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RESEARCH PAPER

Stochastic Programming Models for Dynamic Facility Layout Problem in Flexible Manufacturing Systems

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Abstract

An appropriate facility layout is required to reduce total manufacturing cost, especially in uncertain environments. The design of a desirable facility layout is essential when the rearrangement of the facilities is expensive. Using Routing Flexibility (RF) as a principle of the Flexible Manufacturing System (FMS) can lead to the fulfillment of this need. This paper propounds two new mathematical models for the Dynamic Facility Layout Problem (DFLP) with stochastic approaches. The RF is considered when the demands of the independent parts follow Exponential and Normal distributions in which their parameters randomly alter from period to period. The primary nonlinear models are first linearized by the proposed innovative technique. Then, the performance of the proposed models and the linearization technique is assessed by solving two test problems. Next, the RF effect on the manufacturing system is analyzed. The obtained results verify the validity and applicability of the proposed models. It is also shown that the suggested linearization technique is an efficient technique with 99% accuracy, even if convexity conditions are not met.

Keywords:
Facility Layout;
Flexible Manufacturing System;
Linearization Technique;
Stochastic Approaches;
Uncertain Environments

Introduction

To consider a proper design for the facility layout in any production system is inevitable. On the other hand, Material Handling Cost (MHC) is induced by 20% –50% of the Total Manufacturing Cost (TMC), while an effectual facility layout can diminish the cost from 10% to 30% [1]. So, the MHC is one of the most proper measures to assess the performance of a facility layout. The aim of the Facility Layout Problem (FLP) is to determine the locations of the facilities to reduce the MHC [2]. A facility can be defined as follows: a physical operational portion like a department in an organization, a machine in a manufacturing system, and a manufacturing cell [3]. The facility layout can affect the Key Performance Indexes (KPI) like Work-In-Process (WIP), throughput rate, and manufacturing lead time [4]. In FLP, it is assumed that the material flows between the centroid of two facilities.

In general, there are three different approaches to deal with a multi-period FLP: I) static, II) dynamic, and III) robust approaches. If the facility Rearrangement Cost (RC) is low, then the static approach will be appropriate, called Static Facility Layout Problem (SFLP). When the

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mass flow of materials does not alter from period to period, the SFLPs could be applied. If demands vary in the planning horizon which is usual in dynamic environments, then the materials will flow differently. Because of the inefficient layout, rearrangement of the facility layout is essential, where the SFLP is no longer useful and is converted into Dynamic Facility Layout Problem (DFLP) [5]. As previously mentioned, the implementation of a facility layout is a significant part of the initial stages of creating a production system that is typically very costly. An inefficient facility layout can impose at least 36% of the TMC [6]. In DFLP, the optimal layout is not necessarily constant and can alter from period to period. However, sometimes the facility rearrangement may not be possible for technical reasons in the real world.

In the FLP with the robust approach, neither a layout is optimal for each period separately, nor the layout changes from period to period. Instead, a stable facility layout with the minimum MHC under different stochastic demand scenarios is determined as the best layout in the whole planning horizon [2]. If the uncertainty is considered, then the stochastic approach will apply to all three aforementioned types of FLPs. Thus, the SFLP and DFLP are converted into Stochastic Static Facility Layout Problem (SSFLP) and the Stochastic Dynamic Facility Layout Problem (SDFLP), respectively. In both of them, the Decision Maker (DM) presents an opinion on uncertainty in part demands by confidence level $(1-\alpha)$. The part demands are independent random variables with known means and variances and alter from period to period.

Routing Flexibility (RF) is one of the necessities of each modern system such as a flexible manufacturing system (FMS). Flexible design is defined as the capability to modify the changes without significant impact on performance. The RF is specified as the average number of choices of a machine that a commodity can choose. Generally, the RF in a facility layout is the capability to manufacture a throughput by alternative routes within the system [7]. Flexibility is a valuable attribute and should be considered in a dynamic competitive environment. In the performance assessment, the value of flexibility should be considered [8]. The solution method for the FLP is determined by the features of the model like size problem, linearity, non-linearity, and modelling approach. The solution procedures can be generally classified into three types: a) exact, b) heuristic, and c) intelligent methods [9]. On the other hand, the Quadratic Assignment Problem (QAP) is NP-complete [10]. The optimization methods are unable to solve the problems with 15 or more facilities, within a reasonable time [11]. So, in addition to the three above-mentioned methods, there is a need for approximated algorithms that can present reasonable suboptimal solutions. However, the proposed models in this research are solved using an exact method, after applying a novel linearization technique.

This paper has two major contributions. First, the RF is appended to the DFLP with the stochastic approaches under Normal and Exponential distribution functions. Second, a novel innovative linearization technique is introduced for the first time, which can be utilized for other complex cases with a high degree of non-linearity. The remainder of the paper is organized as follows: In Section 2, the most related research in the literature are briefly reviewed. Section 3 describes the discussed problem followed by Section 4 where two stochastic models are presented when independent part demands are randomly distributed with the specified mean and variance. In Section 5, the proposed linearization technique is introduced. To validate the developed models as well as the linearization technique, some experiments are conducted including two sensitivity analyses in Section 6. Some managerial insights have presented in Section 7. Finally, the obtained results are presented in Section 8.

Literature review

In recent studies, flexibility and robustness are two favorite subjects for FLP [12]. They declared the optimization approaches are one of the most main used methods to get the optimal solutions for small-sized problems. They also stated that the approximated approaches are widely classified such as improvement algorithms, construction algorithms, and meta-heuristic algorithms. The studies related to DFLP are summarized in Table 1.

Table 1. Literature review of DFLP in terms of formulation and resolution approach

(Reference)	Resolution Approach				Uncertain environment	Problem formulation			
						Constraints		Model	
	Exact	Approximated	Stochastic	Intelligent		Non-overlapping	Principal of Flexibility	Deterministic	Un Deterministic
[13]			S T			√	√	Non L	
[14]	D P							Non L	
[15]	D P							Non L	
[16]	B&B								Non L
[17]		√			√				Non L
[18]			S T		√				Non L
[19]		√				√	√		Non L
[20]				S A					Non L
[21]		S T			√	√			Non L
[22]		√							Non L
[23]				GA					Non L
[11]		√						Non L	
[6]		√						Non L	
[5]				S A		√	√		Non L
[24]		√							Non L
[25]		√				√			Non L
[26]			H						Non L
[27]	B&B								Non L
[28]				TSA					Non L
[29]				GA	√	√			Non L
[2]		DP& SA							Non L
[30]		C M			√				Non L
[31]				SA	√	√		Non L	
[32]				SA	√	√		Non L	
[33]				GA	√		√		Non L
[34]				SA	√	√			Non L
[35]				GA	√		√		Non L
This research	✓				✓	✓	✓		Linear

S T: Scenario Tree, Non L: Nonlinear, DP: Dynamic Programming, B&B: Branch and Bound, S A: Simulated Annealing, GA: Genetic Algorithm, H: Heuristic, TSA: Tabu Search Algorithm, C M: Combined Metaheuristic

Upon reviewing the literature, it seems that there are two genuine research gaps in designing a facility layout in dynamic environments. To the best of the authors' knowledge, no research has investigated RF in the DFLP, so far. Accordingly, two integrated mathematical models based on QAP for the DFLP with the stochastic approaches are presented for the first time, where independent part demands follow the Exponential and Normal distribution functions. One of the reasons for using Normal distribution for demands is that most of the real-world phenomena are compatible with Normal distribution [7]. Furthermore, to the best of our knowledge, this paper is the first to propose a new linearization technique for the nonlinear zero-one polynomial programming problems and utilize it for the presented models whereas [36] and [37] refer to linearization techniques.

Problem description

The existence of uncertainty in the DFLP creates the SDFLP. Moreover, the confidence level $(1-\alpha)$ determines the uncertainty in part demands specifying by the DM. In this paper, a new nonlinear zero-one polynomial programming formulation is proposed for the DFLP with the stochastic approaches. In the proposed mathematical models, the RF is appended when the independent part demands follow the Exponential and Normal distributions in which their parameters are randomly altered from period to period.

Notations

In this subsection, the assumptions, indices and parameters and the decision variables of the proposed models are presented as follows.

Assumptions

The assumptions of the proposed models are as follows:

1. The facilities are equal-sized, and the number of facilities is equal to the number of locations.
2. The discrete representation is the same as the SDFLP.
3. The part demands are independent random variables with specified variance and mean in each period; however, they are randomly altered from period to periods.
4. The confidence level is specified. It shows the DM's opinion on the uncertainty of demands.
5. The parts with known sizes are transmitted as a batch from a facility to another.
6. The interest rate is clear for each period (year, month).
7. The distance between the center of facility locations (computed by the rectilinear distance), the number of total periods, the present value of part movement cost, and the facility sequence in each period are all given and specified as input parameters in the model.

