Advances in Industrial Engineering, Winter 2021, 55(1): 47-68
DOI: 10.22059/jieng.2021.325375.1776
RESEARCH PAPER

An L-Shaped Method to Solve a Stochastic Blood Supply Chain Network Design Problem in a Natural Disaster

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Received: 19 June 2021, Revised: 17 July 2021, Accepted: 17 July 2021

Abstract

The level of blood and blood products in the human body is very important. Lack of blood and its products in a body can have a deadly effect on health. Due to the explanation, managing the supply of blood and blood products is a critical issue in the healthcare system, especially when the system is faced with high demand for the product. In natural disasters, demand for blood units increases sharply because of injuries. Hence, efficiency in blood supply chain management plays a significant role in this situation in supplying blood for transfusion centers and hospitals. It is vital to supply at in right time to prevent casualties. The present paper proposes an optimization model for designing a blood supply chain network in case of an earthquake disaster. The proposed two-stage stochastic model is programmed based on scenarios for an earthquake in a populated mega-city. The designed network has three layers; the first layer is donation areas, the second layer consists of distribution centers and facilities, and the last layer is transfusion centers. In the present study, a two-stage stochastic optimization model based on real scenarios is proposed. Decisions of locating permanent collection facilities and the amount of each blood type pre-inventory are made in the first stage, and operation decisions that have dependent on possible scenarios are made in the second stage. The model also considers the possibility of blood transfusion between different blood groups and its convertibility to blood derivatives regarding medical requirements. In order to solve the proposed two-stage stochastic model, an L-shaped algorithm, an efficient algorithm to solve scenario-based stochastic models, has been used. In addition, the application of the model and the algorithm tests with real data of likely earthquakes in the Tehran mega-city (the densest city of Iran).

Keywords:
Supply Chain Network Design;
Blood Supply Chain;
Stochastic Optimization;
Natural Disaster;
Blood Type;
L-Shaped Exact Method

Introduction

Design and management of blood supply networks are among the most important topics studied in supply chain management and modeling literature. Important decisions are made in the process of supply chain design, such as optimal locating, optimal capacity planning in each location, and demand market's optimal locations allocation, assuring the minimum costs [1]. It is supposed in static supply chains that all the decisions, such as capacities and locations, are determined and fixed during the planning phase, despite the change in other parameters. However, in dynamic supply chains, some of the decisions change during the planning

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regarding the demand in different periods. The planning itself is divided into several periods [2]. Blood supply chains are among the most important issues concerning a crisis or a natural disaster. A crisis is defined as "any event leading to environmental damage, destruction, human life loss, or human suffering and endangers the health or health services in a way that the affected areas by the event would require relief and response provided by their surrounding areas" [3]. The crises have several phases, including prevention, preparation, response, and recovery.

Recent crises have shown various effects on blood use and supply in critical situations. For instance, in the previous earthquake in Bam, Iran, the blood supply was very inadequate compared to the demand, and the blood supply chain's functionality was very ineffective. Other cases of the blood supply chain inadequacy in critical circumstances are found in Sri Lanka [4] and Tokyo [5].

Critical situations and their intensity highly influence the function of the chain. Therefore, it is vitally important to design a supply chain network in which crisis conditions and the unique nature of the blood are considered, especially to reduce post-disaster casualties.

Human blood has unique features differentiating it and its supply chain from other products and supply chains. The first and foremost feature of blood is its limited durability; its second important characteristic is that it can be supplied only by donation. Another structural feature of this product is its derivatives, including platelets, red blood cells, and plasma, which can be extracted from it. Each of these components is valuable and has various uses. For example, platelets can be used for cancer patients [6].

