a}_{ij}\right)$ is a 2×2 square matrix. The fuzzy determinants of the matrix $\tilde{A}$ are represented by the symbol $det\left(\tilde{A}\right)$ and are defined according to the following equation [36].

$$det\left(\tilde{A}\right) = (\tilde{a}_{11} \otimes \tilde{a}_{22}) \ominus (\tilde{a}_{12} \otimes \tilde{a}_{21}) \tag{8}$$

Ranking function

To determine that the fuzzy number $\tilde{A}$ is greater than, equal to or less than the fuzzy number $\tilde{B}$, we use the ranking function introduced by Fortemps and Roubens [37]. Based on $C\left(\tilde{A} \geq \tilde{B}\right)$, the comparison between $\tilde{A}$ and $\tilde{B}$ is according to the following equations.

$$\begin{array}{ll} C\left(\tilde{A} \geq \tilde{B}\right) > 0 & IF\ \tilde{A} > \tilde{B} \\ C\left(\tilde{A} \geq \tilde{B}\right) \geq 0 & IF\ \tilde{A} \geq \tilde{B} \\ C\left(\tilde{A} \geq \tilde{B}\right) = 0 & IF\ \tilde{A} = \tilde{B} \end{array} \tag{9}$$

So:

$$C\left(\tilde{A} \geq \tilde{B}\right) = R\left(\tilde{A}\right) - R\left(\tilde{B}\right) \tag{10}$$

$$R\left(\tilde{A}\right) = \frac{1}{2}\int_0^1 \left(\tilde{A}^L(\alpha) + \tilde{A}^R(\alpha)\right) d\alpha \tag{11}$$

It should be noted that for a triangular fuzzy number $\tilde{A} = T(a, b, c)$, Eq. 11 is written as $R\left(\tilde{A}\right) = \frac{a+2b+c}{4}$, with the value being equal to the middle number $\tilde{A}$ is.

Multivariate Process incapability index considering the measurement error in certain mode

The $MIC'''_{pp}(u, v)$ index was presented by Abbasi Ganji [28]; in the present article, this index is shown as $MIC'''^{G}_{pp}(u, v)$ in the presence of the measurement error. So, *u* and *v* are the weighting factors for the deviation of the mean vector from the target vector and the variability of the process, and G indicates the measurement error.

To calculate this index, we use the division of the tolerance region into the process region. Because the $MIC'''_{pp}(u, v)$ index consists of two indices of inaccuracy and imprecision, the effect of measurement error on both indices must be considered. The imprecision index is calculated by considering the measurement error according to Eq. 12. It should be noted that the following equations, which are the basis for fuzzy index calculation, have been developed by the authors. The difference between the following equations and the equations presented in Abbasi Ganji's research [28] is that Abbasi Ganji did not consider measurement error in her equations and to further develop, we added it to her equations.

$$MC''''^{G}_{ip}(u) = \left(\frac{vol.(99.73\%\ of\ process\ and\ measurement\ error\ region\ )}{vol.(ellipsoid\ with\ radius\ r_i)}\right)^2$$
$$= \left(\frac{|\mathbf{\Sigma}_G|^{\frac{1}{2}}\left(\pi\chi^2_{0.0027,v}\right)^{\frac{p}{2}}\left(\Gamma\left(\frac{p}{2}+1\right)\right)^{-1}}{\pi r_1(u) r_2(u) \ldots r_p(u)}\right)^2 \quad (12)$$
$$= \left(\frac{|\mathbf{\Sigma}+\mathbf{\Sigma}_E|^{\frac{1}{2}}\left(\pi\chi^2_{0.0027,v}\right)^{\frac{p}{2}}\left(\Gamma\left(\frac{p}{2}+1\right)\right)^{-1}}{\pi r_1(u) r_2(u) \ldots r_p(u)}\right)^2 ; \quad u \geq 0$$

In Eq. 12, p is the number of qualitative characteristics under consideration. Also, $r_i(u)$ is an ellipsoid radius and $\Gamma\left(\frac{p}{2}+1\right)$ is a gamma function. λ^M is the multivariate gauge capability and calculated according to Eq. 13; by adding this relation to the previous equations, a new index is created.

$$\lambda^M = \frac{vol.(99.73\%\ of\ process\ region)}{vol.(ellipsoid\ with\ radius\ r_i)} = \frac{|\mathbf{\Sigma}_E|^{\frac{1}{2}}\left(\pi\chi^2_{0.0027,v}\right)^{\frac{p}{2}}\left(\Gamma\left(\frac{p}{2}+1\right)\right)^{-1}}{\pi r_1(u) r_2(u) \ldots r_p(u)} \quad (13)$$