Indices and parameters

The all indices and parameters used in the proposed nonlinear models have presented in Table 2.

Table 2. Indices and parameters of the proposed nonlinear models.

Indices	Parameters
i,j Indices of facilities $(\boldsymbol{i,j=1,2,\dots,M});\boldsymbol{i\neq j}$	d_{lq} Distance between facility locations *l*, *q* D_{kt} Demand for part *k* in the period *t* I_r Interest rate f_{ijktn} The flow of materials for part *k* between facilities *i* and *j* in route production *n* in the period *t*.
l,q Indices of facility locations $(\boldsymbol{l,q=1,2,\dots,M});\boldsymbol{l\neq q}$ *M* Total number of facilities or facility locations *t* Index of the period $(\boldsymbol{t=1,2,\dots,T})$	f_{ijkt} The flow of materials for part *k* between facilities i and *j* in the period *t* f_{ijk} Materials flow for part *k* between facilities *i* and *j* f_{ij} Materials flow for all parts between facilities *i* and *j* *E()* Expected value (mean) of a parameter *Var()* Variance of a parameter
T Number of periods under consideration	$Z_{1-\alpha}$ The value of standard normal *Z* for $(1-\alpha)$ confidence interval C_{kt} The movement cost of each batch size per unit distance for part *k* in the period *t*
k Index for parts $(\boldsymbol{k=1,2,\dots,K})$	C_k Present value of the movement cost for each batch size per unit distance for part *k*
K Number of parts	$\beta_{ijktn}\begin{cases}1; & \text{If facility } j \text{ appears immediately after facility } i \text{ in period } t \text{ for part } k \text{ in route } n\\ 0; & \text{Otherwise.}\end{cases}$ B_k Transfer batch size for part *k*
n Index of the route of production $(\boldsymbol{n=1,2,\dots,N})$	P_{kn} Passing probability of part *k* from route *n* $C(\pi_{rm})$ Total MHC for layout π
N Number of production routes.	$U(\pi_{rm},1-\alpha)$ Maximum value (upper bound) of $C(\pi_{rm})$ with the confidence level $(1-\alpha)$ OFV_{rm} Total cost of facility layout π_{rm}

Decision variable

The used decision variables in the proposed models are as follows:

$$x_{il}=\begin{cases}1 & ;\ \text{If facility } i \text{ is assigned to location } l\\ 0 & ;\ \text{Otherwise}\end{cases}$$

Quadratic Assignment Problem (QAP)

Two mathematical models are presented in this paper based on QAP for the DFLP with the stochastic approaches, where the independent part demands follow the Exponential and Normal distribution functions. The following zero-one programming is a QAP formulation suggested by [38] used to extend the DFLP in this paper:

$$\text{Minimize } Z=\sum_{i,j,k,l} f_{ik}d_{jl}x_{ij}x_{kl} \tag{1}$$

subject to:

$$\sum_i x_{ij}=1 \quad ;\ \forall j \tag{2}$$

$$\sum_j x_{ij}=1 \quad ;\ \forall i \tag{3}$$

$$x_{ij}\in\{0,1\} \quad ;\ \forall i,j \tag{4}$$

Where n, f_{ik} and d_{jl} are the total number of locations, the flow of material from plant (facility) i to plant k, and the distance from location j to location l, respectively. If plant (facility) i is assigned to location j, then x_{ij} would be one; otherwise zero. In Eq 1, $i\neq$ k implies that $j\neq l$, $j\neq l$ implies $i\neq k$, $i=k$ implies $j=l$, and $j=l$ implies $i=k$ due to Eqs. 2 and 3. It should be noted that in the presented formulation, the assumptions of subsection 3.1.1 are applied.

The proposed mathematical models

In this section, we define the proposed objective function and present two new mathematical models.

Determining the new objective function

Usually, when a machine breakdown happens, uncertainty occurs in production. Under these circumstances, production routing flexibility is needed [7]. Also, since RF is one of the principles of FMS, the P_{kn} is added to the model for taking the RF into account. The P_{kn} is the passing probability of part *k* from route *n*. So, the material flow of part *k* between facilities *i* and *j* in the route of production *n* in period *t* (f_{ijktn}) is calculated as follows:

$$f_{ijktn} = \beta_{ijktn} \frac{C_{kt} D_{kt}}{B_k} P_{kn} \quad ; \quad \forall i,j,k,t,n \tag{5}$$

In Eq. 5, β_{ijktn} is a zero-one variable that ensures two consecutive operations can be done on part *k* in facilities *i* and *j* in period *t* in production route *n*. Regarding the considered assumptions, D_{tk} is an independent random variable and therefore, f_{ijktn} is also an independent random variable. The total flow for part *k* between facilities *i* and *j* in period *t* resulting from all routes of production (f_{ijktn}) can be written as Eq. 6:

$$f_{ijkt} = \sum_n f_{ijktn} = \sum_n \beta_{ijktn} \frac{C_{kt} D_{kt}}{B_k} P_{kn} \quad ; \quad \forall i,j,k,t \tag{6}$$

Since f_{ijktn} is an independent random variable, f_{ijkt} is also a random variable, as well. The flow of part *k* between facilities *i* and *j* over all periods (f_{ijk}) is given by Eq. 7:

$$f_{ijk} = \sum_t \frac{f_{ijkt}}{T} = \sum_{t,n} \beta_{ijktn} \frac{C_{kt} D_{kt}}{T B_k} P_{kn} \quad ; \quad \forall i,j,k \tag{7}$$

Finally, the total flow between facilities *i* and *j* in the planning horizon resulting from all periods can be written as Eq. 8:

$$f_{ij} = \sum_k f_{ijk} = \sum_{k,t,n} \beta_{ijktn} \frac{C_{kt} D_{kt}}{T B_k} P_{kn} \quad ; \quad \forall i,j \tag{8}$$

Since f_{ijkt} is an independent random variable, f_{ijk} and f_{ij} are also random variables. Eqs. 9 and 10 represent the expected value and variance of f_{ij}, respectively.

$$E(f_{ij}) = \sum_k E(f_{ijk}) = \sum_{k,t,n} \beta_{ijktn} \frac{C_{kt} E(D_{kt})}{T B_k} P_{kn} \quad ; \quad \forall i,j \tag{9}$$

$$Var(f_{ij}) = \sum_k Var(f_{ijk}) = \sum_{k,t,n} \left(\frac{\beta_{ijktn} C_{kt} P_{kn}}{T B_k} \right)^2 Var(D_{kt}) \quad ; \quad \forall i,j \tag{10}$$

Since f_{ij} is an independent random variable, the MHC for π_{rm}, i.e., $C(\pi_{rm})$ is similarly a random variable with the following mean and variance as Eqs. 11 and 12.

$$E(C(\pi_{rm})) = \sum_{i,j,l,q} E(f_{ij}) d_{lq} x_{il} x_{jq} = \sum_{i,j,l,q} \sum_{k,t,n} \beta_{ijktn} \frac{C_{kt} E(D_{kt})}{T B_k} P_{kn} d_{lq} x_{il} x_{jq} \tag{11}$$

$$Var\big(C(\pi_{rm})\big) = \sum_{i,j,l,q} Var(f_{ij})\big(d_{lq}x_{il}x_{jq}\big)^2 = \sum_{i,j,l,q}\sum_{k,t,n}\left(\frac{\beta_{ijktn}C_{kt}P_{kn}}{TB_k}\right)^2 Var(D_{kt}){d_{lq}}^2\big(x_{il}x_{jq}\big)^2 \quad (12)$$

If the decision-maker considers $U(\pi_{rm}, 1-\alpha)$ as the maximum value (upper bound) of $C(\pi_{rm})$ with the confidence level $(1-\alpha)$, then the $U(\pi_{rm}, 1-\alpha)$ can be minimized instead of $C(\pi_{rm})$. So:

$$P\big(C(\pi_{rm}) \leq U(\pi_{rm}, 1-\alpha)\big) = 1-\alpha \quad (13)$$

After standardization, Eq. 13 can be rearranged as Eq. 14:

$$U(\pi_{rm}, 1-\alpha) = E\big(C(\pi_{rm})\big) + Z_{1-\alpha}\left(\sqrt{Var\big(C(\pi_{rm})\big)}\right) \quad (14)$$

Instead of minimizing $C(\pi_{rm})$, its upper bound i.e., $U(\pi_{rm}, 1-\alpha)$ can be minimized. Therefore, OFV_{rm} can be rewritten as Eq. 15:

$$Minimize \quad OFV_{rm} = U(\pi_{rm}, 1-\alpha) \quad (15)$$

Also, the time value of money can be formulated as Eq. 16:

$$C_{kt} = C_k(1+I_r)^t \quad (16)$$

If Eqs. 11, 12, 14, and 16 are inserted into Eq. 15, then we have:

$$\begin{aligned} Minimize \quad OFV_{rm} &= \sum_{i,j,l,q}\sum_{k,t,n}\beta_{ijktn}\frac{C_{kt}P_{kn}}{TB_k}E(D_{kt})d_{lq}x_{il}x_{jq} \\ &+ Z_{1-\alpha}\left(\sqrt{\sum_{i,j,l,q}\sum_{k,t,n}\left(\frac{\beta_{ijktn}C_{kt}P_{kn}}{TB_k}\right)^2 Var(D_{kt}){d_{lq}}^2\big(x_{il}x_{jq}\big)^2}\right) \end{aligned} \quad (17)$$

In the next subsection, Eq. 17 is formulated according to the distribution type of D_{kt} and its $E(D_{kt})$ and $Var(D_{kt})$. It is assumed that the independent part demands follow Normal and Exponential distributions, as discussed in subsections 4.2 and 4.3, respectively. Each of them has their own known expected value and variance.

