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Von Neumann Regular McCoy Rings

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Abstract

A ring R is said to be right McCoy, if for every f(x),g(x) in the polynomial ring R[x], with f(x)g(x)=0 thereexists a nonzero element c϶R with f(x)c=0. In this note, we show that von Neumann regular McCoy rings are abelian. This gives a positiveanswer to the question rised in Comm. Algebra 42 (2014) 1565- 1570."

Keywords: McCoy rings; Von Neumann regular rings; Abelian rings.

Introduction

Throughout this note all rings are associative with unity. According to McCoy [6, Theorem 2], any commutative ring R has the property that, if f(x) is a zero-divisor in R[x], then the ideal generated by the coefficient of f(x) is a zero-divisor in R.

P. P. Neilsen in [2], calls a ring R *right McCoy* (resp. *left McCoy*), if f(x) is a left (resp. right) zero-divisor in R[x], then the left (resp. right) ideal generated by the coefficient of f(x) is a left (resp. right) zero-divisor in R. He showed that, reversible rings (that is, ab=0 implies ba=0 for all a,b϶R) are McCoy [2, Theorem 2]. It is obvious that every commutative ring is reversible. A ring R is called *semi-commutative* if for any a϶R, the right annihilator of it is an ideal of R. Reduced rings are clearly reversible and reversible rings are semi-commutative. In [2], Nielsen provides an example of a semi-commutative ring that is not right McCoy.

A ring R is called 2-*primal* if $Nil_*(R)$=Nil(R). A ring R is *symmetric* if abc=0 implies acb=0, for all a,b,c϶R.

A ring R is called an *Armendariz* ring if whenever polynomials $f(x)=a_0+a_1x+\ldots+a_mx^m$, $g(x)=b_0+b_1x+\ldots+b_mx^m$϶R[x] satisfy f(x)g(x)=0, then $a_ib_j=0$ for each i,j. Armendariz had proved that a reduced ring (i.e., a ring without nonzero nilpotent elements) satisfies this condition.

A ring R is called von Neumann regular for each a϶ R there exists b϶R such that a=aba. In other word, a ring R is von Neumann regular if any finitely generated right ideal of it is a direct summand of it.

In [3], Nasr-Isfahani posed a question whether is it true that any von Neumann regular McCoy ring is abelian?

In this note, we give a positive answer to this question.

Results

An idempotent $e=e^2$϶R is called left (resp. right) *semi central* if ere=re (resp. ere=er)or every element .R϶r

Lemma 1.1. An idempotent e in a ring R is a central idempotent if and only if e is both left semicentral and right semicentral in R.

Theorem 1.2. Von Neumann regular right McCoy rings are abelian.

.Proof Let $e=e^2$϶R. We shall show that eR(1-e)=0=(1-e)Re because in this cases, we have er=ere=re for all r϶R, as stated in Lemma 2.1. Now if eR(1-e)≠0 (the other case is similar), then er(1-e)≠0 for some r϶R.

Thus er(1-e)R=hR for some $h=h^2$϶R by the regular condition on R. It follows that er(1-e)s=h for some s϶R. Let

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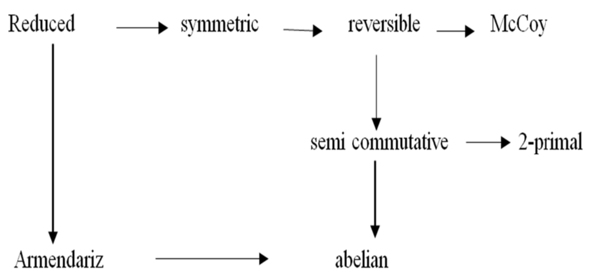
$f(x)=e+er(1-e)x+(1-e)sex^2+(1-e)x^3$
and
$g(x)=-(1-e)sh+hx+(1-e)shx^2-hx^3$.

Since $eh=h=h^2=er(1-e)sh$, we have $f(x)g(x)=0$, right McCoy property grantees that there must exists a nonzero element $c\epsilon R$ such that $f(x)c=0$. As e, (1-e) are coefficients of f(x), we get $ec=(1-e)c=0$, and so $c=1c=((1-e)+e)c=0$, which is a contradiction. So we get $eR(1-e)=0$. In a similar way as above, we can show that $(1-e)Re=0$. So e is both left and right semi central idempotent of R and so by Lemma 2.1, $e\epsilon Cent(R)$. Thus R is an abelian ring.

We have the following diagram:



Corollary 1.3. Let R be a von Neumann regular ring. Then the following statements are equivalent:
(1) R is right McCoy;
(2) R is reduced;
(3) R is symmetric;
(4) R is reversible;
(5) R is semi commutative;
(6) R is 2-primal;
(7) R is Armendariz;
(8) R is abelian.

Proof. (5)→ (4)→ (3)→ (2) follows from [4, P.361] and [4, Proposition 1.3]. (5) (6)→ follows from [5, Lemma 2.7]. As R is von Neumann, its prime radical is zero we have any semiprime 2-primal ring is reduced. Hence we get (6).(2) → Thus we have (2) ↔ (3) ↔ (4) ↔ (5) ↔ (6).

The implication (2) (8)→ (7) is clear and (8) (2) → follows from the fact that any ablian von Neumann regular ring is reduced. So (2)↔(7)↔(8).

(2)(1)→ is clear and (1) (8) → follows from the Lemma 2.2. As (2)↔(8) we get (1)↔(2)↔(8).

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