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RESEARCH PAPER

A More Human-Like Portfolio Optimization Approach: Using Utility Function to Find an Individualized Portfolio

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Abstract

In this paper, a multi-objective model based on the decision maker's (DM) utility function is proposed to find an optimized portfolio that fits with the desires of the DM. The proposed algorithm is developed in three stages. First, fundamental analysis on accounting criteria is done using the TOPSIS-DEA method. By this method, companies' efficiency ranks according their fundamental reporting sheets are achieved. Second, the specific utility function of DM is found using the UTASTAR method. Third, a two-objective model is solved to find the stocks' proportion for the individual investor. In this study, different criteria and decision making tools are used to make human-like decisions that meet investor's expectations as much as possible. This approach is illustrated in this paper by a real-world case study concerning the evaluation of stocks in the Iran stock exchange. The suggested portfolio not only made a higher level of utility with the minimum level of risk but also is consistent with the investor's interests.

Keywords:
Multiple Criteria Decision Making (MCDM);
Portfolio Optimization;
Utility Function;
Fundamental Analysis

Introduction

Decision-making can be considered as a complex task, and everyone is continuously involved in this task. To deal with the complexity of decision making in an academic context, different tools are provided by the operations research to identify preferences, create models, and solve the problem. Portfolio selection as a financial decision-making tool has attracted many researchers, experts and investors for decades. Markowitz [1] is the pioneer in portfolio selection and other researchers extend Markowitz's mean-variance bi-objective optimization problem to make it more realistic. Risk and return are the main factors of any modern portfolio, and researchers trying to incorporate them into their models. However, in the real world, the problem of selecting an attractive portfolio is a multi-criteria issue, so it seems too simplistic to make DM decide based on only two factors.

Focusing on investors' preferences and attitudes can be seen in some studies i.e. [2,3,4,5,6]. Researchers like Ehrgott et al. [7] tried to develop a primary model in different aspects incorporating utility theory into the Markowitz model. Furthermore, Merton [8] maximized the utility of terminal wealth. Fei [9], Buckley et al. [10], Bodnar et al. [11], and Ma et al. [12] selected the utility function in portfolio optimization based on the subjective degree of preferred risk. Commonly used utility functions in financial studies are of the following four forms [13]:

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$$u_1(x) = x^{\alpha} / \alpha (0 < \alpha < 1) \tag{1}$$

$$u_2(x) = \ln(x) \tag{2}$$

$$u_3(x) = (1 - e^{-\alpha x}) / \alpha (\alpha > 0) \tag{3}$$

$$u_4(x) = x - x^2 / \alpha (\alpha > 0) \tag{4}$$

Utility functions $u_1(x)$ and $u_2(x)$ are often used in situations where $c_1, . . . ,c_n$ satisfies $c_i \geq 0$ subject to $\sum_{i=1}^{n} c_i = 1$. The optimal portfolio under either $u_1(x)$ or $u_2(x)$ is $\{c_1 w, ..., c_n w\}$ and w is the initial wealth. The utility function $u_3(x)$ is often used when αw is normally distributed and $u_4(x)$ is often used in the capital asset pricing model [13].

Despite many attempts to develop the Markowitz model, few studies tried to find the DM's unique preferences and apply them in the mean-variance model directly. It is worth mentioning that all the investors are not the same and different models are required for different groups. The Mean-variance basic model is mathematically developed in different ways. Yet, the questions are that how much these models are practical in portfolio selection, and how much they consider investor's preferences and needs. Basically, more interaction between investor and consultant leads to semi human-like models aiming at satisfying the investor. A multi-criteria model based on more than two objective functions combined with an appropriate utility approach allows for higher flexibility in modeling the investors' objectives [7] which can be used to consider DM's preferences. So, a straightforward procedure will be provided in this paper with the goal of discovering and involving DM's interests in the process of decision making.

Literature review

Studies related to portfolio selection are divided based on two categories; single objective models that maximize return or minimize risk, and multi-objective models with both goals [14]. Markowitz [1] in his first work minimized risk in a certain level of return. Then Markowitz et al. [15] developed the primary model by replacing variance with semi-variance as a risk measure. Corraza and Favaretto [16] applied a proper branch-and-bound-based algorithm to solve the problem while Freitas et al. [17] presented the portfolio optimization model based on a neural network. The ‘‘Limited Asset Markowitz'' model was solved by Cesarone et al. [18]. Li et al. [19] considered a possibilistic return and risk, then they solved a fuzzy portfolio selection model. Also, some researchers like sharp [20], Chopra and Ziemba [21], Huang [22], Rios and Sahinidis [23], Zhang et al. [24], Sadjadi et al. [25], and Liu and Zhang [26] proposed portfolio optimization models with the purpose of maximizing the profit.

Instead of analyzing portfolios solely by one goal the researches of second group tried to tackle with multi-objective models (e.g. [27,28,29,30]). The goal programming approach was proposed for portfolio selection by Leung et al. [27]. Armananzas and Lozano [28] applied greedy search, simulated annealing, and ant colony optimization for multi-objective portfolio optimization problems. Chiam et al. [29] suggested an evolutionary approach for multi-objective portfolio optimization. Ammar [30] focused on a fuzzy-based multi-objective model. Greco et al. [31] modified the Mean-variance model under uncertainty and they solved the problem by using multi-criteria decision aiding and a Rough set approach. In a new multi-objective portfolio model Rather et al. [32] maximize entropy as well as gain loss spread of a portfolio. Zhao et al. [33] introduced a mean-conditional value at risk-skewness portfolio optimization model.

Several studies modified the mean-variance model and used utility functions to make the portfolio optimization problem much more similar to the real world. These studies can be categorized into two groups. First, the studies in which some common utility functions like logarithmic, exponential and power were used. The following studies can be categorized in the first group: Akian et al. [34] maximized the long-run average growth of wealth for a logarithmic utility function considering transaction cost. Ferland and Watier [35] worked on a continuous-time utility portfolio selection problem and implemented their model on power and exponential utilities. Yu et al. [36] compared four frequently used utility functions i.e. the power, logarithm, exponential and quadratic utility functions to consider their effects in portfolio selection. Çanakog˘lu and Özekici [37] used some utility functions of the HARA family which includes exponential, logarithmic, and power utility functions. They maximized the expected utility of the terminal wealth. By the assumption that investors have the logarithm utility function, Ma et al. [38] proposed an engineering model with bankruptcy control. Bodnar et al. [11] proposed a multi-period portfolio problem by applying an exponential utility function under return predictability. Sukono et al. [54] suggested a Mean-VaR portfolio optimization by risk tolerance to be used when the utility function is square-shaped.