Modelling under normal distribution

One of the reasons for using Normal distribution for demands is that most of the real-world phenomena are compatible with Normal distribution [7]. Now, if D_{kt} is considered as a normally distributed random variable, its expected value and variance can be calculated as Eqs. 18 and 19:

$$E(D_{kt}) = \mu_{kt} \tag{18}$$
$$Var(D_{kt}) = \sigma^2{}_{kt} \tag{19}$$

When the independent random variables are summed, their properly normalized sum leads to a Normal distribution even if the main variables themselves are not normally distributed (Central Limit Theorem (CLT)). If D_{kt} follows Normal distribution, then it is not necessary for K to tend to infinity. In this case, regardless of K, the distribution of the summation tends to be Normal [39]. By inserting the Eqs. 18 and 19into Eq. 17, the first proposed model is obtained as follows:

$$\begin{aligned} Minimize \quad OFV_{rm} &= \frac{1}{T}\left(\sum_{i,j,l,q}\sum_{k,t,n}\beta_{ijktn}\frac{C_k(1+I_r)^t}{B_k}P_{kn}\,\mu_{kt}\,d_{lq}x_{il}x_{jq}\right) \\ &+ \frac{Z_{1-\alpha}}{T}\left(\sqrt{\sum_{i,j,l,q}\sum_{k,t,n}\left(\frac{\beta_{ijktn}C_k(1+I_r)^tP_{kn}}{B_k}\right)^2\sigma_{kt}{}^2 d_{lq}{}^2\left(x_{il}x_{jq}\right)^2}\right); \quad i \neq j \end{aligned} \tag{20}$$

Eq. 20 shows the objective function for the DFLP with stochastic approaches, when the RF is considered. The OFV_{rm} should be minimized, in which the part demands (D_{kt}) are independent normally distributed variables. The constraints of this model are as follows:

$$\textstyle\sum_i x_{il} = 1 \qquad ; \forall\, l \tag{21}$$

Eq. 21 ensures that each location contains only one facility in the planning horizon.

$$\textstyle\sum_l x_{il} = 1 \qquad ; \forall\, i \tag{22}$$
$$x_{ij} \in \{0,1\} \qquad ; \forall i, l \tag{23}$$

Eq. 22 ensures that each facility is located in only one location over the planning horizon. Though the part demands are random variables, the above-presented model is certain. Thus, the optimal values of the zero-one decision variables and optimal facility locations that were already unknown are now obtained, and the facility layout π_{rm} with stochastic approaches is determined over all periods. In the next subsection, Eq. 17 is rewritten, when the part demands (D_{kt}) follows the Exponential distribution function.

Modelling under Exponential distribution

To evaluate the performance of the proposed model, it is assumed that the demand of part *k* in period *t* (D_{kt}) is a random variable with an Exponential distribution. If λ_t (which is positive and signifies the average number of events within an interval in period *t*) be the parameter of an Exponential distribution for all parts in period *t*, then $D_{kt} \sim Exp(\lambda_t)$ and its expected value and variance are as Eqs. 24-26:

$$E(D_{kt}) = \frac{1}{\lambda_t} \qquad ; \quad \forall t \tag{24}$$
$$Var(D_{kt}) = \frac{1}{\lambda_t{}^2} \qquad ; \quad \forall t \tag{25}$$

According to the CLT, getting:

$$\lim_{k\to\infty}\textstyle\sum_k D_{kt} \sim Normal\left(\frac{K}{\lambda_t}, \frac{K}{\lambda_t{}^2}\right) \qquad ; \quad \forall t \tag{26}$$

If $D_{kt} \sim Exp(\lambda_t)$ and $K \geq 20$, then the CLT could be establish [39]. According to Eq. 26 and by inserting Eqs. 24-25 into Eq. 17, the objective function of the proposed model in this condition can be presented as Eq. 27:

$$Minimize \quad OFV_{rm} = \frac{K}{T}\left(\sum_{i,j,l,q}\sum_{k,t,n}\beta_{ijktn}\frac{C_k(1+I_r)^t P_{kn}}{\lambda_t B_k} d_{lq} x_{il} x_{jq}\right) + \frac{Z_{1-\alpha}\sqrt{K}}{T}\left(\sqrt{\sum_{i,j,l,q}\sum_{k,t,n}\left(\frac{\beta_{ijktn} C_k(1+I_r)^t P_{kn}}{\lambda_t B_k}\right)^2 d_{lq}{}^2 (x_{il}x_{jq})^2}\right); \; i \neq j \tag{27}$$

Eq. 27 represents the Objective Function Value (OFV_{rm}) for the DFLP with stochastic approaches, when routing flexibility is considered. The constraints of this proposed model are the same as ones explained in subsection 4.2.

Linearization

It is often difficult to deal with nonlinear terms due to the complexity of their solving methods. To make the proposed models more tractable, an attempt is made so as to linearize the nonlinear terms as much as possible. In this section, a new linearization technique for the proposed models are presented. The suggested linearization method is based on some theoretical and numerical techniques [40]. In the next section, it is shown that the proposed linearization technique works efficiently and simply with 99% accuracy, even if the convexity condition is not met. Considering Eqs. 19-22, the main steps of the proposed linearization method are as follows:

Step 1) Linearization of the product of two binary variables

As it can be seen, the term $x_{il}{}^2 x_{jq}{}^2$ existing in the OFV_{rm} is nonlinear. Since both variables are binary, without loss of generality, one can omit the power of two, and it consequently can be linearized by the help of an auxiliary binary variable (U_{iljq}) and three extra constraints as Eqs. 28 to 31:

$$x_{il}{}^2 x_{jq}{}^2 = x_{il}x_{jq} \rightarrow U_{iljq} \qquad ; \forall\, i,l,j,q \tag{28}$$
$$U_{iljq} \leq x_{il} \qquad ; \forall\, i,l,j,q \tag{29}$$
$$U_{iljq} \leq x_{jq} \qquad ; \forall\, i,l,j,q \tag{30}$$
$$U_{iljq} \geq x_{il} + x_{jq} - 1 \qquad ; \forall\, i,l,j,q \tag{31}$$

Step 2) Linearization of the radical term

After implementing the first step, Eq. 17 can be rewritten as follows:

$$\begin{aligned} \mathit{Minimize}\quad OFV_{rm} &= \sum_{i,j,l,q}\sum_{k,t,n}\beta_{ijktn}\frac{C_{kt}P_{kn}}{TB_k}E(D_{kt})d_{lq}U_{iljq} \\ &+ Z_{1-\alpha}\left(\sqrt{\sum_{i,j,l,q}\sum_{k,t,n}\left(\frac{\beta_{ijktn}C_k(1+I_r)^t P_{kn}}{TB_k}\right)^2 Var(D_{kt}){d_{lq}}^2 U_{iljq}}\right) \end{aligned} \tag{32}$$

As can be seen in Eq. 32 the last square root is an explicit nonlinear term named here as '*radical*'.

$$radical = \sqrt{\sum_{i,j,l,q}\sum_{k,t,n}\left(\frac{\beta_{ijktn}C_k(1+I_r)^t P_{kn}}{TB_k}\right)^2 Var(D_{kt}){d_{lq}}^2 U_{iljq}} \tag{33}$$