The determinants of the variance-covariance matrix E are obtained according to Eq. 14.

$$|\mathbf{\Sigma}_E|^{\frac{1}{2}} = \frac{\lambda^M\left(\pi r_1(u) r_2(u) \ldots r_p(u)\right)}{\left(\pi\chi^2_{0.0027,v}\right)^{\frac{p}{2}}\left(\Gamma\left(\frac{p}{2}+1\right)\right)^{-1}} \quad (14)$$

According to the theorem of the sum determinants of two matrices A and B [38], the equation $|\Sigma+\Sigma_E|$ is equal to $|\Sigma+\Sigma_E| = |\Sigma| + |\Sigma_E| + C$; also, by considering that the equation $|\Sigma+\Sigma_E| = |\Sigma_G|$, it is established that the value of C will be equal to $C = |\Sigma_G| - (|\Sigma| + |\Sigma_E|)$; by placing the above items in Eq. 12, the $MC''''^{G}_{ip}(u)$ index is calculated as follows:

$$MC''''^{G}_{ip}(u) = MC''''_{ip}(u) \times \left(1 + \frac{(\lambda^M)^2}{MC''''_{ip}(u)} + \frac{C}{|\mathbf{\Sigma}_G - \mathbf{\Sigma}_E|}\right)$$
$$= MC''''_{ip}(u) \times \left(\frac{(\lambda^M)^2}{MC''''_{ip}(u)} + \frac{C + (|\mathbf{\Sigma}_G - \mathbf{\Sigma}_E|)}{|\mathbf{\Sigma}_G - \mathbf{\Sigma}_E|}\right)$$
$$= MC''''_{ip}(u) \times \left(\frac{(\lambda^M)^2}{MC''''_{ip}(u)} + \frac{|\mathbf{\Sigma}_G| - |\mathbf{\Sigma}| - |\mathbf{\Sigma}_E| + (|\mathbf{\Sigma}_G - \mathbf{\Sigma}_E|)}{|\mathbf{\Sigma}_G - \mathbf{\Sigma}_E|}\right) \quad (15)$$
$$= MC''''_{ip}(u) \times \left(\frac{(\lambda^M)^2}{MC''''_{ip}(u)} + \frac{|\mathbf{\Sigma}_G| - |\mathbf{\Sigma}_E|}{|\mathbf{\Sigma}_G - \mathbf{\Sigma}_E|}\right)$$

Finally, the $MC''''^{G}_{ip}(u)$ index is calculated according to Eq. 16.

$$MC''''^{G}_{ip}(u) = (\lambda^M)^2 + \left(MC''''_{ip}(u) \times \frac{|\mathbf{\Sigma}_G| - |\mathbf{\Sigma}_E|}{|\mathbf{\Sigma}_G - \mathbf{\Sigma}_E|}\right) \quad u \geq 0 \quad (16)$$

Considering that there is no standard deviation, σ, in the inaccuracy index $C''''_{ia}(u,v) = \frac{9vA^2}{(d^* - uA^*)^2}$ as introduced by Abbasi Ganji [28], measurement error in this part of the process incapability index formula has no effect and the inaccuracy index $MC''''_{ia}(u,v)$ is calculated according to Eq. 17.

$$MC_{ia}^{\prime\prime\prime}(u,v) = \frac{9v\boldsymbol{A}'\boldsymbol{A}}{\boldsymbol{r}'(u)\boldsymbol{r}(u)}; \quad u,v \geq 0 \tag{17}$$

In Eq. 17, A' represents the transposition of the vector A and this vector is calculated according to the transpose matrix equations in mathematics. In the above equations, the following equations are established.

$$d_i^* = \min\{D_{l_i}, D_{u_i}\}, D_{l_i} = T_i - LSL_i, D_{u_i} = USL_i - T_i \tag{18}$$

$$d_i = \frac{USL_i - LSL_i}{2} \tag{19}$$

$$A_i = \frac{d_i(\mu_i - T_i)}{D_{u_i}} I\{\mu_i > T_i\} + \frac{d_i(T_i - \mu_i)}{D_{l_i}} I\{\mu_i \leq T_i\} \tag{20}$$

$$A_i^* = \frac{(\mu_i - T_i)^2}{D_{u_i}} I\{\mu_i > T_i\} + \frac{(T_i - \mu_i)^2}{D_{l_i}} I\{\mu_i \leq T_i\} \tag{21}$$

$$r_i(u) = |d_i^* - uA_i^*| \tag{22}$$

$$\boldsymbol{d}^* = \begin{pmatrix} d_1^* \\ d_2^* \\ \vdots \\ d_p^* \end{pmatrix}, \boldsymbol{A} = \begin{pmatrix} A_1 \\ A_2 \\ \vdots \\ A_p \end{pmatrix}, \boldsymbol{A}^* = \begin{pmatrix} A_1^* \\ A_2^* \\ \vdots \\ A_p^* \end{pmatrix}, \boldsymbol{r}(u) = \begin{pmatrix} r_1(u) \\ r_2(u) \\ \vdots \\ r_p(u) \end{pmatrix} \tag{23}$$