In the second group of studies, the Multi-criteria approach is used to model the investor's preferences. Bouri et al. [4] employed 5 criteria; including the return, risk, liquidity, size, and price-earnings ratio (PER), aggregating them with PROMETHEE II. They tried to illustrate that using a multi-criteria approach can be integrated into the portfolio selection process. Ehrgott et al. [7] proposed a model based on Multi-attribute utility theory and the classical mean-variance model of Markowitz for portfolio optimization. They considered five sub-objectives for risk and return, then generated DM's specific utility functions for each of the five objectives. After adding some constraints on lower and upper bounds of assets' percentages, they also applied 2PLS, SA, TS, and a GA algorithm to solve this problem. Ehrgott et al. [5] implemented two approaches for portfolio optimization; multi-attribute utility theory (MAUT) and multi-objective programming (MOP). They found that these approaches show different results in the sequence of investor's preferences and optimization. Hurson et al. [39] applied a synergy of MACBETH and MAUT multi-criteria methods for portfolio selection. They claimed that combining these two methods improves the investor's decision-making process. Lopes and Almeida [40] focused on oil and gas development projects by employing an additive multi-attribute utility function. They considered the possibility of considering the DM's preferences and behaviors toward risk. Touni et al. [41] used the UTASTAR method to understand the DM's behavior and his preferences to give a specific insight into the financial consultant. They ranked stocks using the DM utility function. The previously mentioned studies are some examples of the second group of studies. Mendonça et al. [53], proposed an integer multi-objective mean-CVaR portfolio optimization model to approximate the investor's behavior.

Table 1 illustrates the previous researches in the area of portfolio optimization. As the Portfolio optimization problem is inherently a multi-objective problem, we decided to study both single objective and multi-objective studies in this area. To consider behavioral aspects of investor some researchers tried to use different common utility functions for investors. Indeed, MCDM approaches provide some sophisticated tools to find investors' attitudes towards every criterion.

Table 1. Selected relevant literature review

Authors	Objective Function	Utility Approach	MCDM Approach	Other Approaches	Criteria
Markowitz et al. [1952]	Min	-----	-----	-----	Variance, Return
Sharp [1967]	Max	-----	-----	Linear programming algorithm	Variance, Return
Markowitz et al. [1993]	Min	-----	-----	Critical line algorithm	Semi-variance, Return
Akian et al. [2000]	Max	Well-known utility function	-----	Logarithmic utility function	Variance, Return, Transaction costs
Leung et al. [2001]	MODM	-----	*	Goal programming	Variance, Return, Skewness
Bouri et al. [2002]	MODM	-----	*	PROMETHEE II	Return, Risk, Liquidity, Size,(PER)
Ehrgott et al. [2004]	Min	-----	*	Simulated annealing, tabu search and genetic algorithm	Five sub-objectives for risk and return
Armananzas, Lozano [2005]	MODM	-----	-----	Greedy search, simulated annealing, and ant colony optimization	Variance, Return
Corazza, Favaretto [2006]	Min	-----	-----	Branch and bound	Variance, Return
Chiam et al. [2007]	MODM	-----	-----	Evolutionary approach	Variance, Return
Ferland and Watier [2008]	Min	Well-known utility function	-----	Power and exponential utilities	Variance, Return
Ehrgott et al. [2009]	MODM	-----	*	UTADIS method	-----
Yu et al. [2009]	Min	Well-known utility function	-----	Power, logarithm, exponential and quadratic utilities	Variance, Return
Rios and Sahinidis [2010]	Max	Well-known utility function	-----	Indefinite quadratic	Variance, Return
Zhang [2011]	Max	-----	-----	Sequential minimal optimization	Possibilistic mean, Variance, Transaction costs
Hurson et al. [2012]	Min	-----	*	MACBETH and MAUT utility function	β coefficient, Marketability, Return on equity, Dividend yield, Price earnings ratio
Ma et al. [2012]	Min	Well-known utility function	-----	Logarithm utility function	Variance, Return, Transaction costs, no-shorting Constraints
Greco et al. [2013]	MODM	-----	*	Multi-criteria decision aiding and Rough set approach	VaR, CVaR, Expected shortfall
Rather et al. [2014]	MODM	-----	-----	Genetic algorithm	Entropy, Gain loss spread, Return,
Li et al. [2015]	Min	-----	-----	Fuzzy portfolio selection	Possibilistic return and risk
Sukono [2017]	Max	Well-known utility function	-----	Square-shaped utility functions	VaR, Return

Authors	Objective Function	Utility Approach	MCDM Approach	Other Approaches	Criteria
Touni et al. [2019]	Min	-----	*	UTASTAR approach	Beta, Return, Liquidity
Mendonça et al. [2020]	MODM	-----	*	Evolutionary algorithm	CVaR, Return, Cardinality constraint, Rebalancing, Transaction cost
Current research	MODM	Flexible utility function	*	UTASTAR method, TOPSIS DEA, ε-constraint method	Return, Beta, Liquidity, Efficiency, Safety, and Foreign currency dependency

The proposed algorithm in this paper is an improvement to Touni et al.'s algorithm [41], where a more comprehensive MCDM approach is developed to find the investor's utility function towards different criteria. The focus of this paper is on finding DM's preferred portfolio based on his unique utility function. Different multi-criteria methods are applied to incorporate the investor's preferences in the decision-making process. Additionally, the proposed model considers the inherent multi-dimensional nature of the problem using the augmented ε-constraint method. To illustrate the model, it is applied in a case study, including 20 firms listed in the Iran stock exchange market.

The rest of this paper is organized as follows: In Section 3, the method is explained and the proposed two-objective model is presented based on Markowitz and UTASTAR model. In Section 4, augmented the ε-constraint method is described. In Section 5, the proposed model is applied to actual data and the results are shown in Section 6. The conclusions of this manuscript are summarized in Section 7.

Method and data analysis

In this manuscript, Markowitz and UTASTAR methods are combined to allow the financial consultant to consider the investors' preferences in the process of decision making. At first, DM's utility function is discovered and his final utility function is applied as the objective of the model. Fig. 1 illustrates the proposed procedure in detail. The first step aims to assess stocks fundamentally using different accounting criteria. In the second step some market values are added to create a comprehensive criteria group, and DM's specific preferences towards each criterion are found by the UTASTAR. The third step presents a utility-risk model that is solved by the ε-constraint solution method allowing the investor to get as close as possible to his preferred stock portfolio.

Criteria selection

Nowadays, portfolio optimization models featured by criteria other than risk and return have become more common. In the real world, different people have different characteristics, so there is no standard investor. Two groups of criteria were selected based on literature review and by consulting with financial experts with more than 10 years of experience in portfolio selection/optimization. Those that are based on market value (i.e. mean return, total risk (variance), systematic risk (beta), and the stock liquidity) and those that are called the accounting criteria (ratios used by the analysts or managers) [4].