To linearize the radical term, a new integer variable $A \geq 0$ is introduced as Eq. 34:

$$A = \lceil radical \rceil \tag{34}$$

Where ⌈ ⌉ is the bracket sign and rounds the *radical* up to the nearest integer A. In fact, the *radical* expression is a positive real number. Ignoring the fraction part of that (if any), we assume *A* as a positive integer number, so *A* approximates the *radical* with a strictly lower than 1-unit absolute error. It is quite good when the *radical* takes large values. Hence, the *radical* expression is replaced by *A* in the model. *A* is a positive integer number, and according to *Theorem 1*, it can be rewritten as Eq. 35:

$$A = \sum_{v=0}^{m-1} 2^v\, y_v + (UB - 2^m + 1)\, y_m \tag{35}$$

Where y_v is a binary variable (c.f. *Theorem 1*). Also, *UB* is the upper bound of *A*, which can be obtained by setting all binary variables to one, while *m* is attained by Eq. 36:

$$m = \lfloor log_2(UB + 1) \rfloor \tag{36}$$

$$UB = \sqrt{\sum_{i,j,l,q}\sum_{k,t,n}\left(\frac{\beta_{ijktn}C_k(1+I_r)^t P_{kn}}{TB_k}\right)^2 Var(D_{kt}){d_{lq}}^2} \tag{37}$$

After rising the two sides of Eq. 34 to power of two, we have:

$$A^2 = \sum_{i,j,l,q}\sum_{k,t,n}\left(\frac{\beta_{ijktn}C_k(1+I_r)^t P_{kn}}{TB_k}\right)^2 Var(D_{kt}){d_{lq}}^2 U_{iljq} \tag{38}$$

It should be noticed that the right-hand side of Eq. 38 is linear. Therefore, suffice it to linearize A^2. According to *Theorem 2*, the Eq. 56 is proved to precisely simulate the quadratic term A^2. Inequalities (39-42) are the linear equivalent of Eq. 38, which should be added as new constraints to the model.

$$\sum_{i,j,l,q}\sum_{k,t,n}\left(\frac{\beta_{ijktn}C_k(1+I_r)^t P_{kn}}{TB_k}\right)^2 {\sigma_{kt}}^2\, {d_{lq}}^2 U_{iljq} \quad \leq \sum_{v=0}^{m-1} 2^{2v}\, y_v + \sum_{v=0}^{m-2}\sum_{v<w}^{m-1} 2^{v+w+1}\, y_{vw} + (UB - 2^m + 1)\sum_{v=0}^{m-1} 2^{v+1}\, y_{vm} + (UB - 2^m + 1)^2 y_m \tag{39}$$

$$y_{vw} \leq y_v \qquad ; \forall v,w \in \{0,\dots,m\} \tag{40}$$
$$y_{vw} \leq y_w \qquad ; \forall v,w \in \{0,\dots,m\} \tag{41}$$
$$y_{vw} \geq y_v + y_w - 1 \qquad ; \forall v,w \in \{0,\dots,m\} \tag{42}$$

Where y_v and y_{vw} are binary variables.

According to steps 1 and 2, one can rewrite Eqs. 20-23 as Eqs. 43-54:

$$Minimize\ OFV_{rm} = \sum_{i,j,l,q}\sum_{k,t,n} \beta_{ijktn} \frac{C_k(1+I_r)^t}{TB_k} P_{kn}\, \mu_{kt}\, d_{lq} U_{iljq} + Z_{1-\alpha}\, A \quad ; \qquad i \neq j \tag{43}$$

Subject to:

$$\sum_{i,j,l,q}\sum_{k,t,n}\left(\frac{\beta_{ijktn} C_k (1+I_r)^t P_{kn}}{TB_k}\right)^2 \sigma_{kt}{}^2\, d_{lq}{}^2 U_{iljq} \leq \sum_{v=0}^{m-1} 2^{2v}\, y_v + \sum_{v=0}^{m-2}\sum_{v<w}^{m-1} 2^{v+w+1}\, y_{vw} + (UB - 2^m + 1)\sum_{v=0}^{m-1} 2^{v+1}\, y_{vm} + (UB - 2^m + 1)^2 y_m \tag{44}$$

$$A = \sum_{v=0}^{m-1} 2^v\, y_v + (UB - 2^m + 1) y_m \tag{45}$$

$$\sum_{i} x_{il} = 1 \qquad ; \forall\, l \in \{1,\dots,M\} \tag{46}$$

$$\sum_{l} x_{il} = 1 \qquad ; \forall\, i \in \{1,\dots,M\} \tag{47}$$

$$U_{iljq} \leq x_{il} \qquad ; \forall\, i,l,j,q \in \{1,\dots,M\} \tag{48}$$
$$U_{iljq} \leq x_{jq} \qquad ; \forall\, i,l,j,q \in \{1,\dots,M\} \tag{49}$$
$$U_{iljq} \geq x_{il} + x_{jq} - 1 \qquad ; \forall\, i,l,j,q \in \{1,\dots,M\} \tag{50}$$
$$y_{vw} \leq y_v \qquad ; \forall v,w \in \{0,\dots,m\} \tag{51}$$
$$y_{vw} \leq y_w \qquad ; \forall v,w \in \{0,\dots,m\} \tag{52}$$
$$y_{vw} \geq y_v + y_w - 1 \qquad ; \forall v,w \in \{0,\dots,m\} \tag{53}$$
$$y_v, y_{vw}, U_{iljq}, x_{il}, x_{jq} \in \{0,1\} \qquad ; \forall i, l, j, q, v, w \tag{54}$$

Theorem 1: If A is an integer variable by upper bound UB and $m = \lfloor log_2(UB+1) \rfloor$, then A can be converted to a binary equation as Eq. 55:

$$A = \sum_{v=0}^{m-1} 2^v\, y_v + (UB - 2^m + 1)\, y_m \tag{55}$$

Proof 1: See Appendix A.

Theorem 2: If A is an integer variable by upper bound UB, then A^2 can be linearized by the help of binary variables y_i and y_{ij} as Eq. 56:

$$A^2 = \sum_{v=0}^{m-1} 2^{2v}\, y_v + \sum_{v=0}^{m-2}\sum_{v<w}^{m-1} 2^{v+w+1}\, y_{vw} + (UB - 2^m + 1)\sum_{v=0}^{m-1} 2^{v+1}\, y_{vm} + (UB - 2^m + 1)^2 y_m \tag{56}$$

Proof 2: See Appendix B.

In the next section, the proposed linearization technique is evaluated.

Computational experiments

The aims of this section are twofold: 1) In order to evaluate the performance of the proposed model and the suggested linearization technique, two test problems are generated and solved, and 2) the sensitivity of the model to the three critical aspects, i.e., routing flexibility, confidence level and batch size is evaluated. The exact approaches are often found not to be suited for large size problems. That's why only small problem instances are studied.

Validation and verification of the proposed models

In this subsection, twelve randomly generated test problems are solved to validate and verify the proposed model. Moreover, the optimal objective function value of the nonlinear model is compared with the linear model of the same problem to verify the precision of the proposed linearization technique. All computations are coded in GAMS 24.1.3 software and run on a PC Core i7, RAM 8GB, and 2.4 GHz CPU. In problem set (*I*), it is supposed that a manufacturing system consists of three equal-sized facilities and there are three facility locations. The problem set (*II*) consists of five facilities and five facility locations. Both of them are examined in three periods (T=3, T=6 and T=9) with three parts and six routes of production, such that the facilities should be arranged on the shop floor. The part demands follow the Normal distribution function with known mean and variance and alter from period to period shown in Table 3. The distance between facility locations for both test problems is also given in Table 4.

Table 3. The expected values (means) and variances of parts for the problem sets *(I)*and *(II)*.

Parts (k)	Period 1		Period 2		Period 3		Period 4		Period 5		Period 6	
	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
1	6.22	1.07	5.65	1.11	3.76	2.58	6.51	2.62	6.51	1.55	5.46	1.71
2	2.56	2.82	8.86	2.44	6.63	1.61	9.11	1.34	7.58	2.94	7.61	1.91
3	7.62	1.89	9.12	2.318	3.54	1.37	4.55	1.66	5.94	1.38	7.54	2.46

Table 4. Distance between facility locations for the problem sets *(I)* and *(II)*.

From \ To	1	2	3	4	5
1	0	10	20	15	25
2	10	0	10	20	5
3	20	10	0	20	10
4	15	20	20	0	20
5	25	5	10	20	0

The parts flow between the facilities are given in terms of batches. In both test problems, the present value of the movement cost for each batch size per unit as well as the transfer batch size for part k is given in Table 5. Moreover, Table 5 presents the routes of the parts and its probability for the problem set *(I)*. Table 6 also gives the routes of the parts and its probability for the problem set *(II)*.

Table 5. Primary value of the parameters, the route of the parts and its probability for the problem set (I).

Parts (k)	Facility sequence	Probability of the route (P_{kn})	C_k	B_k
1	2-3-1	0.5	30	10

	2-3	0.2		
	2-1	0.3		
2	3-1-2	1	50	5
3	1-2	0.7	10	25
	1-3	0.3		

Table 6. The route of the parts and its probability for the the problem set *(II)*.