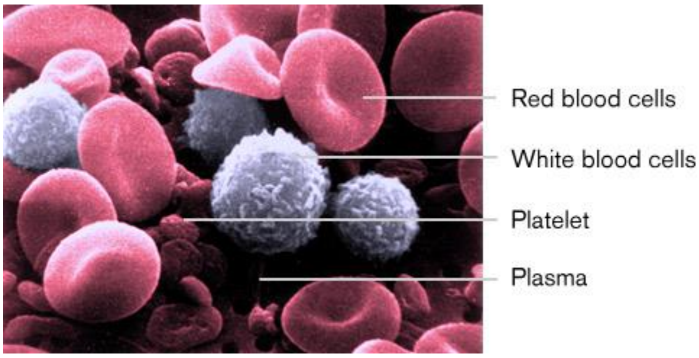

Fig 1. Microscopic photograph of blood components [7]

Another characteristic of blood is that it is a multi-product such that there are various blood types, and the inter-transfusion between these types is unique. Blood types are classified based on the existence or the absence of certain inheritable antigens on the surface of the red blood cells. According to the commonest classifications, human blood is divided into four types, i.e., AB, B, A, and O. According to another classification based on the existence of Rh molecule, there are positive and negative blood types [7].

Same as other products, blood has a supply chain whose management consists of collecting and delivering it to the customers at the right amount, right place, and the right time. The importance of time and amount is immediate, as it is related to human life. Another distinctive feature of blood is its supply. Since it is donated by humans voluntarily, its supply chain and distribution are quite different from other products [8].

A substantial challenge following an earthquake is providing the injured with blood in affected areas. Iran is among the most earthquake-prone countries [9]. Iran's capital, Tehran, is the third-largest city in the Middle East and has a population of 16 million in the metropolitan areas, which is divided into 22 municipal districts. The city is considered an impending place for severe earthquakes. Given its population density, size, and complexity, blood supply chain management during a severe earthquake and emergency situation is absolutely necessary [10].

Literature review

According to a literature review in 2013 on blood and blood products models optimization, the studies in this field are concentrated on optimizing the blood supply chain at the first two levels, i.e., hospitals or local blood databases, focusing on stock optimization and stock control models. However, models of blood supply chains are rare, and among which, the papers that study the network design are even rarer. Furthermore, according to the literature review of the dynamic location, very few papers have studied the supply chain network utilizing a dynamic approach [11].

Among supply chains, blood supply chain management is highly important; the reason is obvious: the product involved in this system is human blood, which can only be provided by collecting the blood from the voluntary donation (human blood cannot be produced in a laboratory outside the human body yet). Therefore, its appropriate management is vital. Another distinguishing characteristic is its limited lifetime. It expires after a certain time and has to be disposed of, as it has no other use [12].

Human blood has a lifetime of 21 days, which can be increased to 35 days by special packaging. After this period, it has to be disposed of. Therefore, its usage and storage must be managed appropriately.

Managing a blood bank includes different stages, such as collection, testing, separation, and distribution. The first stage, i.e., collecting the donors' blood, happens after a thorough examination of donors by physicians. One blood unit is taken as well as a few test tubes of blood used to define the blood type of the donor. After the donation, the donor's health is again examined.

A blood unit refers to a blood package, which holds about 0.5 liters of blood. Each of the models proposed in the literature is usually for one product, which is often the whole blood or red blood cells. Most of the studies on blood bank management fall into three categories [13]:

1. Managing the blood bank of a hospital
2. Managing local blood bank
3. Managing blood banks at the supply chain level

Based on the explanations above, one of the most important perishable products is human blood, which is classified into eight groups, and the transfusion compatibility between the groups is known. Blood consists of different components, the most important of which are red blood cells, platelets, and plasma. These components can be derived from the whole blood by an operation. Blood has a fixed, limited lifetime of 35 days, and after this period, it has to be discarded. The products studied by most researchers are the whole blood and the red blood cells, both with a lifetime of 35 days. The only source of blood in the human body and has to be taken from the volunteers. This product is collected and kept in blood packages in blood centers, hospitals, or mobile units in the form of blood units. After the collection, certain tests are conducted on it, and if it passes the tests, it is sent to the demand centers, that is, the hospitals.