The first group of criteria can be widely used when DM wants to buy a stock and sell it after a short time. Naturally, it can be claimed the DM just want to gain some profit by stock's ups and downs in a short period. For DMs who can be attributed as real investors, the second group of criteria is also important. These DMs are interested in finding out more about the firm and its fundamental features. As traders typically adopt some combination methods to make their buy or sell decisions, this study aims to consider both groups of criteria which are valid and apprehensible for the investor.

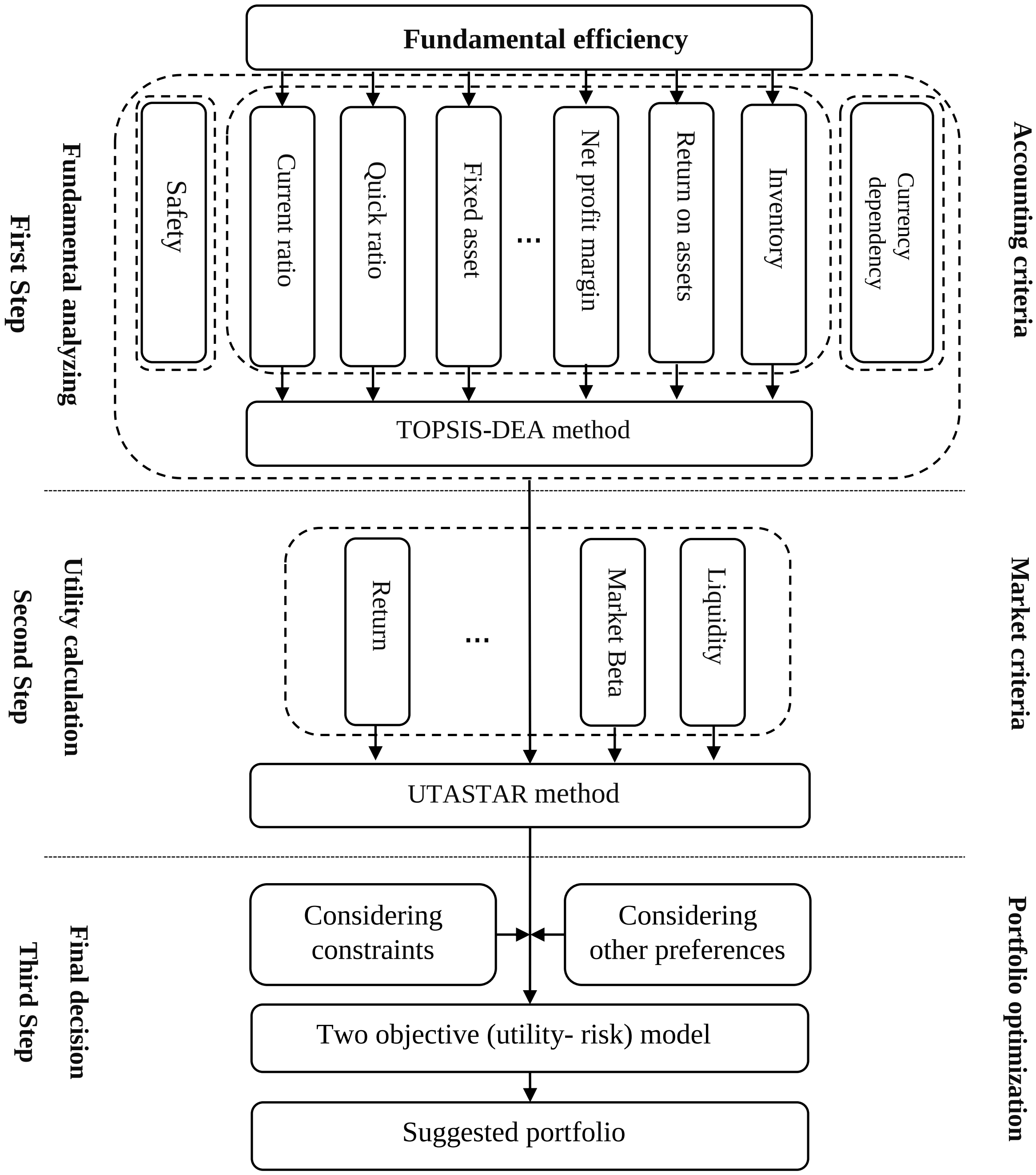

Fig. 1. Portfolio selection flowchart

By consulting with some finance experts in investment agencies we determined to consider risk, return and liquidity as the most common and necessary criteria for analyzing stocks, and decided to involve financial ratios to assess the firms fundamentally. Finally, six different criteria based on two groups (market values and accounting criteria) are considered and the importance of each category can be illustrated based on the investor's willingness for investment. Since understanding the ratios extracted from accounting sheets may not be easy for individuals, an integrated concept will be suggested in this research. Actually, without a good understanding, neither the investor can recognize if the criterion is consistent with his preferences nor consultants can expect a reliable ranking based on DM's preferences. Including the concept of efficiency would help to understand financial ratios more easily and provides a chance to integrate several criteria as an integrated concept. An explanation for each criterion is given in the following:

Return: To calculate return following formula has been employed:

$$r_{i,t+1} = \frac{p_{i,t+1} - p_{i,t} + D}{p_{i,t}} \tag{5}$$

Where $r_{i,t+1}$ is the return of stock i in the period of t, and $p_{i,t}$ refers to the price of i at the beginning of the period. Also $p_{i,t+1}$ refers to the price of i at the end of the period, D is the dividend, if during the considered period the firm divide profit it gets the amount of D, otherwise, it is 0.

Risk: There are two kinds of risks [4]; systematic risk which refers to the inherent risk of the entire market that affects the overall market, not just a particular stock or industry, and unsystematic risk the one that is unique to a specific company and can be reduced through diversification. In this manuscript, both of these risks are incorporated into the model. First, the systematic risk is calculated for each stock by the β formula to find out DM's behavior towards risk. Then the unsystematic risk is minimized in the proposed model that can be calculated using the following model where r_i refers to the return of stock i , and r_m is market return.

$$\beta = \frac{\operatorname{cov}(r_i, r_m)}{\operatorname{var}(r_m)} \tag{6}$$

Liquidity: Liquidity has become a common criterion in many papers (i.e. [4,2,42,43,44]) simply because it makes the model more realistic. The more stock transaction is in a market, the more liquidity is expected for the stock. A higher amount of liquidity is supposed to be a reliable guarantee to buy or sell the stock comfortably. In this manuscript, the transaction volume rate was considered to measure liquidity.