Parts (k)	Facility sequence	Probability of the route (P_{kn})
1	2-3-1-5	0.5
	2-3-4	0.2
	4-1-2	0.3
2	3-1-2-5	1
3	1-2-4	0.7
	1-3-5	0.3

As an example, the facility sequence for the third part in the problem set *(I)* is 1 →2 with the probability of 0.7 and *1→3* with the probability of 0.3. This example shows that the first and second operations on the third part are accomplished with the probability of 0.7 through facilities 1 and 2, respectively. The first and third operations on the third part are completed by facilities 1 and 3 with the probability of 0.3. In both test problem sets the interest rate are considered 0.2. The confidence level also are 0.75, 0.85, 0.9 and 0.99.

After solving the problems optimally, the objective function value of the linear and nonlinear models for the problem sets *(I), (II)* are presented and compared in Table 7 and then depicted in Figs. 1 and 2, respectively. It should be noted that the elapsed time-lapse of both of these problems is less than one minute for all calculations.

Table 7. Comparison of the objective function value of the linear and nonlinear models for the problem sets *(I)* and *(II)*.

Period	α	Test problem *(I)*			Test problem *(II)*		
		Nonlinear	Linear	Difference (%)	Nonlinear	Linear	Difference (%)
T=3	0.75	2467.86	2467.948	3.57×10^{-3}	3304.53	3304.662	1.3×10^{-3}
	0.85	2887.16	2887.996	8.3×10^{-3}	3730.74	3730.994	2.5×10^{-3}
	0.9	3327.54	3327.79	2.5×10^{-3}	4234.23	4234.873	6.4×10^{-3}
	0.99	3710.43	3710.84	4.1×10^{-3}	4651.41	4651.89	8.4×10^{-3}
T=6	0.75	3848.114	3848.307	1.93×10^{-3}	4948.54	4948.914	3.38×10^{-3}
	0.85	4434.14	4434.307	4.34×10^{-3}	5248.74	5248.914	3.38×10^{-3}
	0.9	4862.31	4862.89	5.8×10^{-3}	5543.11	5549.74	1.7×10^{-3}
	0.99	5103.24	5103.91	6.8×10^{-3}	5850.42	5850.79	3.8×10^{-3}
T=9	0.75	5262.33	5262.97	6.4×10^{-3}	6109.27	6109.81	5.4×10^{-3}
	0.85	5790.12	5790.86	7.4×10^{-3}	6632.15	6632.67	5.2×10^{-3}
	0.9	6103.19	6103.68	4.9×10^{-3}	7002.35	7002.961	6.11×10^{-3}
	0.99	6832.32	6832.87	5.5×10^{-3}	7556.62	7556.99	3.8×10^{-3}

According to Table 7, it is evident that as the confidence level (α) increases, the Objective Function Value increases, too. In other words, the confidence level is directly related to the Objective Function Value. This means that managers have to pay more to be more confident.

Also, according to Figs. 1 and 2, the values of both objective functions are very similar (error $\leq 0.01\%$) in both sample problems, and the performance of the proposed linearization technique verifies what is proved by *Theorem* 2.

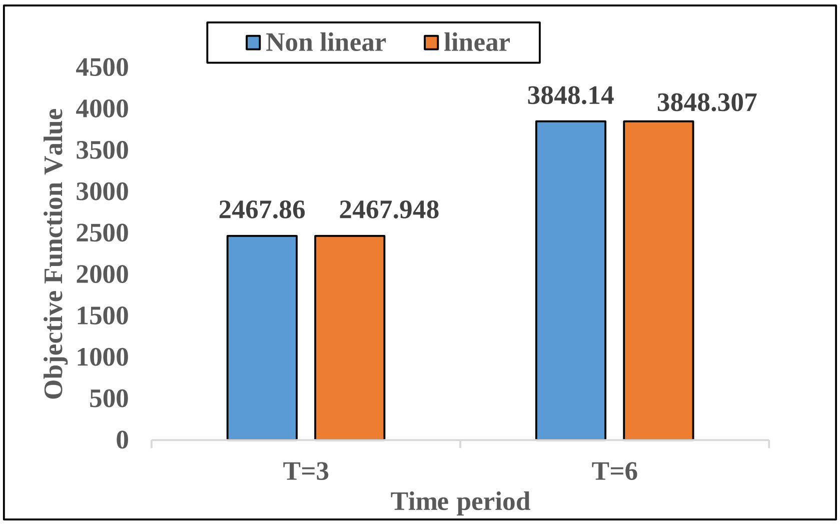

Fig. 1. Comparison of the objective function value of the linear and nonlinear models for the problem set (I).

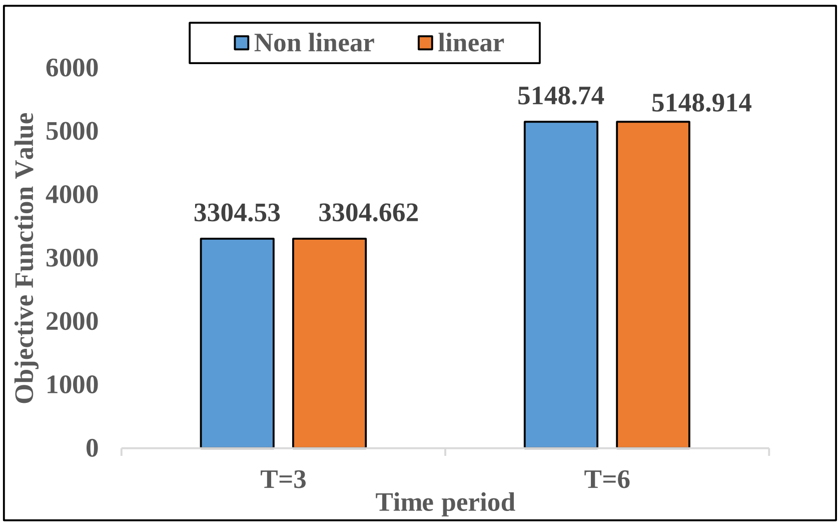

Fig. 2. Comparison of the objective function value of the linear and nonlinear models for the problem set (II).

Sensitivity analysis

We first study the effect of routing flexibility on the manufacturing system. The sensitivity of the proposed model with "the confidence level $(1-\alpha)$" and "transfer batch size for part k (B_k)" is also evaluated in subsections 6.2.1 and 6.2.2, respectively. To clarify the advantage of routing flexibility, the producibility is also discussed. According to [41], the producibility of a system is the ability to perform the intended task. The overall producibility of the system is calculated as Eq. 57:

$$R(z) = \sum_{k,t} R_{kt}(z) \qquad (57)$$

Where $R_{kt}(z)$ is the expected producibility of the system for part k in period t. According to [41], $R_{kt}(z)$ can be calculated as Eq. 58:

$$R_{kt}(z) = E(R_{kt}(z)|x) = \sum_x (R_{kt}(z)|x)\, p(x) \qquad (58)$$

Where x represents the state of the system and $p(x)$ signifies its probability. Also, z equals to $\frac{c}{d}$ so that c and d are the capacity and load of a facility, respectively. In an inflexible system, the production may come to a grinding halt whenever a machine breaks down. $R_{kt}(z)|x$ shows total producibility of the system in the x state. When there is no flexibility, $R_{kt}(z)|x$ can be calculated as Eq. 59:

$$R_{kt}(z)|x = \sum_j w_j \; r[z_j] \tag{59}$$

In Eq. 59, w_j is the weight of facility j which is chosen so as to sum to unity. For $r[z_j]$, the function r will give a measure of the producibility with respect to the amount of work achieved by any system. In other words, a substantial feature of flexibility is that it enables the system to adjust itself to different changes. This ability helps the system to maintain a high level of producibility. An advantage of flexibility is that it can handle such emergency situations. According to [41], when there is routing flexibility, $R_{kt}(z)|x$ can be calculated as Eq. 60:

$$R_{kt}(z)|x = \sum_{j\neq i} w_j \; r[z_j] + w_i \, r[\xi_i] \tag{60}$$

Where ξ_i can be computed by Eq. 61 as follows:

$$\xi_i = \frac{l_i}{d_i}\left(\sum_j P_{ij}\right) r\left[\frac{c_j - d_j}{P_{ij} l_i + P_{kj} l_k}\right] \tag{61}$$

Where l_i is an excess load of facility i so that $l_i = d_i - c_i$, and if $d_i < c_i$ then $l_i = 0$. Also, P_{ij} is the proportion of the load that cannot be launched to another facility. It represents a measure of the facility's inflexibility. The proportions are chosen so as to sum to unity, since a job exclusively can be diverted to one facility. According to [41], assuming the independence of all n facilities, the probability $p(x)$ is calculated as Eq. 62:

$$p\,(x = x_1, x_2, \dots, x_n) = p_i \prod_{j\neq i} (1 - p_j) \tag{62}$$

Where p_i is the probability that facility i is down. Also $x = (x_1, x_2, \dots, x_n)$ defines the state of the system, where $x_i = 1$, if facility i is in operating condition, and $x_i = 0$, if it is down.