In Eqs. 20 and 21, $I\{x\}$ is an indicator function defined according to Eq. 24.

$$I\{x\} = \begin{cases} 1; & x \geq 0 \\ 0; & x < 0 \end{cases} \tag{24}$$

To summarize, the error-affected multivariate incapability index $MIC_{pp}^{\prime\prime\prime G}(u,v)$ is in accordance with Eq. 25.

$$\begin{aligned} MIC_{pp}^{\prime\prime\prime G}(u,v) &= MC_{ia}^{\prime\prime\prime}(u,v) + MC_{ip}^{\prime\prime\prime G}(u) \\ &= \frac{9v\boldsymbol{A}'\boldsymbol{A}}{\boldsymbol{r}'(u)\boldsymbol{r}(u)} + (\lambda^M)^2 + \left(MC_{ip}^{\prime\prime\prime}(u) \times \frac{|\boldsymbol{\Sigma}_G| - |\boldsymbol{\Sigma}_E|}{|\boldsymbol{\Sigma}_G - \boldsymbol{\Sigma}_E|}\right) \quad u,v \geq 0 \end{aligned} \tag{25}$$

Multivariate process incapability index considering measurement error in fuzzy environment

In this section, according to the fuzzy concepts presented at the beginning of Section 3, the multivariate process incapability index is calculated by considering the measurement error in the fuzzy state, as indicated by the symbol $\tilde{MIC}_{PP}^{\prime\prime\prime G}(u,v)$, in accordance with Eq. 26. To calculate this index, both inaccuracy and imprecision indices in fuzzy mode must be calculated and added up.

To calculate the index in the fuzzy mode, the α-cut method is used and for each index, its LR mode is estimated.

$$\begin{aligned} \tilde{MIC}_{PP}^{\prime\prime\prime G}(u,v) &= \tilde{MC}_{ia}^{\prime\prime\prime}(u,v) + \tilde{MC}_{ip}^{\prime\prime\prime G}(u,v) \\ &= \left\{ MC_{ia}^{\prime\prime\prime L}(\alpha) + MC_{ip}^{\prime\prime\prime G^L}(\alpha), MC_{ia}^{\prime\prime\prime R}(\alpha) + MC_{ip}^{\prime\prime\prime G^R}(\alpha) \right\} \end{aligned} \tag{26}$$

The index $MC_{ia}'''(\alpha)$ is a fuzzy multivariate inaccuracy index calculated according to Eq. 27. It should be noted that as mentioned in the previous section, measurement error has no effect on this index.

$$MC_{ia}'''(\alpha)=\left\{MC_{ia}'''^{L}(\alpha),MC_{ia}'''^{R}(\alpha)\right\}=\left\{\frac{9v\otimes \boldsymbol{A}'^{L}(\alpha)\otimes \boldsymbol{A}^{L}(\alpha)}{\tilde{\boldsymbol{r}}'^{R}(\alpha)\otimes \tilde{\boldsymbol{r}}^{R}(\alpha)},\frac{9v\otimes \boldsymbol{A}'^{R}(\alpha)\otimes \boldsymbol{A}^{R}(\alpha)}{\tilde{\boldsymbol{r}}'^{L}(\alpha)\otimes \tilde{\boldsymbol{r}}^{L}(\alpha)}\right\} \quad (27)$$

The index $MC_{ip}'''^{G}(u,v)$ is a multivariate imprecision index considering the measurement error in the fuzzy mode; it is calculated according to Eq. 28. To obtain the fuzzy variance-covariance matrix of the data, mixed with errors, represented by the symbol $\boldsymbol{\Sigma}_G$, the fuzzy variance - covariance matrix of the original data ($\boldsymbol{\Sigma}$) must be combined with the fuzzy-covariance matrix of the error ($\boldsymbol{\Sigma}_E$).