Efficiency: Getting information about efficiency is for the sake of investors who are committed to diving deeper into stocks information and analyzing them fundamentally. The concept of efficiency makes it easier for an investor to use accounting criteria and is helpful to reach a clear idea about the firm's financial situation. The efficiency is determined and ranked through the TOPSIS-DEA technique. TOPSIS-DEA provides two ideal unrealistic points, the best and the worst, through which we can compare realistic units and rank their efficiency based on the distances between these two ideal units. This approach seems to be very similar to human-like decision-making. The underlying assumption behind this method, which is close to the rationality of the human mind when it comes to selecting the best choice, is to opt for alternatives with the shortest distance from the best ideal solution while having the longest distance from the worst ideal solution [57]. There is a large number of financial criteria and using each of them depends on the manager's attitude and objectives. In this paper, we used debt-equity ratio, fixed asset, current asset, and cost of goods sold as the DEA inputs, and the current ratio, quick ratio, fixed asset turnover, return on assets, return on capital employed, net profit margin, and inventory turnover as the DEA outputs. TOPSIS-DEA method is as follows:

IDMU:	The best ideal solution
ADMU:	The worst ideal solution
X:	Inputs criteria
Y:	Outputs criteria

i:	Number of inputs
r:	Number of outputs
j:	Number of stocks
$v_{i:}$	The weights of the inputs
$u_{r:}$	The weights of the outputs
$y_{r:}$	Refer to r^{th} output
$x_{i:}$	Refer to i^{th} input
$y_{rj:}$	The amount of r^{th} output of j^{th} stock
$x_{ij:}$	The amount of i^{th} input of j^{th} stock
θ_I^* :	IDMU's efficiency
φ_A^* :	ADMU's efficiency
θ_j^* :	The best efficiency of stock i
φ_j^* :	The worst efficiency of stock i

$$IDMU = (X^{\min}, Y^{\max}) \tag{7}$$

$$ADMU = (X^{\max}, Y^{\min}) \tag{8}$$

Where

$$X_i^{\min} = \min\{x_{ij}\},\ \ y_r^{\max} = \max\{y_{rj}\}\ \ \forall i, r \tag{9}$$

$$X_i^{\max} = \max\{x_{ij}\},\ \ y_r^{\min} = \min\{y_{rj}\}\ \ \forall i, r \tag{10}$$

Where IDMU and ADMU refer to virtual DMUs, X and Y refer to inputs and outputs, respectively. i and r are the number of inputs and outputs, and j is the number of stocks. The first stage after creating IDMU and ADMU is to calculate their efficiency using the following models:

Model1

$$\theta_I^* = \max \sum_{r=1}^{s} u_r y_r^{\max} \tag{11}$$

$$s.t. \sum_{i=1}^{m} v_i x_i^{\min} = 1 \tag{12}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0,\ \forall j \tag{13}$$

$$u_r, v_i \ge \varepsilon \quad \forall r, i. \tag{14}$$

θ_I^* is IDMU's efficiency where v_i and u_r are the weights of the inputs and outputs, respectively.

Model2

$$\varphi_A^* = \min \sum_{r=1}^{s} u_r y_r^{\min} \tag{15}$$

$$s.t.\sum_{i=1}^{m} v_i x_i^{\max} = 1 \tag{16}$$

$$\sum_{r=1}^{s} u_r y_r^{\max} - \sum_{i=1}^{m} v_i x_i^{\min} \theta_i^* \le 0,\ \forall j \tag{17}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0,\ \forall j \tag{18}$$

$$u_r, v_i \ge \varepsilon \quad \forall r, i. \tag{19}$$

φ_A^* is ADMU's efficiency where, v_i and u_r are the weights of inputs and outputs, and constraint (17) guarantees that θ_I^* 's efficiency remains positive. Then, we have to find the best and the worst efficiency for each DMU by some changes in the DEA model, which is as follows:

Model3

$$\theta_j^* = \max \sum_{r=1}^{s} u_r y_{rj} \tag{20}$$

$$s.t.\sum_{i=1}^{m} v_i x_{ij} = 1 \tag{21}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0,\ \forall j \tag{22}$$

$$\sum_{r=1}^{s} u_r y_r^{\max} - \sum_{i=1}^{m} v_i x_i^{\min} \theta_i^* \le 0,\ \forall j \tag{23}$$

$$u_r, v_i \ge \varepsilon \quad \forall r, i. \tag{24}$$

The best efficiency for each stock must be maximized whilst the IDMU's efficiency does not change. Moreover, the worst efficiency must be minimized whilst the ADMU's efficiency does not change.

Model4

$$\varphi_j^* = \min \sum_{r=1}^{s} u_r y_{rj} \tag{25}$$

$$s.t.\sum_{i=1}^{m} v_i x_{ij} = 1 \tag{26}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0,\ \forall j \tag{27}$$

$$\sum_{r=1}^{s} u_r y_r^{\min} - \sum_{i=1}^{m} v_i x_i^{\max} \varphi_A^* \le 0,\ \forall j \tag{28}$$

$$u_r, v_i \ge \varepsilon \quad \forall r, i. \tag{29}$$

The last stage is to find the proximity of each DMU to the ideal DMU that can be measured by the following formula:

$$RC_j = \frac{\varphi_j^* - \varphi_A^*}{(\varphi_j^* - \varphi_A^*) + (\theta_I^* - \theta_j^*)}, \quad \forall \text{j} \tag{30}$$

Safety: It can be important due to creating a sense of trustworthiness in an investor that may motivate him for buying the stock. Companies release their earning per share (EPS) profit, and also they forecast it periodically. Higher realized profit shows more reliable anticipation and better performance of the firm. If there is no difference or a modest amount of difference between expected profit and realized one, the stock will probably be considered as a safer one for investment.

Foreign currency dependency: The effect of the exchange rate as a macro-economic factor on stock prices has been analyzed by many researchers (i.e. [45,46,47,48,49]). Macro-economic analysis studies companies' decisions and the behavior of those decisions on the market to determine the way in which the stock price is constructed [48]. Lam [49] used companies' financial and macroeconomic data to find out their financial performance. For this purpose, she selected 16 financial statement variables and 11 macroeconomic variables including the effective exchange rate. Naturally, the price of stocks in countries like Iran are quite dependent on common currencies like the Dollar, Pound, and Euro. Basically, some factors like oil prices, sanctions, the stability of a country's government, and inflation lead to fluctuation in foreign currency prices and will influence DM's financial decisions. In the case of oil prices, Ebrahimi [56] studied the volatility of oil price in global markets which is one of the factors that influence the capital markets of the countries of which their economy is based on oil revenues.