In this section, the expected producibility of the system is only calculated for the third part in the first period under the condition of the problem set *(I)* named $R_{31}(z)$. The $R_{31}(z)$ can be obtained using total producibility of the system named $R_{31}(z)|x$, where x represents the state of the system. In this paper, d_i as the load of facility i is equavelent to the mean of demand. Also, c_1 is the capacity of the first facility considered as $c_1 = 4$ while the capacity of the second and third facilities are $c_2 = 5$ and $c_3 = 10$, respectively. The facility weights are set as $w_1 = 0.4$, $w_2 = 0.3$, and $w_3 = 0.3$ and P_{ij} is set as $P_{ij} = \frac{1}{3}$ for all i and j. According to the above description, the obtained results are presented in Table 8.

Table 8. The obtained results of $R_{31}(z)|x$ and $R_{31}(z)$

		$R_{31}(z)\|x$		$R_{31}(z)$	
x	$p(x)$	Inflexibility	Flexibility	Inflexibility	Flexibility
(1,1,1)	0.027	0.7	0.88	0.0189	0.024
(0,1,1)	0.063	0.5	0.5	0.0315	0.032

(1,0,1)	0.063	0.5	0.54	0.0315	0.034
(1,1,0)	0.063	0.4	0.404	0.0252	0.025
(0,0,1)	0.147	0.3	0.3	0.0441	0.044
(0,1,0)	0.147	0.2	0.2	0.0294	0.029
(1,0,0)	0.147	0.2	0.2	0.0294	0.029
(0,0,0)	0.343	0	0	0	0
$R_{kt}(z) = E(R_{kt}(z)\|x) = \sum_x (R_{kt}(z)\|x)\, p(x)$				0.21	0.22

It could be interesting to note that Table 8 is only for the third part. However, only 1% improvement in the producibility can be very effective in a large-size production system. To exemplify Table 8, the state of $x = (x_1 = 1, x_2 = 1, x_3 = 0)$ is considered that only the third facility is down. So, c_3 is equal to zero. Also, the excess load of the third facility would be equal to d_3. In this calculation, p_i is set as $p_i = 0.3$, so:

$$p\ (1,1,0) = p_i \prod_{j \neq i} (1 - p_j) = 0.3 * 0.3 * 0.7 = 0.063 \tag{63}$$

Also, $R_{31}(z)|(1,0,1)$ is calculated as a sample from Table 8 for the inflexibility and flexibility states using Eqs. 64 and 65, respectively:

$$R_{31}(z)|(1,0,1) = 0.4 * r\left[\frac{4}{7.62}\right] + 0 + 0.3 * r\left[\frac{10}{7.62}\right] = 0.5 \tag{64}$$

$$R_{31}(z)|(1,0,1) = 0.5 + 0.4 * r\left(\frac{7.62 - 4}{7.62} * \frac{1}{3} * r\left[\frac{10 - 7.62}{\frac{1}{3} * (7.62 - 4) + \frac{1}{3} * 7.62}\right]\right) = 0.54 \tag{65}$$

It is assumed that $C_d, C_{RF}, C_V, z^*, P_U, P_E$, and Q are the total cost belonging to down of a facility (monetary unit), the cost of routing flexibility, variable cost, the cost of facility layout (monetary unit), price of per unit, the expected producibility of the system, and quantity of production, respectively. The break-even point calculation scheme is presented as Eq. 66:

$$P_U Q P_E = z^* + C_{RF} + C_d + QC_V \quad \rightarrow \quad Q = \frac{z^* + C_{RF} + C_d}{P_U P_E - C_V} \tag{66}$$

Assuming the equality of z^*, Table 9 presents the other components of the break-even point analysis in two conditions; without and with routing flexibility for the data of the problem set (I). The last column of Table 9 is obtained by Eq. 66.

Table 9. The components of the break-even point analysis

Components / Type of layout	$z^*(\$)$	$C_{RF}(\$)$	$C_d(\$)$	$C_V(\$)$	P_E	$P_U(\$)$	Q^*
Stochastic layout without routing flexibility	2467	0	500	15	0.21	100	495

Stochastic layout with routing flexibility	2467	150	500	15	0.22	100	445

The observed 50 units difference in Q^* indicates that considering just one additional route in the production system of the problem set (I) could result in an increase in production by 10%. Fig. 3 demonstrates the above economic interpretation of the break-even point.

To specify the privilege of the routing flexibility, the break-even point analysis is used here. There are many factors (such as product type, the volume of the production and product diversity) in choosing the type of the production system. However, the first issue is their cost-effectiveness. So, the cost of routing flexibility (C_{RF}) is one of the most important components when making a decision. This paper formulates C_{RF} using Eq. 66. According to Fig. 3, if C_{RF} becomes more than 150$, then the routing flexibility is not economical.

Confidence level $(1-\alpha)$

Table 10 presents the results of solving the proposed linearized model for the problem set *(I)* when the confidence level $(1-\alpha)$ alters. Fig. 4 displays the relationship between the confidence level $(1-\alpha)$ and the objective function value.

Table 10. The sensitivity analysis of the proposed linearized model to the confidence level $(1-\alpha)$ for the problem set *(I)*.

Confidence level $(1-\alpha)$	Period	
	T=3	T=6
0.75	2429.08	3809.43
0.85	2473.66	3853.65
0.95	2549.2	3928.59

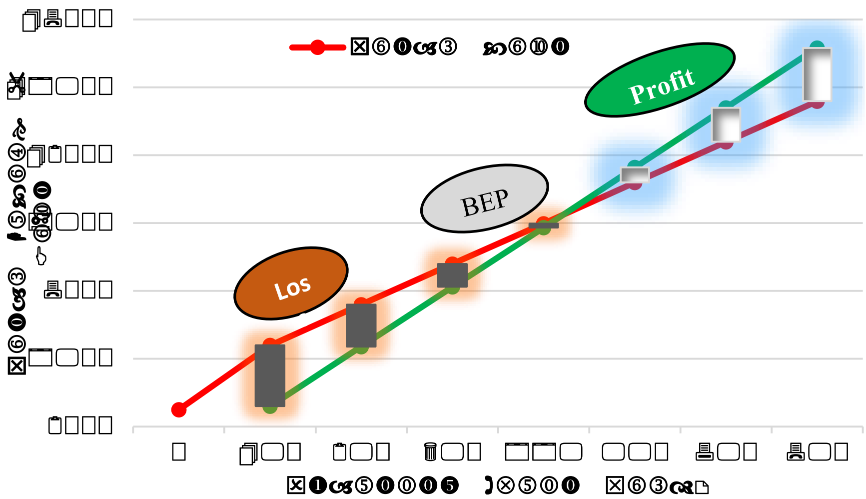

Fig. 3. Graphical representation of the break-even point

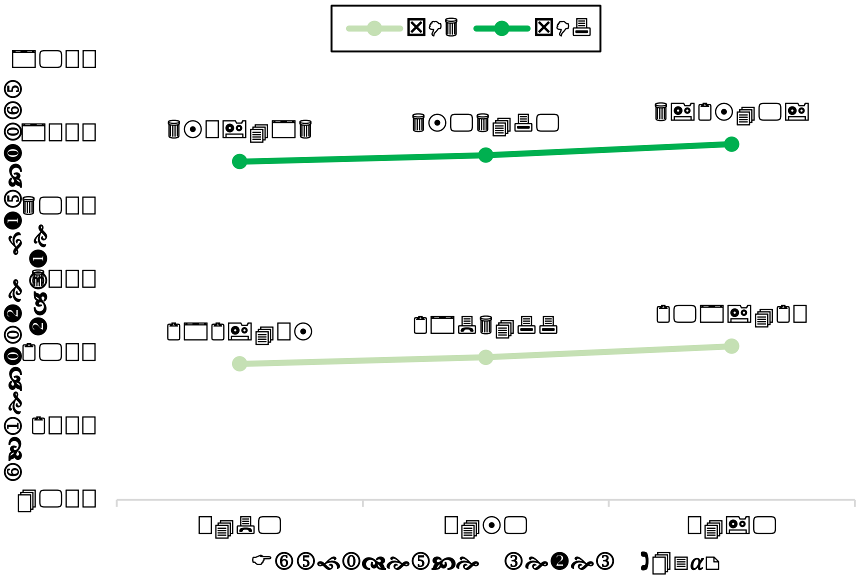

Fig. 4. The sensitivity of the proposed linearized model to the confidence level

As expected, when the confidence level $(1-\alpha)$ increases, the objective function value also increases. In other words, the confidence level $(1-\alpha)$ has a direct relationship with the objective function value.