Uncertainty, unpredictability, and dynamic decision-making are the major issues in supply chain problems. Unverified or insufficient information about an earthquake's time, magnitude, and actual location, as well as road access to the affected areas, the severity of the injury, and the consequent amount of blood required makes the decision-making more difficult than ever [14].

Hospitals have some demands every day that they have to meet them. When demand is sent to a hospital, the required number of units with the suitable blood type is chosen and undergo a matching test. After passing the test, they are queued for injection. Some of the blood units are chosen and tested are not used; thus, they are sent back to the blood bank. Since blood products are critical in treating patients, management of blood banks is of utmost significance, both socially and economically, for both patients and hospitals. The present issue is highlighted when realizing that studies have shown the blood availability should be improved in the near future, particularly in the developed countries, due to its increasing demand due to population aging [7]. Therefore, authorities and researchers are focusing on two ways to fulfill this need: (I) effective bloodstock management and (II) improvement of blood production policies. WHO examined 124 countries, among which only in 49 countries, 100% of the donated blood units were received for free [8]. It shows the need for changing blood collection policies. Conversely, common techniques, including the just-in-time technique, are not appropriate for blood collection management. The blood shortage consequences are quite different from other products shortage due to its direct connection with human life. Managing and supplying blood components can be examined at three different levels: the hospital level where a decision-maker is a person such as the one in charge of the hospital blood bank; the local level where several hospitals are covered by a blood center, and the decisions are made by the manager of the center [7]; and finally, the trans-local level which actually manages a supply chain and is responsible for providing coordination among a number of local blood centers, each of which supplies some hospitals covered by it. At each level, the decision-maker faces some problems, which lead to many problems in modeling subsequently. Perishability of blood and the way to collect blood are among such problems. There are also other issues mentioned in the literature, e.g., the need for using blood components or answering the question of whether the collected blood should be decomposed into its components or not. Another issue is the existence of various blood types, which, if considered, makes the model multi-product and adds to the problems. There are much more problems in modeling the issues of blood management that are out of the scope of this paper. One of the major problems facing the researchers is to define the criterion or the aim of the problem.

This section discusses a few studies performed on the blood supply chain considering all of its stages, such as blood collection, stock tests, and location. Moreover, considering that mobile facilities are used in many areas, a short review of this issue is also provided. As an example, [11] has conducted such a study. He investigated the supply chain and used each area's population as the demand; he supposed that the demand of each area is related to the populations and the proposed model consists of three hierarchical problems. The first one was a model of the central location in which the mean weight minimized the distance between the points and the blood center. The weight used in this model was each area's population. The second subproblem was a set cover problem determining the new blood center's location. With solving these problems, the number of existing and new blood centers are determined, and the aim of the last problem is to reallocating the existing and new units to each demand area. Thus, allocation to the areas is according to the area. In another study [12] proposed a blood center location model, which is similar to the set cover problem. He suggested three objective functions for the problem. The first was minimizing the fixed costs needed for blood centers establishment. The second was minimizing the total covered distances between the blood centers and hospitals, which were calculated as Euclidean distances. The third objective function was minimizing the coefficient of variation to uniform the blood centers distribution. This objective function to locate the blood bank sought minimization of the distance between blood centers and the hospitals as much as possible. The three objective functions were modeled by goal programming, whose limitation was included in the model. In a study by Gunpinar [13], not only the method of allocating the stock of the blood center to different hospitals was determined, but also a vehicle routing problem was solved to examine the blood collection method by mobile units. Jabbarzadeh [15] suggested a model for designing a supply chain network considering the crisis situation and scenario-making. The model used real data of Tehran city while locating a blood collection center in a crisis situation. The facilities were also considered to have two types, and the mobile facilities were also used in the model. Nagurney [16] studied the blood supply chain network generally and considered all of the chain's components, including collection points, processing and storage facilities, and distribution facilities and hospitals. The models also considered different aspects, such as optimum allocation, supply risk that can influence the system, and the amount of waste that could enter the system by extra supply [17] considered a model of locating blood supply chain networks allocation in emergency states. The model included a fine for the shortage state and stock state. Mobile facilities were also considered for blood collection. Farrokhizadeh et al. [14] suggested a multi-period, bi-objective, mixed-integer mathematical model under multiple scenarios, considering blood group compatibility matrix in a case study on a possible earthquake in the European Side of Istanbul. The model is designed to minimize the unsatisfied blood demand and total cost of the network.