As, many companies in these countries have to supply the raw material in the dollar, therefore they will have to convert their local currency into dollars to make the payment, and may export their products to countries where the dollar is the common currency. So we use β to find the co-efficiency between stocks and dollar price, and shareholders can say their preferences towards it, and how many dependency they prefer or whether their portfolio should include these kinds of stocks or not.

UTASTAR

The UTASTAR algorithm [50] is an improvement of the original UTA method that infer decision models and turn preferences' data into a ranked list of options. This approach is called preference disaggregation in literature. The LP model is as follows:

Model5

$$\min z = \sum_{k=1}^{m} (\sigma^+(a) + \sigma^-(a)) \tag{31}$$

$s.t.$

$$\begin{cases} \Delta(a_k, a_{k+1}) \geq \delta & if \;\; a_k > a_{k+1} \\ \Delta(a_k, a_{k+1}) = 0 & if \;\; a_k \sim a_{k+1} \end{cases} \tag{32}$$

$$\sum_{i-1}^{n} \sum_{j-1}^{a_i -1} w_{ij} = 1 \tag{33}$$

$$\sigma^+ \geq 0 \;\; , \;\; \text{w}_{ij} > 0 \tag{34}$$

For more details on UTA family algorithms, one may refer to the monograph by Siskos et al. [50]. Chapter 8 of the mentioned book is devoted entirely to UTA methods.

The Markowitz risk-utility model

In this section, the ultimate utility function combined with the previously mentioned criteria is maximized while the unsystematic risk of the portfolio is minimized. In the proposed model, i and j indicates the stocks, where x_i shows the percentage of the money needed for investment in these stocks, p_i can be zero or one. And constraint (40) illustrates the logical relationship between p_i and x_i. The model will be as follows:

Z_1:	Utility function
Z_2:	Unsystematic risk
i , j:	Refer to stocks
x_i:	the percentage of the money to invest in stock i
u_i:	Utility of i^{th} stock for the investor
r_i:	Return of i^{th} stock
p_i:	zero or one variable
σ_{ij} :	Covariance i and j
n:	The number of stocks in portfolio
Return:	Investor's desired return of portfolio

Model6

$$\max z_1 = \sum_{i=1}^{n} x_i u_i \tag{35}$$

$$\min z_2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \tag{36}$$

s.t.

$$\sum_{i=1}^{n} x_i r_i \geq return \tag{37}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{38}$$

$$\sum_{i=1}^{n} p_i = n \tag{39}$$

$$p_i \geq x_i \quad \forall i \tag{40}$$

$$x_i \geq 0 \quad \text{i=1,2,...,n} \quad p_i = 0 or 1 \tag{41}$$

Solution approach

The augmented ε-constraint method (AUGMECON) which is introduced by Mavrotas [51] is one of the most practical algorithms in Multi-Objective Mathematical Programming (MOMP). Generally, these problems do not have a single optimal solution that optimizes all the objective functions at the same time. In MOMP there is not optimality, instead, we have Pareto optimality or efficiency [51].

Case study

The data of the Iran stock exchange market were used as a data source. There are some criteria through which the best firms are announced every season. This manuscript used 20 of them based on whether all the required data is accessible to implement the proposed model. Return, liquidity, and β are forecasted by the artificial neural network for the next 30 days, and a three-layer perceptron 4-15-1 is employed. Besides, last year information extracted from the financial statements was used to measure efficiency and safety. Table 2 gives information about stock's efficiency ranking where stocks are ranked by TOPSIS-DEA method:

Table 2. TOPSIS-DEA information

Firms ID	θ_j^*	φ_j^*	RC	rank	Firms ID	θ_j^*	φ_j^*	RC	rank
1	0.052	0.01	0.000224	12	**12**	0.011	0.007	0.000139	16
2	0.053	0.007	0.00014	15	**13**	0.383	0.055	0.00149	1
3	0.017	0.006	0.000112	17	**14**	0.04	0.01	0.00022	13
4	0.559	0.054	0.001475	2	**15**	1	0.002	0	20
5	0.411	0.004	0.000057	19	**16**	0.687	0.04	0.00108	4
6	1	0.038	0.001	5	**17**	0.053	0.026	0.000671	7
7	0.009	0.005	0.000084	18	**18**	0.122	0.026	0.000673	6
8	1	0.007	0.00014	14	**19**	0.148	0.013	0.0003	10
9	0.376	0.011	0.00025	11	**20**	0.053	0.051	0.0013	3
10	0.085	0.017	0.00042	9	**IDMU**	35.751			
11	1	0.018	0.00046	8	**ADMU**		0.002		

Table 3. Stocks' values of the criteria

firms ID	return	Beta	liquidity	Efficiency rank	safety	FCD	firms ID	return	Beta	liquidity	Efficiency rank	safety	FCD
1	0.0172	0.12	5.7543	12	0.448	0.564	**11**	0.0701	0.16	0.0579	8	1.4496	0.545
2	0.0007	0.44	2.1527	15	1.589	0.962	**12**	0.0511	0.69	52.946	16	0.6713	0.568
3	0	0.46	46.212	17	0	0.723	**13**	0.459	0.45	0.8399	1	0.6214	0.222
4	0.1275	0.05	3.094	2	0.588	0.670	**14**	0.1122	0.78	17.777	13	0.756	0.322
5	0.0015	0.08	3.2332	19	0.752	0.936	**15**	0.0198	0.3	1.2266	20	0.9593	0.649
6	0.0904	0.18	1.3709	5	0.674	0.504	**16**	0.0009	0.58	0.8001	4	0.7518	0.094
7	0.0133	0.42	21.167	18	0	0.020	**17**	0.2578	0.68	12.344	7	0.7340	0.744
8	0	0.28	2.6143	14	0.194	0.292	**18**	0.0009	0.19	0.0432	6	0.8525	0.830
9	0.0133	0.42	0.9292	11	0.411	0.221	**19**	0.2885	0.21	27.596	10	0.9318	0.891
10	0.0006	0.58	21.59	9	0	0.604	**20**	0.1306	0.14	1.4053	3	0.6506	0.969

The criteria are separated into two groups to be more comprehensible for the investor to compare and rank, and also, he is able to mention his interests towards each group by weighting them. Then, for each group of criteria, a sample is considered to start finding utility function by the UTASTAR method. The following equation shows the weights of each market criteria resulted from the investor's ranking preferences:

$$U_M(g) = 0.265 \times U_1(g_1) + 0.331 \times U_2(g_2) + 0.402 \times U_3(g_3) \qquad (42)$$

By using CurveExpert, the marginal utility functions are approximated as follows:

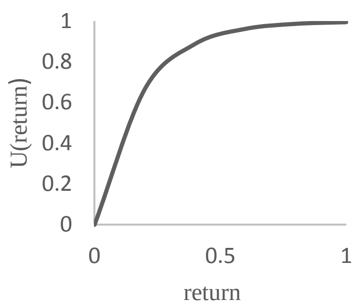

Fig. 2. Return marginal utility

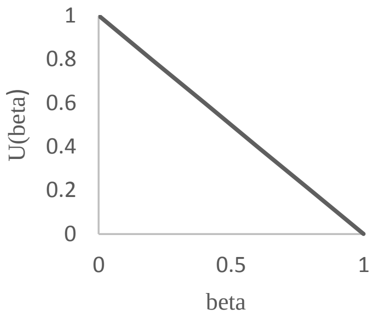

Fig. 3. Beta marginal utility

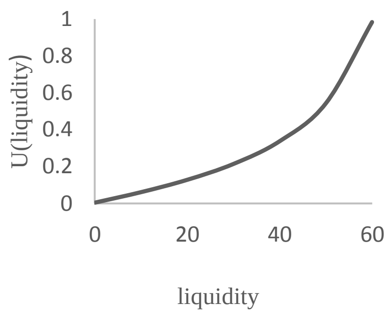

Fig. 4. Liquidity marginal utility

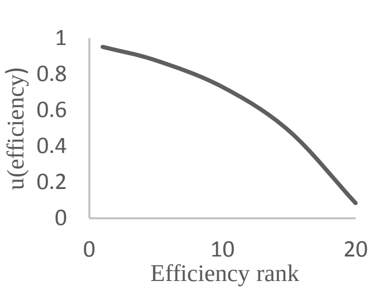

Fig. 5. Efficiency rank marginal utility

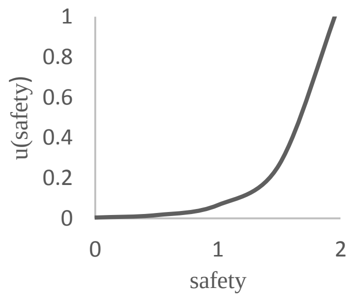

Fig. 6. Safety marginal utility

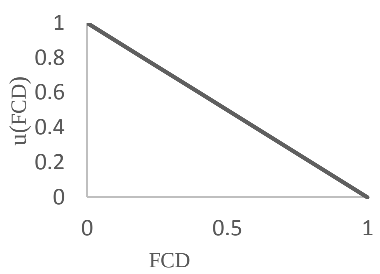

Fig. 7. FCD marginal utility

$$U_M(g) = 0.265 \times (1 - e^{-5.5R}) + 0.331 \times (1 - B) + 0.402 \times \frac{0.0056(1+L)}{1-0.004L} \tag{43}$$

Eq. 43 is the ultimate market utility function related to return, beta, and liquidity criteria, this procedure is done for the remained criteria which reaches to the Eq. 44:

$$U_F(g) = 0.331\big(0.138 \times (8 - e^{0.1 \times FER})\big) + 0.264(0.004e^{2.8 \times S}) + 0.402(1 - FCD) \tag{44}$$

Figs. 2, 3, 4, 5, 6 and 7 illustrate the estimated marginal utility function of each criterion for the investor. In Eq. 43 R, B, and L refer to return, beta, and liquidity, respectively. Also, FER, S, and FCD are fundamental efficiency rank, safety, and foreign currency dependency respectively. The final utility is as following equation, where w can be determined based on investor's attitudes towards each group of criteria:

$$U_{final}(g) = w \times U_F(g) + (1 - w) \times U_M(g) \tag{45}$$

Computational Results and discussion

The first step is to calculate the payoff table, this is done through lexicographic optimization and the results are shown in Table 4. First, we optimize utility (higher priority), then the unsystematic risk objective is optimized by adding the constraint f1=z1*.

Table 4. Lexicographic optimization results

	f_1	f_2
f_1	0.896	$910.9607*10^{-6}$
f_2	0.845	$4*10^{-6}$

After calculating the payoff table, we divide objective functions' ranges to eight equal intervals, then nine grid points are used as the values of e_2 in the augmented ε-constraint method. The results are shown in Table 5:

Table 5. Pareto optimization results

row	F2	F1	row	F2	F1	row	F2	F1
1	$910.96*10^{-6}$	0.896	4	$570.82*10^{-6}$	0.887	7	$230.68*10^{-6}$	0.872
2	$797.58*10^{-6}$	0.893	5	$457.44*10^{-6}$	0.882	8	$117.3*10^{-6}$	0.866
3	$682.20*10^{-6}$	0.89	6	$344.06*10^{-6}$	0.878	9	$4*10^{-6}$	0.846

For each Pareto solution the investment percentages are presented in Table 6:

Table 6. Final results

row	Unsystematic risk	Utility	Stock proportion				
1	910.96*10-6	0.896	X4=0.01	X6=0.01	X9=0.01	X11=0.02	X13=0.95
2	797.58*10-6	0.893	X4=0.01	X6=0.01	X9=0.01	X11=0.081	X13=0.889
3	682.20*10-6	0.89	X4=0.01	X6=0.01	X9=0.01	X11=0.147	X13=0.823
4	570.82*10-6	0.887	X4=0.01	X6=0.01	X9=0.01	X11=0.217	X13=0.753
5	457.44*10-6	0.882	X4=0.01	X6=0.01	X9=0.01	X11=0.295	X13=0.675
6	457.44*10-6	0.878	X4=0.01	X6=0.01	X9=0.01	X11=0.385	X13=0.585
7	457.44*10-6	0.872	X4=0.01	X6=0.01	X9=0.01	X11=0.491	X13=0.479
8	457.44*10-6	0.866	X4=0.01	X6=0.01	X9=0.01	X11=0.62	X13=0.35
9	457.44*10-6	0.846	X4=0.171	X6=0.01	X11=0.721	X13=0.088	X20=0.01

As Table 5 illustrates, firms 4, 6, 9, 11, and 13 are in many efficient portfolios. Let us analyze these stocks and see if the suggested portfolios are consistent with DM's preferences. Basically, if an individual's utility function is concave, linear, or convex, then the individual is risk-averse, risk-neutral, or risk-seeking, respectively [52]. So Figs. 2, 3 and 4 show that investor is strongly risk-averse towards return, risk-neutral to systematic risk, and mildly risk-seeking to liquidity. In terms of return, the utility of the middle is much more than 0.5, and the average of return values is about 0.074, it can be understood that even a low amount of return may satisfy the investor. Unlike return, the investor tends to have a higher level of liquidity in his portfolio due to the risk-seeking aspect of his behavior towards this criterion. By comparing selected stocks' values with the average of liquidity (i.e. 3.58) it can be concluded that except stock 11 other ones got good values. For both systematic risk and foreign currency dependency with an average of 0.13 and 0.76 respectively, the investor tends to be risk-neutral. As we can see most of the selected stocks are near or lower than averages which shows the stocks are correctly selected by the model. According to Fig. 6 investor is strongly risk-seeking to safety with an average of 0.54. Table 7 shows that safety quantities in the selected stocks are quite consistent with his preferences (higher quantities for safety are more preferable for the investor). Even investor is almost risk-averse towards efficiency, so the model suggested the efficient stocks to investors to increase his total utility value of the portfolio.