Transfer batch size for the part k (B_k)

In order to shed more light on the proposed model, the sensitivity of the proposed linearized model to the transfer batch size (B_k) is analyzed. Table 11 presents the results of solving the proposed linearized model for the problem set *(II)*. Fig. 5 displays the relationship between the transfer batch size (B_k) and the objective function value.

Table 11. The sensitivity analysis of the proposed linearized model to increase of transfer batch size (B_k) for the problem set *(II)*.

Increment percentage of the transfer batch size (B_k)	Period	
	T=3	T=6
0%	3258.99	5103.24
50%	2190.07	3420.03
90%	1739.54	2709.88

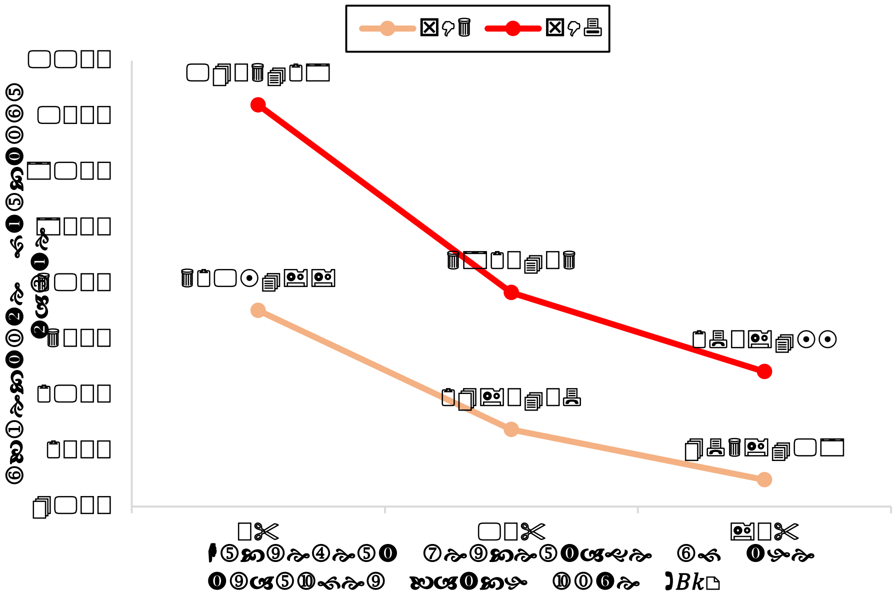

Fig. 5. The sensitivity of the proposed linearized model to the transfer batch size (B_k)

According to Fig. 5, when the transfer batch size (B_k) increases, the objective function value decreases. So, it could be said that they have inverse relationship.

Managerial insights

The facility layout affects significantly the efficiency of the production system in the real world. It is a critical matter in every new facilities construction or the rearranging of existing ones. According to the necessity of a desirable arrangement in any layout, especially in industrial zones with an uncertain environment, the impact of the proposed models is egregious.

The managers can take these models to the industrial floor to adapt to the potential alters in the competitive world's today. These can have applied in any industrial system. This paper has specified the privilege of routing flexibility. Also, the break-even point analysis has used here. So, the cost of routing flexibility is one of the most important components when making a decision.

This paper can have created practical insight for managers such as decision making about product type and product diversity, estimating the volume of the production, evaluating producibility, etc. Also, managers can make an economical decision by the proposed model and concept's this paper.

Conclusion and future works

In this paper, two new integrated mathematical models were proposed for designing a facility layout in an uncertain environment. Exploring the literature review, a considerable amount of effort was made so as to cover two found important research gaps in the Dynamic Facility Layout Problem (DFLP) area through stochastic approaches. To do so, first, the routing flexibility (RF) in the basic model was considered in which the independent part demands followed Exponential and Normal distribution functions. It is often difficult to deal with nonlinear terms due to the complexity of their solving methods. Accordingly, the proposed model was linearized with a new linearization technique based on some theoretical and numerical methods which can be enumerated as the second contribution of this paper. The suggested linearization technique works well with an average error of less than 1%. Solving the DFLP as the NP-complete problem by the exact method can be inspirational for use in other similar models. Also, two randomly generated test problems were solved to validate and verify the proposed models. The obtained results demonstrated that the discussed models have acceptable performance. Moreover, the routing flexibility effects on the manufacturing system was discussed and the sensitivity of the proposed models was then analyzed. This paper showed that the use of RF as the flexible manufacturing system (FMS) principle could reduce production costs, especially in uncertain environments. Generally, the main contributions of the paper can be summarized as follows:

- Adding the RF to the DFLP using stochastic approaches.
- Proposing a new exact linearization technique for nonlinear zero-one polynomial programming problems.
- Considering the Exponential and Normal distributions for random variables of the independent part demands, where their parameters randomly alter from period to period.
- Analyzing the RF effect on the manufacturing system.

Finally, this research can be continued in future works in the following streams:

- Considering some real constraints such as unequal-sized machines, closeness ratio, aisles as well as a budget constraint.
- To propose a new exact linearization technique for nonlinear zero-one polynomial programming
- Further investigation of the stability of the output layout by considering the confidence level as a fuzzy variable.
- Product demands can be considered dependent.
- The proposed models can be used for concurrently design of inter-cell and intra-cell layout design in the FMS.

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Appendices

Appendix A

Proof 1: According to Number theory, each positive integer number lower than or equal to $2^n - 1$ can be built by the sum of a subset of the set $\{2^0, 2^1, 2^2, \dots, 2^{n-1}\}$ which can be proved by induction. For example, 15 is equal to $2^4 - 1$ and can be built as 1+2+4+8=15. In other words, every positive integer number *x* with the upper bound of the form $UB = 2^n - 1$ can be written with the help of *n* binary variables y_i as follows:

$$x = 2^0 y_0 + 2^1 y_1 + \cdots + 2^{n-1} y_{n-1}$$
$$x = 0 \rightarrow y_0 = \cdots = y_{n-1} = 0$$
$$x = 1 = 2^0 \rightarrow y_0 = 1, y_1 = \cdots = y_{n-1} = 0$$
$$x = 2 = 2^1 \rightarrow y_1 = 1, y_0 = y_2 = \cdots = y_{n-1} = 0$$
$$x = 3 = 2^0 + 2^1 \rightarrow y_0 = y_1 = 1, y_2 = \cdots = y_{n-1} = 0$$
$$x = 4 = 2^2 \rightarrow y_2 = 1, y_0 = y_1 = y_3 = \cdots = y_{n-1} = 0$$
$$\vdots$$
$$x = 14 = 2^1 + 2^2 + 2^3 \rightarrow y_0 = 0, y_1 = \cdots = y_{n-1} = 1$$
$$x = 15 = 2^0 + 2^1 + 2^2 + 2^3 \rightarrow y_0 = y_1 = y_2 = \cdots = y_{n-1} = 1$$

Now, one can generalize the above observation to the case that the upper bound is not of the form $2^n - 1$, but it falls within $2^n - 1 < UB < 2^{n+1} - 1$. In this case, one must prove that every integer number *x* with an upper bound of UB, can be built by the help of Eq. A.1:

$$x = \underbrace{2^0 y_0 + 2^1 y_1 + \cdots + 2^{n-1} y_{n-1}}_{first\ part} + \underbrace{(UB - 2^n + 1) y_n}_{second\ part} \tag{A.1}$$

For x lower than or equal to $2^n - 1$, it has already shown that it can be built by the first part of the equation, $x = 2^0 y_0 + 2^1 y_1 + \cdots + 2^{n-1} y_{n-1}$. So, suffice it to show that every x greater than $2^n - 1$, is also possible to be built by the help of the second part, i.e., $(UB - 2^n + 1)y_n$.
It is also easy to show that the second part is always lower than or equal to the maximum value of the first part, since the maximum value of the first part is $2^n - 1$ and the supremum value of the $UB - 2^n + 1$ is $2^{n+1} - 1 - 2^n + 1 = 2^{n+1} - 2^n = 2^n$, given that x is an integer and $UB < 2^{n+1} - 1$, so, $UB - 2^n + 1 \leq 2^n - 1$.
Now, suppose that x is an integer number greater than $2^n - 1$. In order to complete the proof, suffice it to demonstrate that x can be built by Eq. A.1.
Suppose that $y_n = 1$; it means that we need to build $x - (UB - 2^n + 1)$ by the first part to complete the x. Since the upper bound of x is UB, so the term $x - (UB - 2^n + 1)$ is always lower than or equal to $2^n - 1$, therefore it is clearly possible to be built by the first part, and consequently the proof is completed.
For instance, if $UB = 20$, one can build every $x \leq 20$ by the help of the equation below:

$$x = \underbrace{2^0 y_0 + 2^1 y_1 + 2^2 y_2 + 2^3 y_3}_{first\ part} + \underbrace{(20 - 2^4 + 1) y_4}_{second\ part} = 2^0 y_0 + 2^1 y_1 + 2^2 y_2 + 2^3 y_3 + 5 y_4$$

As already depicted, for $x \leq 15$, it can be built by the first part, and for $x > 15$ we have:
$x = 16 = 11 + 5 \rightarrow y_4 = 1$, where $11 (<= 15)$ itself can be built by the first part.
$x = 17 = 12 + 5 \rightarrow y_4 = 1$, where $12 (<= 15)$ itself can be built by the first part.
$\vdots$
$x = 20 = 15 + 5 \rightarrow y_4 = 1$, where $15 (<= 15)$ itself can be built by the first part.