$$MC_{ip}'''^{G}(u,v)=\left\{MC_{ip}'''^{G^L}(\alpha),MC_{ip}'''^{G^R}(\alpha)\right\}=\left\{\left[\frac{\left|\boldsymbol{\Sigma}_G{}^{L}\right|^{\frac{1}{2}}\otimes\left(\pi\otimes\chi^2_{0.0027,v}\right)^{\frac{p}{2}}\otimes\left(\Gamma\left(\frac{p}{2}+1\right)\right)^{-1}}{\pi\otimes r_1^R(\alpha)\otimes r_2^R(\alpha)\ldots r_p^R(\alpha)}\right]^2,\left[\frac{\left|\boldsymbol{\Sigma}_G{}^{R}\right|^{\frac{1}{2}}\otimes\left(\pi\otimes\chi^2_{0.0027,v}\right)^{\frac{p}{2}}\otimes\left(\Gamma\left(\frac{p}{2}+1\right)\right)^{-1}}{\pi\otimes r_1^L(\alpha)\otimes r_2^L(\alpha)\ldots r_p^L(\alpha)}\right]^2\right\} \quad (28)$$

Eq. 28 is used to calculate the fuzzy variance-covariance matrix determinants of the data, mixed with errors, ($|\boldsymbol{\Sigma}_G|$). For bivariate indices, $MC_{ip}'''^{G}(u,v)$ is calculated according to Eq. 29.

$$MC_{ip}'''^{G}(u,v)=\left\{MC_{ip}'''^{G^L}(\alpha),MC_{ip}'''^{G^R}(\alpha)\right\}=\left\{\frac{\left|\boldsymbol{\Sigma}_G{}^{L}\right|\otimes(11.829)^2}{\left(r_1^R(\alpha)\otimes r_2^R(\alpha)\right)^2},\frac{\left|\boldsymbol{\Sigma}_G{}^{R}\right|\otimes(11.829)^2}{\left(r_1^L(\alpha)\otimes r_2^L(\alpha)\right)^2}\right\} \quad (29)$$

The multivariate gauge capability is indicated by the symbol λ^M and is a crisp number. The fuzzy variance-covariance error matrix ($\boldsymbol{\Sigma}_E$) is a diagonal matrix in which the entries outside the main diagonal are zero and the entries of the main diagonal are the square root of the determinant of the fuzzy error matrix $\sqrt{|\boldsymbol{\Sigma}_E|}$ [39-42]; it is calculated according to Eq. 30.

$$\sqrt{|\Sigma_E|}(\alpha)=\left\{\sqrt{|\Sigma_E|}^L(\alpha),\sqrt{|\Sigma_E|}^R(\alpha)\right\}$$
$$=\left\{\frac{\lambda^M(\alpha)\otimes\left(\pi\otimes r_1^L(\alpha)\otimes r_2^L(\alpha)\ldots r_p^L(\alpha)\right)}{\left(\pi\otimes\chi^2_{0.0027,v}\right)^{\frac{p}{2}}\otimes\left(\Gamma\left(\frac{p}{2}+1\right)\right)^{-1}},\frac{\lambda^M(\alpha)\otimes\left(\pi\otimes r_1^R(\alpha)\otimes r_2^R(\alpha)\ldots r_p^R(\alpha)\right)}{\left(\pi\otimes\chi^2_{0.0027,v}\right)^{\frac{p}{2}}\otimes\left(\Gamma\left(\frac{p}{2}+1\right)\right)^{-1}}\right\} \tag{30}$$

For indices that depend on two qualitative characteristics (bivariate indices), the entries of the main diagonal of the fuzzy variance-covariance error matrix are calculated according to Eq. 31.

$$\sqrt{|\Sigma_E|}(\alpha)=\left\{\sqrt{|\Sigma_E|}^L(\alpha),\sqrt{|\Sigma_E|}^R(\alpha)\right\}$$
$$=\left\{\frac{\lambda^M\otimes r_1^L(\alpha)\otimes r_2^L(\alpha)}{11.829},\frac{\lambda^M\otimes r_1^R(\alpha)\otimes r_2^R(\alpha)}{11.829}\right\} \tag{31}$$

To calculate the indices presented in Eqs. 27 to 29, the following equations should be used. To calculate Eqs. 32, 33, and 34 should be first calculated; finally, their minimum is selected according to Eq. 32.

$$d_i^*=\min\left\{D_L,D_U\right\}$$
$$=\min\left\{T_i\,!\ \ LSL_i,USL_i\,!\ \ T_i\right\},i=1,2,\ldots,P$$
$$=\min\left\{d^{*L},d^{*R}\right\} \tag{32}$$

$$d^{*L}(\alpha)=\left\{T^L\,!\ \ LSL^R,T^R\,!\ \ LSL^L\right\} \tag{33}$$

$$d^{*R}(\alpha)=\left\{USL^L\,!\ \ T^R,USL^R\,!\ \ T^L\right\} \tag{34}$$

The α cut of a triangular fuzzy number, d_i , is calculated according to Eq. 35.