Table 7. Selected stocks' values

Firm ID	return	beta	liquidity	efficiency	safety	FCD
4	0.1275	0.05	3.094	2	0.588325	0.6707
6	0.0904	0.18	1.3709	5	0.674133333	0.5045
9	0.0133	0.42	0.9292	11	0.4115	0.2215
11	0.0701	0.16	0.0579	8	1.449625	0.5452
13	0.459	0.45	0.8399	1	0.621475	0.2228
20	0.1306	0.14	1.4053	3	0.6506	0.9693

Conclusion

This paper proposed a reliable procedure to make a portfolio optimization model similar to the real world. Besides, in this research, we tried to make financial concepts easier to understand for an investor who plays an important role in the investment process. Then investor's preferences are recognized through precise and comprehensive methods. The proposed model considers the return, systematic risk, liquidity, efficiency, safety, and foreign currency dependency as important factors in stock selection.

The proposed decision-making method and its combination with the optimization process is the main innovation of this manuscript. The results showed a high level of utility with minimized unsystematic risk at a certain level of appropriate return. Besides, the suggested portfolios showed complete compatibility with investor's preferences. Since investors in the stock market always decide to choose a portfolio for the uncertain future [55], so the uncertainty of variables for future decisions should also be considered in the model. Furthermore, using dynamic optimization or multi-period approaches and comparing the results can be worthwhile.

References

[1] H. Markowitz, (1952). Portfolio selection*, J. Finance 7, 77–91.
[2] Xidonas, P., Mavrotas, G., and Psarras, J. (2010). A multiple criteria decision-making approach for the selection of stocks. Journal of the Operational Research Society, 61(8), 1273-1287.
[3] Raei, R., and Jahromi, M. (2012). Portfolio optimization using a hybrid of fuzzy ANP, VIKOR and TOPSIS. Management Science Letters, 2(7), 2473-2484.
[4] Bouri, A., Martel, J. M., and Chabchoub, H. (2002). A multi-criterion approach for selecting attractive portfolio. Journal of Multi-Criteria Decision Analysis, 11(4-5), 269-277.
[5] Ehrgott, M., Waters, C., Kasimbeyli, R., and Ustun, O. (2009). Multiobjective programming and multiattribute utility functions in portfolio optimization. INFOR: Information Systems and Operational Research, 47(1), 31-42.
[6] Tamiz, M., and Azmi, R. A. (2019). Goal programming with extended factors for portfolio selection. International Transactions in Operational Research, 26(6), 2324-2336.
[7] Ehrgott, M., Klamroth, K., and Schwehm, C. (2004). An MCDM approach to portfolio optimization. European Journal of Operational Research, 155(3), 752-770.
[8] Morton, A. J., and Pliska, S. R. (1995). Optimal portfolio management with fixed transaction costs. Mathematical Finance, 5(4), 337-356.
[9] Fei, W. (2007). Optimal consumption and portfolio choice with ambiguity and anticipation. Information Sciences, 177(23), 5178-5190.
[10] Buckley, I., Saunders, D., and Seco, L. (2008). Portfolio optimization when asset returns have the Gaussian mixture distribution. European Journal of Operational Research, 185(3), 1434-1461.
[11] Bodnar, T., Parolya, N., and Schmid, W. (2015). On the exact solution of the multi-period portfolio choice problem for an exponential utility under return predictability. European Journal of Operational Research, 246(2), 528-542.
[12] Ma, G., Siu, C. C., and Zhu, S. P. (2019). Dynamic portfolio choice with return predictability and transaction costs. European Journal of Operational Research, 278(3), 976-988.
[13] Yu, B. W. T., Pang, W. K., Troutt, M. D., and Hou, S. H. (2009). Objective comparisons of the optimal portfolios corresponding to different utility functions. European Journal of operational research, 199(2), 604-610.
[14] Rather, A. M., Sastry, V. N., and Agarwal, A. (2017). Stock market prediction and Portfolio selection models: a survey. Opsearch, 54(3), 558-579.
[15] Markowitz, H., Todd, P., Xu, G., and Yamane, Y. (1993). Computation of mean-semivariance efficient sets by the critical line algorithm. Annals of Operations Research, 45(1), 307-317.

[16] Corazza, M., and Favaretto, D. (2007). On the existence of solutions to the quadratic mixed-integer mean–variance portfolio selection problem. European Journal of Operational Research, 176(3), 1947-1960.

[17] Freitas, F. D., De Souza, A. F., and de Almeida, A. R. (2009). Prediction-based portfolio optimization model using neural networks. Neurocomputing, 72(10-12), 2155-2170.

[18] Cesarone, F., Scozzari, A., and Tardella, F. (2013). A new method for mean-variance portfolio optimization with cardinality constraints. Annals of Operations Research, 205(1), 213-234.

[19] Li, T., Zhang, W., and Xu, W. (2015). A fuzzy portfolio selection model with background risk. Applied Mathematics and Computation, 256, 505-513.

[20] Sharpe, W. F. (1967). A linear programming algorithm for mutual fund portfolio selection. Management Science, 13(7), 499-510.

[21] Chopra, V. K., and Ziemba, W. T. (1993). The Effect of Errors in Means, Variances, and Covariances on Optimal Portfolio Choice. The Journal of Portfolio Management, 19(2), 6-11.

[22] Huang, X. (2007). Two new models for portfolio selection with stochastic returns taking fuzzy information. European Journal of Operational Research, 180(1), 396-405.

[23] Rios, L. M., and Sahinidis, N. V. (2010). Portfolio optimization for wealth-dependent risk preferences. Annals of Operations Research, 177(1), 63-90.

[24] Zhang, X., Zhang, W. G., and Xu, W. J. (2011). An optimization model of the portfolio adjusting problem with fuzzy return and a SMO algorithm. Expert Systems with Applications, 38(4), 3069-3074.

[25] Sadjadi, S. J., Gharakhani, M., and Safari, E. (2012). Robust optimization framework for cardinality constrained portfolio problem. Applied Soft Computing, 12(1), 91-99.

[26] Liu, Y. J., and Zhang, W. G. (2015). A multi-period fuzzy portfolio optimization model with minimum transaction lots. European Journal of Operational Research, 242(3), 933-941.