Fazli-Khalaf et al. [18] proposed a model with robust possibilistic flexible chance constraint programming (RPFCCP) to optimize three objectives, i.e., minimizing total costs of the supply chain, minimizing total transportation time between facilities, and maximizing the total reliability of the tested blood at laboratories. The presented mathematical had five 5s: blood collection centers, blood donors, blood centers, laboratories, and hospitals. Ghasemi et al. [19] offered mixed integer mathematical programming to minimize the shortage, as well as the total costs of the location and facilities allocation. The Modified Multi-Objective Particle Swarm Optimization (MMOPSO) approach was used as a solution to solve the problem in a real case study on Tehran, including five levels, i.e., the affected areas, hospitals, distribution, temporary care centers, and temporary accommodation. Khalilpourazari et al. [9] presented a blood supply chain model for a case study on the deadliest earthquake of 2017 (in the Iran–Iraq border) with six levels and two kinds of collection facilities, that is, temporary and permanent centers. The research used a neural-learning process on the aforesaid data obtained from the experiences of past real earthquakes. Asadpour et al. [20] proposed a bi-objective Mixed Integer Programming (MIP) solution to design a green blood supply chain. Investigations on COVID-19 demonstrate that the plasma of the people who have recovered from the virus infection can help other patients in this case. The perishability of plasma leads to constraints in this study, and the model attempts to minimize the total cost and the waste in the designed network.

Karadağ et al. [21] presented a novel multi-objective mixed-integer location-allocation model for the BSC network design problem. Their considered objective was minimizing distances between the blood supply chain elements and the length of the mobile unit routes. The objectives are prioritized by experts using the Analytical Hierarchical Process (AHP). GhatrehSaman et al. [22] proposed a dynamic, robust location-allocation model for the problem under uncertainty and with disruptions in the operational phase, respectively.

Kamyabniya et al. [23] propose an integrated logistics network to share multi-type platelet. This model allocates multi-type platelets to patients according to ABO/Rh(d)-compatible blood substitutions and considers a three-layer logistics network for platelets that accounts for the impact of the age of the platelets on the suitability for different types of injuries with Lagrangian relaxation and the augmented -constraint method. Multiple scenarios with a case study based on data about a possible earthquake in Tehran were considered. Soltani et al. [24] developed a blood supply chain network in disasters using the hub location approach. A mixed-integer linear programming model based on hub location is proposed for intercity transportation. Two new methods were developed based on Particle Swarm Optimization and Differential Evolution algorithms to solve practical-sized problems and presented a two-stage stochastic programming approach for blood supply planning following disasters which may be helpful in inventory decisions in hybrid uncertainty, to minimize the wastages and shortages. Hamdan and Diabat [25] proposed a bi-objective robust optimization model for the design of the blood supply chain.

Two-stage stochastic optimization model with Lagrangian relaxation-based algorithm presented in a real case study of blood banks in Jordan.

A summary of the relevant studies to this paper has been presented in Table 1 to clear contributions related to the considered problem and solution approaches.

This study proposes a two-stage scenario-based optimization model for disaster management based on the historical features of a potential earthquake in Tehran, Iran. The model was established for multi-periods and multi-products where plasma and platelet are two important products that may be separated from the whole blood units when the disaster happens. Moreover, the compatibility of blood groups is considered.