$$d_i(\alpha)=\left\{d_i^L,d_i^R\right\}$$
$$=\left\{\frac{USL^L\,!\ \ LSL^R}{2},\frac{USL^R\,!\ \ LSL^L}{2}\right\} \tag{35}$$

The set of α-cuts is in the form $A(\alpha)=\left(A^L(\alpha),A^R(\alpha)\right)$; to calculate it, one of the propositions of each of the Eqs. 36 to 38 must be selected. To do this, the ranking function is used to determine $\max\left\{\bar{X}_i,T_i\right\}$.

$$A=\begin{cases}\dfrac{d_i\otimes\left(\bar{X}_i - T_i\right)}{USL_i - T_i}; \max\left\{\bar{X}_i, T_i\right\}=\bar{X}_i\\[2ex] \dfrac{d_i\otimes\left(T_i - \bar{X}_i\right)}{T_i - LSL_i}; \max\left\{\bar{X}_i, T_i\right\}=T_i\end{cases} \tag{36}$$

$$A^L(\alpha)=\begin{cases}\dfrac{d_i^L\otimes\left(\bar{X}_i^L - T_i^R\right)}{USL^R - T_i^L}; \max\left\{\bar{X}_i, T_i\right\}=\bar{X}_i\\[2ex] \dfrac{d_i^L\otimes\left(T_i^L - \bar{X}_i^R\right)}{T_i^R - LSL^L}; \max\left\{\bar{X}_i, T_i\right\}=T_i\end{cases} \tag{37}$$

$$A^R(\alpha)=\begin{cases}\dfrac{d_i^R\otimes\left(\bar{X}_i^R - T_i^L\right)}{USL^L - T_i^R}; \max\left\{\bar{X}_i, T_i\right\}=\bar{X}_i\\[2ex] \dfrac{d_i^R\otimes\left(T_i^R - \bar{X}_i^L\right)}{T_i^L - LSL^R}; \max\left\{\bar{X}_i, T_i\right\}=T_i\end{cases} \tag{38}$$

The set of α-cuts is in the form $A^*(\alpha)=\left(A^{*L}(\alpha), A^{*R}(\alpha)\right)$; to calculate it, one of the propositions of each of the Eqs. 39 to 41 must be selected. To do this, the ranking function is used to determine the $\max\left\{\bar{X}_i, T_i\right\}$.

$$A^*=\begin{cases}\dfrac{\left(\bar{X}_i - T_i\right)^2}{USL_i - T_i}; \max\left\{\bar{X}_i, T_i\right\}=\bar{X}_i\\[2ex] \dfrac{\left(T_i - \bar{X}_i\right)^2}{T_i - LSL_i}; \max\left\{\bar{X}_i, T_i\right\}=T_i\end{cases} \tag{39}$$

$$A^{*L}(\alpha)=\begin{cases}\dfrac{\left(\bar{X}_i^L - T_i^R\right)^2}{USL_i^R - T_i^L}; \max\left\{\bar{X}_i, T_i\right\}=\bar{X}_i\\[2ex] \dfrac{\left(T_i^L - \bar{X}_i^R\right)^2}{T_i^R - LSL_i^L}; \max\left\{\bar{X}_i, T_i\right\}=T_i\end{cases} \tag{40}$$

$$A^{*R}(\alpha)=\begin{cases}\dfrac{\left(\bar{X}_i^R - T_i^L\right)^2}{USL_i^L - T_i^R}; \max\left\{\bar{X}_i, T_i\right\}=\bar{X}_i\\[2ex] \dfrac{\left(T_i^R - \bar{X}_i^L\right)^2}{T_i^L - LSL_i^R}; \max\left\{\bar{X}_i, T_i\right\}=T_i\end{cases} \tag{41}$$

Eq. 42 is used to calculate the ellipsoid radius in the multivariate mode.

$$r_i(\alpha)=\{r_i^L(\alpha),r_i^R(\alpha)\} =\{d_i^{*L} \ominus (u\otimes A_i^{*R}),d_i^{*R} \ominus (u\otimes A_i^{*L})\} \tag{42}$$

Multivariate Process incapability index without considering measurement error in the fuzzy mode

The multivariate imprecision index without considering the measurement error in the fuzzy state is indicated by the symbol $MC_{ip}'''(u,v)$; it is estimated according to Eq. 43.