[27] Leung, M. T., Daouk, H., and Chen, A. S. (2001). Using investment portfolio return to combine forecasts: a multiobjective approach. European Journal of Operational Research, 134(1), 84-102.

[28] Armananzas, R., and Lozano, J. A. (2005, September). A multiobjective approach to the portfolio optimization problem. In 2005 IEEE Congress on Evolutionary Computation (Vol. 2, pp. 1388-1395). IEEE.

[29] Chiam, S. C., Al Mamun, A., and Low, Y. L. (2007, September). A realistic approach to evolutionary multiobjective portfolio optimization. In 2007 IEEE Congress on Evolutionary Computation (pp. 204-211). IEEE.

[30] Ammar, E. E. (2008). On solutions of fuzzy random multiobjective quadratic programming with applications in portfolio problem. Information Sciences, 178(2), 468-484.

[31] Greco, S., Matarazzo, B., and Słowiński, R. (2013). Beyond Markowitz with multiple criteria decision aiding. Journal of Business Economics, 83(1), 29-60.

[32] Rather, A. M., Sastry, V. N., and Agarwal, A. (2014, September). Portfolio selection using maximum-entropy gain loss spread model: a GA based approach. In 2014 International Conference on Advances in Computing, Communications and Informatics (ICACCI) (pp. 400-406). IEEE.

[33] Zhao, S., Lu, Q., Han, L., Liu, Y., and Hu, F. (2015). A mean-CVaR-skewness portfolio optimization model based on asymmetric Laplace distribution. Annals of Operations Research, 226(1), 727-739.

[34] Akian, M., Sulem, A., and Taksar, M. I. (2001). Dynamic Optimization of Long-Term Growth Rate for a Portfolio with Transaction Costs and Logarithmic Utility. Mathematical Finance, 11(2), 153-188.

[35] Ferland, R., and Watier, F. (2008). FBSDE approach to utility portfolio selection in a market with random parameters. Statistics and probability letters, 78(4), 426-434.

[36] Yu, B. W. T., Pang, W. K., Troutt, M. D., and Hou, S. H. (2009). Objective comparisons of the optimal portfolios corresponding to different utility functions. European Journal of operational research, 199(2), 604-610.

[37] Çanakoğlu, E., and Özekici, S. (2010). Portfolio selection in stochastic markets with HARA utility functions. European Journal of Operational Research, 201(2), 520-536.

[38] Ma, Q. H., Yao, H. X., and Li, S. Y. (2012). Logarithm Utility Maximization Portfolio Engineering with Bankruptcy Control: a Nonparametric Estimation Framework. Systems Engineering Procedia, 5, 150-155.
[39] Hurson, C., Mastorakis, K., and Siskos, Y. (2012). Application of a synergy of MACBETH and MAUT multicriteria methods to portfolio selection in Athens stock exchange. International Journal of Multicriteria Decision Making 7, 2(2), 113-127.
[40] Lopes, Y. G., and de Almeida, A. T. (2015). Assessment of synergies for selecting a project portfolio in the petroleum industry based on a multi-attribute utility function. Journal of Petroleum Science and Engineering, 126, 131-140.
[41] Touni, Z., Makui, A., and Mohammadi, E. (2019). A MCDM-based approach using UTA-STAR method to discover behavioral aspects in stock selection problem. International Journal of Industrial Engineering and Production Research, 30(1), 93-103.
[42] Gupta, P., Mehlawat, M. K., and Saxena, A. (2010). A hybrid approach to asset allocation with simultaneous consideration of suitability and optimality. Information Sciences, 180(11), 2264-2285.
[43] Barak, S., Abessi, M., and Modarres, M. (2013). Fuzzy turnover rate chance constraints portfolio model. European Journal of Operational Research, 228(1), 141-147.
[44] Li, J., and Xu, J. (2013). Multi-objective portfolio selection model with fuzzy random returns and a compromise approach-based genetic algorithm. Information Sciences, 220, 507-521.
[45] Acikalin, S., Aktas, R., and Unal, S. (2008). Relationships between stock markets and macroeconomic variables: an empirical analysis of the Istanbul Stock Exchange. Investment Management and Financial Innovations, 5(1), 8-16.
[46] Peiro, A. (2016). Stock prices and macroeconomic factors: Some European evidence. International Review of Economics and Finance, 41, 287-294.
[47] Owusu-Nantwi, V., and Kuwornu, J. K. (2011). Analyzing the effect of macroeconomic variables on stock market returns: Evidence from Ghana. Journal of Economics and International Finance, 3(11), 605-615.
[48] Tirea, M., and Negru, V. (2014, September). Intelligent stock market analysis system-a fundamental and macro-economical analysis approach. In 2014 16th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing (pp. 519-526). IEEE.
[49] Lam, M. (2004). Neural network techniques for financial performance prediction: integrating fundamental and technical analysis. Decision support systems, 37(4), 567-581.
[50] Siskos, Y., and Yannacopoulos, D. (1985). UTASTAR: An ordinal regression method for building additive value functions. Investigaçao Operacional, 5(1), 39-53.
[51] Mavrotas, G. (2009). Effective implementation of the ε-constraint method in multi-objective mathematical programming problems. Applied mathematics and computation, 213(2), 455-465.
[52] Greco, S., Figueira, J., and Ehrgott, M. (2016). Multiple criteria decision analysis. New York: Springer.
[53] Mendonça, G. H., Ferreira, F. G., Cardoso, R. T., and Martins, F. V. (2020). Multi-attribute decision making applied to financial portfolio optimization problem. Expert Systems with Applications, 158, 113527.
[54] Sukono, Sidi, P., Bon, A. T. B., and Supian, S. (2017, March). Modeling of Mean-VaR portfolio optimization by risk tolerance when the utility function is quadratic. In AIP Conference Proceedings (Vol. 1827, No. 1, p. 020035). AIP Publishing LLC.
[55] Raeiszadeh, S., Dehghan Dehnavi, M., Bahrololoum, M., Peymany Foroushany, M. (2020). Portfolio Selection Optimization Problem Under Systemic Risks. *Advances in Industrial Engineering*, 54(2), 121-140. doi: 10.22059/jieng.2021.321882.1759.
[56] Ebrahimi, S. (2016). Robust Estimation in Nonlinear Modeling of Volatility Transmission in Stock Market. Advances in Industrial Engineering, 50(2), 165-176. doi: 10.22059/jieng.2016.60722.
[57] Aria, S., Torabi, S., Nayeri, S. (2020). A Hybrid Fuzzy Decision-Making Approach to Select the Best online-taxis business. Advances in Industrial Engineering, 54(2), 99-120. doi: 10.22059/jieng.2021.320051.1754.