It is also worth noting that if x is larger than 20, it cannot be built by the equation above and therefore the upper bound, UB, is never violated.
On the other hand, since $2^n - 1 < UB < 2^{n+1} - 1$, so n can be obtained by $\lfloor log_2 (UB + 1) \rfloor$.

Appendix B

Proof 2: According to *Theorem* 1, each positive integer number x with an upper bound of UB can be written in the form of Eq. A.1:

$$x = 2^0 y_0 + 2^1 y_1 + \cdots + 2^{n-1} y_{n-1} + (UB - 2^n + 1) y_n = \sum_{i=0}^{n-1} 2^i y_i + (UB - 2^n + 1) y_n \qquad \text{(B.1)}$$

So, if we raise the two sides of the equation to power of two, then we have:

$$x^2 = \left(\sum_{i=0}^{n-1} 2^i y_i + (UB - 2^n + 1) y_n \right)^2$$

The right-hand side is obviously a binomial term which can be factorized.
After factorization, some terms of the form ${y_i}^2$ or $y_i y_j$ appear. Since y_i is a binary variable, one can replace ${y_i}^2$ and $y_i y_j$ by their linear equivalent terms y_i, and $min\ (y_i, y_j)$, respectively. The last term can be simplified by introducing a new binary variable y_{ij} and adding the below constraints:
$y_i y_j \rightarrow y_{ij}$
$y_{ij} \leq y_i$
$y_{ij} \leq y_j$
$y_{ij} \geq y_i + y_j - 1$

So, the factorization can be simplified as Eq. B.2:

$$x^2 = \sum_{i=0}^{n-1} 2^{2i} y_i + \sum_{i=0}^{n-2} \sum_{i<j}^{n-1} 2^{i+j+1} y_{ij} + (UB - 2^n + 1) \sum_{i=0}^{n-1} 2^{i+1} y_{in} + (UB - 2^n + 1)^2 y_n \quad \text{(B.2)}$$

It is straightforward to show that it is correct for $n \leq 4$.

$$\begin{aligned}
(y_0 + 2y_1 + 4y_2 + 8y_3 + Ay_4)^2 & \\
&= y_0^2 + 4y_0y_1 + 4y_1^2 + 8y_0y_2 + 16y_1y_2 + 16y_2^2 + 16y_0y_3 + 32y_1y_3 \\
&+ 64y_2y_3 + 64y_3^2 + 2Ay_0y_4 + 4Ay_1y_4 + 8Ay_2y_4 + 16Ay_3y_4 + A^2y_4^2 \rightarrow \\
&= y_0 + 4y_{01} + 4y_1 + 8y_{02} + 16y_{12} + 16y_2 + 16y_{03} + 32y_{13} + 64y_{23} \\
&+ 64y_3 + 2Ay_{04} + 4Ay_{14} + 8Ay_{24} + 16Ay_{34} + A^2y_4 \\
&= (y_0 + 4y_1 + 16y_2 + 64y_3) \\
&+ (4y_{01} + 8y_{02} + 16y_{03} + 16y_{12} + 32y_{13} + 64y_{23}) \\
&+ A(2y_{04} + 4y_{14} + 8y_{24} + 16y_{34}) + A^2y_4
\end{aligned}$$

$$= \sum_{i=0}^{3} 2^{2i} y_i + \sum_{i=0}^{2} \sum_{i<j}^{3} 2^{i+j+1} y_{ij} + A \sum_{i=0}^{3} 2^{i+1} y_{i4} + A^2 y_4$$

Where $A = UB - 2^n + 1$,

If we assume that for a given n,

$$f_n = \sum_{i=0}^{n-1} 2^i y_i + Ay_n \Rightarrow {f_n}^2 = \sum_{i=0}^{n-1} 2^{2i} y_i + \sum_{i=0}^{n-2} \sum_{i<j}^{n-1} 2^{i+j+1} y_{ij} + A \sum_{i=0}^{n-1} 2^{i+1} y_{in} + A^2 y_n \quad \text{(B.3)}$$

By using the induction proof, one can show that it is also correct for $n + 1$.

$$\begin{aligned}
{f_{n+1}}^2 &= (y_0 + 2y_1 + 4y_2 + \cdots + 2^{n-1}y_{n-1} + 2^n y_n + Ay_{n+1})^2 \\
&= (y_0 + 2y_1 + 4y_2 + \cdots + 2^{n-1}y_{n-1} + 2^n y_n + Ay_n - Ay_n + Ay_{n+1})^2 \\
&= (y_0 + 2y_1 + 4y_2 + \cdots + 2^{n-1}y_{n-1} + Ay_n + 2^n y_n - Ay_n + Ay_{n+1})^2 \\
&= (f_n + 2^n y_n - Ay_n + Ay_{n+1})^2 = (f_n + 2^n y_n + A(y_{n+1} - y_n))^2 \rightarrow \\
{f_{n+1}}^2 &= f_n^2 + 2^{n+1} f_n y_n - 2Af_n y_n + 2^{2n} y_n^2 - 2^{n+1} Ay_n^2 + A^2 y_n^2 + 2Af_n y_{n+1} \\
&+ 2^{n+1} Ay_n y_{n+1} - 2A^2 y_n y_{n+1} + A^2 y_{n+1}^2 \rightarrow \\
{f_{n+1}}^2 &= f_n^2 + 2^{n+1} f_n y_n - 2Af_n y_n + 2^{2n} y_n - 2^{n+1} Ay_n + A^2 y_n + 2Af_n y_{n+1} + 2^{n+1} Ay_{n(n+1)} \\
&- 2A^2 y_{n(n+1)} + A^2 y_{n+1} \\
&= f_n^2 + f_n y_n (2^{n+1} - 2A) + 2Af_n y_{n+1} + (A^2 - 2^{n+1} A + 2^{2n}) y_n + A^2 y_{n+1} \\
&+ y_{n(n+1)} (2^{n+1} A - 2A^2)
\end{aligned}$$

Where,

$$f_n y_n = \left(\sum_{i=0}^{n-1} 2^i y_i + Ay_n \right) y_n \rightarrow \sum_{i=0}^{n-1} 2^i y_{in} + Ay_n$$

$$f_n y_{n+1} = \left(\sum_{i=0}^{n-1} 2^i y_i + Ay_n \right) y_{n+1} \rightarrow \sum_{i=0}^{n-1} 2^i y_{i(n+1)} + Ay_{n(n+1)}$$

By substitution of the equations, we have:

$$
\begin{aligned}
{f_{n+1}}^2 = {} & \sum_{i=0}^{n-1} 2^{2i}\, y_i + \sum_{i=0}^{n-2}\sum_{i<j}^{n-1} 2^{i+j+1}\, y_{ij} + 2A\sum_{i=0}^{n-1} 2^i\, y_{in} + A^2 y_n \\
& + (2^{n+1} - 2A)\left(\sum_{i=0}^{n-1} 2^i y_{in} + A y_n\right) + A\sum_{i=0}^{n-1} 2^{i+1} y_{i(n+1)} + 2A^2 y_{n(n+1)} \\
& + (A^2 - 2^{n+1}A + 2^{2n})y_{\mathrm{n}} + A^2 y_{n+1} + y_{n(n+1)}(2^{n+1}A - 2A^2)
\end{aligned}
$$

After simplification we have:

$$
\begin{aligned}
\rightarrow {f_{n+1}}^2 = {} & \left(\sum_{i=0}^{n-1} 2^{2i}\, y_i + 2^{2n} y_{\mathrm{n}}\right) + \left(\sum_{i=0}^{n-2}\sum_{i<j}^{n-1} 2^{i+j+1}\, y_{ij} + 2^{n+1}\sum_{i=0}^{n-1} 2^i y_{in}\right) \\
& + A\left(\sum_{i=0}^{n-1} 2^{i+1}\, y_{i(n+1)} + 2^{n+1} y_{n(n+1)}\right) + A^2 y_{n+1} \\
& = \sum_{i=0}^{n} 2^{2i}\, y_i + \sum_{i=0}^{n-1}\sum_{i<j}^{n} 2^{i+j+1}\, y_{ij} + A\sum_{i=0}^{n} 2^{i+1}\, y_{i(n+1)} + A^2 y_n
\end{aligned}
$$

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