$$MC_{ip}'''=\{MC_{ip}'''^{L},MC_{ip}'''^{R}\} =\left\{\left[\frac{|\Sigma^L|^{\frac{1}{2}}\otimes(\pi\otimes\chi^2_{0.0027,v})^{\frac{p}{2}}\otimes\left(\Gamma\left(\frac{p}{2}+1\right)\right)^{-1}}{\pi\otimes r_1^R(\alpha)\otimes r_2^R(\alpha)\ldots r_p^R(\alpha)}\right]^2,\left[\frac{|\Sigma^R|^{\frac{1}{2}}\otimes(\pi\otimes\chi^2_{0.0027,v})^{\frac{p}{2}}\otimes\left(\Gamma\left(\frac{p}{2}+1\right)\right)^{-1}}{\pi\otimes r_1^L(\alpha)\otimes r_2^L(\alpha)\ldots r_p^L(\alpha)}\right]^2\right\} \tag{43}$$

For bivariate indices, $MC_{ip}'''(u,v)$ is calculated according to the following equation.

$$MC_{ip}'''=\{MC_{ip}'''^{L},MC_{ip}'''^{R}\} =\left\{\frac{|\Sigma^L|\otimes(11.829)^2}{(r_1^R(\alpha)\otimes r_2^R(\alpha))^2},\frac{|\Sigma^R|\otimes(11.829)^2}{(r_1^L(\alpha)\otimes r_2^L(\alpha))^2}\right\} \tag{44}$$

The inaccuracy index $MC_{ia}'''(u,v)$ is calculated according to Eq. 27; the multivariate process incapability index is calculated according to Eq .45 without considering the measurement error in the fuzzy mode, $MIC_{PP}'''(u,v)$, is estimated according to the Eq .45.

$$MIC_{PP}'''(u,v)=MC_{ia}'''(u,v)+MC_{ip}'''(u,v) =\{MC_{ia}'''^{L}(\alpha)+MC_{ip}'''^{L}(\alpha),MC_{ia}'''^{R}(\alpha)+MC_{ip}'''^{R}(\alpha)\} \tag{45}$$

Practical example

Example 1: Jackson's example

The first example presented in this section is the one provided by Jackson [43]. He studied the process of Film-developing solution and examined the two components of Elon (E) and Hydroguinone (H). Process information including specification limits and target value for both factors are considered as symmetric triangular fuzzy numbers, as shown in Table 2; to solve this example, the numbers $\lambda^M = 0.1$, u = 1 and v = 1 are used.

Table 2. Fuzzy information of the film-developing solution

	LSL			USL			T		
	L	M	R	L	M	R	L	M	R
E	234	235	236	234	235	236	234	235	236
H	439	440	441	439	440	441	439	440	441

Based on a random sample of size 75, the fuzzy sample mean vector and the fuzzy sample variance-covariance matrix are as follows.

$$\tilde{\tilde{\boldsymbol{X}}} = \begin{pmatrix} 263.32 & 264.32 & 265.32 \\ 470.48 & 471.48 & 472.48 \end{pmatrix} \quad (46)$$

$$\tilde{\mathbf{S}} = \begin{pmatrix} 101.65 \;\; 102.65 \;\; 103.65 & 67.87 \;\; 68.87 \;\; 69.87 \\ 67.87 \;\; 68.87 \;\; 69.87 & 106.96 \;\; 107.96 \;\; 108.96 \end{pmatrix} \quad (47)$$

The process incapability index is calculated by considering the measurement error in the fuzzy mode using Eq. 26. The value of this index, the approximate value of the fuzzy index and the value of the index in the crisp state are presented in Table 3.

Table 3. Process incapability index with measurement error

Value of the fuzzy index	Approximate value of the fuzzy index	Value of the index in crisp state
$\widetilde{\widetilde{MC_{\iota a}^{'''}(u,v)}} = [0.0072 \;\; 0.1391]$	$\widehat{MC_{\iota a}^{'''}(u,v)} \cong 0.0732$	$\widehat{MC_{\iota a}^{'''}(u,v)} = 0.0133$
$\widetilde{\widetilde{MC_{\iota p}^{'''G}}}(u) = [0.9879 \;\; 2.0585]$	$\widehat{MC_{\iota p}^{'''G}}(u) \cong 1.5232$	$\widehat{MC_{\iota p}^{'''G}}(u) = 1.3891$
$\widetilde{\widetilde{MIC_{pp}^{'''G}}}(u,v) = \widetilde{\widetilde{MC_{\iota a}^{'''}(u,v)}} + \widetilde{\widetilde{MC_{\iota p}^{'''G}}}(u) = [0.9952 \;\; 2.1977]$	$\widehat{MIC_{pp}^{'''G}}(u,v) \cong 1.5964$	$\widehat{MIC_{pp}^{'''G}}(u,v) = 1.4024$

Table 4 shows the process incapability index without considering the measurement error in the fuzzy state, the approximate value of the fuzzy index and the value of the index in the crisp mode.

Table 4. Process incapability index without considering measurement error

Value of the fuzzy index	Approximate value of the fuzzy index	Value of the index in crisp state
$\widetilde{\widetilde{MC_{\iota a}^{'''}(u,v)}} = [0.0072 \;\; 0.1391]$	$\widehat{MC_{\iota a}^{'''}(u,v)} \cong 0.0732$	$\widehat{MC_{\iota a}^{'''}(u,v)} = 0.0133$
$\widetilde{\widetilde{MC_{\iota p}^{'''}(u)}} = [0.80251 \;\; 1.5996]$	$\widehat{MC_{\iota p}^{'''}(u)} \cong 1.2011$	$\widehat{MC_{\iota p}^{'''}(u)} = 1.1015$
$\widetilde{\widetilde{MIC_{pp}^{'''}}}(u,v) = \widetilde{\widetilde{MC_{\iota a}^{'''}(u,v)}} + \widetilde{\widetilde{MC_{\iota p}^{'''}(u)}} = [0.8098 \;\; 1.7388]$	$\widehat{MIC_{pp}^{'''}}(u,v) \cong 1.2743$	$\widehat{MIC_{pp}^{'''}}(u,v) = 1.1148$

To decide on the capability or incapability of the process, the value of the process incapability index must be compared to one. Thus, if the value of the index is greater than one, the process is incapable; if the value of the index is less than one, the process is capable. In the fuzzy mode, the incapability index must be compared by a ranking function with the triangular fuzzy number $\tilde{1}$.

According to Table 3, $R\left(\widehat{MIC_{pp}^{'''G}}(u,v)\right) = 1.5964$, which is greater than $R(\tilde{1}) = 1$; so, the process becomes incapable. If this index were also calculated in the case of crisp data, its value would be equal to $\widehat{MIC_{pp}^{'''G}}(u,v) = 1.4024$; in this case, the process is incapable. The advantage of considering the fuzzy mode is that it helps us when the data is crisp and no exact boundary can be drawn for it.

Also, according to Table 4, $R\left(\widetilde{MIC_{pp}^{\prime\prime\prime}(u,v)}\right) = 1.2743$, which is greater than $R(\tilde{1}) = 1$, so the process is incapable. If this index is also calculated in the case of crisp data, its value is equal to $\widetilde{MIC_{pp}^{\prime\prime\prime}(u,v)} = 1.1148$; in this case, the process is incapable.

Example 2: Sultan's example

Sultan [44], in his research on the production process of a type of raw material, examined the two characteristics of Brinell hardness (BH) and Tensile strength (TS). The specification limits of the triangular fuzzy and the value of the triangular fuzzy target for both factors are shown in Table 5. To solve this example, the numbers $\lambda^M = 0.1$, $u = 1$ and $v = 1$ were used.

Table 5. Fuzzy information the production process of a type of raw material

	LSL			USL			T		
	L	M	R	L	M	R	L	M	R
BH (no units)	111.7	112.7	113.7	111.7	112.7	113.7	111.7	112.7	113.7
TS (MPa)	31.7	32.7	33.7	31.7	32.7	33.7	31.7	32.7	33.7

The fuzzy sample mean vector and the fuzzy sample variance-covariance matrix are as follows.

$$\tilde{\bar{X}} = \begin{pmatrix} 176.2 & 177.2 & 178.2 \\ 51.33 & 52.33 & 53.33 \end{pmatrix} \tag{48}$$

$$\tilde{S} = \begin{pmatrix} 336.8 \;\; 337.8 \;\; 338.8 & 84.3308 \;\; 85.3308 \;\; 86.3308 \\ 84.3308 \;\; 85.3308 \;\; 86.3308 & 32.6247 \;\; 33.6247 \;\; 34.6247 \end{pmatrix} \tag{49}$$

The process incapability index is presented in Table 6, taking into account the measurement error in the fuzzy state, the approximate value of the fuzzy index and the value of the index in the crisp mode.

Table 6. Process incapability index considering measurement error

Value of the fuzzy index	Approximate value of the fuzzy index	Value of the index in crisp state
$\widetilde{\widetilde{MC_{ia}^{\prime\prime\prime}(u,v)}} = [0.0078 \;\; 0.0323]$	$\widetilde{MC_{ia}^{\prime\prime\prime}(u,v)} \cong 0.0200$	$\widetilde{MC_{ia}^{\prime\prime\prime}(u,v)} = 0.0009$
$\widetilde{\widetilde{MC_{ip}^{\prime\prime\prime G}(u)}} = [0.4582 \;\; 1.0624]$	$\widetilde{MC_{ip}^{\prime\prime\prime G}(u)} \cong 0.7603$	$\widetilde{MC_{ip}^{\prime\prime\prime G}(u)} = 0.6825$
$\widetilde{\widetilde{MIC_{pp}^{\prime\prime\prime G}(u,v)}} = \widetilde{\widetilde{MC_{ia}^{\prime\prime\prime}(u,v)}} + \widetilde{\widetilde{MC_{ip}^{\prime\prime\prime G}(u)}} = [0.4661 \;\; 1.0947]$	$\widetilde{MIC_{pp}^{\prime\prime\prime G}(u,v)} \cong 0.7804$	$\widetilde{MIC_{pp}^{\prime\prime\prime G}(u,v)} = 0.6835$

The process incapability index without considering the measurement error in the fuzzy state, the approximate value of the fuzzy index and the value of the index in the crisp mode are presented in Table 7.

Table 7. Process incapability index without considering measurement error

Value of the fuzzy index	Approximate value of the fuzzy index	Value of the index in crisp state
$\widetilde{\widetilde{MC_{ia}^{\prime\prime\prime}(u,v)}} = [0.0078 \;\; 0.0323]$	$\widetilde{MC_{ia}^{\prime\prime\prime}(u,v)} \cong 0.0200$	$\widetilde{MC_{ia}^{\prime\prime\prime}(u,v)} = 0.0009$
$\widetilde{\widetilde{MC_{ip}^{\prime\prime\prime}(u)}} = [0.2282 \;\; 0.5205]$	$\widetilde{MC_{ip}^{\prime\prime\prime}(u)} \cong 0.3744$	$\widetilde{MC_{ip}^{\prime\prime\prime}(u)} = 0.3355$
$\widetilde{\widetilde{MIC_{pp}^{\prime\prime\prime}(u,v)}} = \widetilde{\widetilde{MC_{ia}^{\prime\prime\prime}(u,v)}} + \widetilde{\widetilde{MC_{ip}^{\prime\prime\prime}(u)}} = [0.2360 \;\; 0.5528]$	$\widetilde{MIC_{pp}^{\prime\prime\prime}(u,v)} \cong 0.3944$	$\widetilde{MIC_{pp}^{\prime\prime\prime}(u,v)} = 0.3365$

According to Table 6, $R\left(\widehat{MIC_{pp}^{'''G}(u,v)}\right) = 0.7804$, which is less than $R(\tilde{1}) = 1$, the process is capable. If this index is also calculated in the case of crisp data, its value is equal to $\widehat{MIC_{pp}^{'''G}}(u,v) = 0.6835$ so, in this case, the process is capable.

Also, according to Table 7, $R\left(\widehat{MIC_{pp}^{'''}(u,v)}\right) = 0.3944$, which is less than $R(\tilde{1}) = 1$, the process is capable. If this index is also calculated in the case of crisp data, its value is equal to $\widehat{MIC_{pp}^{'''}(u,v)} = 0.3365$; so, in this case, the process is capable.

According to the description given, the fuzzy index presented in determining the capability or incapability of the process acts as in the crisp mode, correctly detecting the incapability.

According to the above explanations, it is clear that considering the measurement error leads to an increase in the value of the fuzzy index, as compared to the case in which the measurement error is not considered. It should be noted that this non-consideration of measurement error leads to the incorrect estimation of process capability; this is because the process may be incapable in reality, but when the measurement error is not taken into account in the calculations, the process is considered capable.

Conclusion

For processes in which the production of products is very sensitive and sophisticated, the use of process incapability indices is essential. Process incapability indices are numerical criteria for determining the degree to which process performance conforms to process specifications. In many real-world processes, the measurement of quality characteristics is affected by an error, which leads to increased incapability.

In this research, for the first time, a multivariate process incapability index was presented by considering the measurement error in the fuzzy mode; the advantage of considering the fuzzy mode was that when the data were not crisp and no precise boundary could be determined for them, this method could be helpful. The fuzzy mode also helps managers make better, more practical decisions.

For two practical examples, the index presented in the fuzzy state was calculated and its performance was examined to determine the capability and incapability of the process. It was shown that the new index presented in the fuzzy environment could determine the capability or incapability of the process as done in a crisp state, thus showing a good performance. This increases managers' confidence in the fuzzy index and they can trust the results of the fuzzy index and make decisions to improve the process.

Ignoring the measurement error also leads to the incorrect calculation of process capability, causing a lot of damage to manufacturing industries, especially high-tech ones. The results of this paper showed that not considering the measurement error leads to less estimation of the process incapability index and this issue causes managers to make wrong decisions about the process.

As suggestions for future research, the process distribution may not be considered normal, the measurement error distribution may not be assumed to be normal, or Bootstrap can be used to estimate the confidence interval of the index.

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