Table 1. Summary of literature on BSC network design

Articles	*Network design*	*Blood groups*	*compatibility blood groups*	*Blood derivatives*	*Real case*	*Disaster*	*Multi periods*	*Multi products*	*Uncertainty*	*Scenario-making*	*# Of layers*	*Cost*	*Dynamic transportation*	*Unsatisfied demands*
Sahin, 2007	☐				☐	☐		☐	☐		3		☐	
Cetin E. 2009						☐			☐		2	☐		
Gunpinar 2013)	☐			☐				☐			4		☐	
Farrokhizadeh et al., 2021	☐					☐	☐		☐		3		☐	☐
Williamson, 2013				☐		☐	☐	☐			-		☐	☐
Rabinowits 1973		☐		☐		☐		☐			-			
(Jabbarzadeh 2014	☐				☐	☐	☐		☐	☐	3	☐	☐	☐
M. A. Nagurney A. 2012	☐					☐			☐		7		☐	☐
Sha Y. 2012	☐	☐				☐	☐				3	☐		☐
Fazli-Khalaf et al. (2017)	☐				☐	☐			☐	☐	5	☐	☐	
Ghasemi et al. (2019)	☐				☐	☐			☐		5	☐	☐	☐
Khalilpourazari et al. (2019)	☐				☐	☐	☐		☐		6	☐	☐	☐
Asadpour et al. (2020)	☐	☐		☐	☐	☐					3	☐		☐
Kamyabniya, et al. 2021	☐	☐		☐	☐	☐		☐	☐	☐	3	☐	☐	☐
Soltani et al. 2021	☐	☐			☐	☐		☐	☐		4		☐	
Hamdan and Diabat 2020	☐				☐		☐	☐			4	☐	☐	
Karadağ et al. 2021	☐	☐			☐	☐		☐	☐	☐	4	☐	☐	
GhatrehSaman et al., 2020	☐				☐	☐	☐		☐		4	☐	☐	☐
Present study	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	3	☐	☐	☐

A review of the previous related studies indicates that (as the best knowledge of authors), no study has been done on using an L-Shaped algorithm to solve the mentioned defined problem. In order to solve the proposed two-stage stochastic model, the L-shaped algorithm, an efficient algorithm to solve scenario-based stochastic models, has been used considering the possibility of blood transfusion between different blood groups and its convertibility to blood derivatives under 8 real scenarios for an earthquake in Tehran.

Proposed mathematical model

Two-stage stochastic model and robust optimization

The proposed model exploits uncertainty. Uncertainty in some parameters can affect the variables of the decision-making model in a way that the created demand for blood affects the decisions of the supply chain network when a crisis starts. Various scenarios are used to classify this uncertainty [26]. One of the most common stochastic models is the two-staged scenario-based stochastic planning. The most important feature of this model is the division of decisions into two stages. The decision-maker makes a decision in the first stage; then, a random event happens, and according to the decision of the first stage, a return decision (second stage decision) is made, which tries to compensate for the potential undesired effects of the decision in the first stage. The reason why it is called a two-staged planning problem is that it is not necessary to make the decisions of the first and second stages simultaneously [27]. The second stage decisions can be delayed till an event happens, and the uncertainty is removed.

Formulating the model of designing the blood supply chain network

The supply chain consists of numerous facilities, such as the blood donators in blood donation areas, the blood collection facilities, and the centers of blood transfusion. The blood donation areas constituting the supply chain's first layer include various areas in Tehran; Also, blood supply is done based on the calculated data for the crisis circumstances. The supply chain's second layer involves the blood collection facilities of various donation areas. The collection facilities may be classified as 1) High-capacity centers for blood collection (permanent facilities) with a certain distance from each other, and 2) mobile or temporary facilities including the buses of the Iranian Blood Transfusion Organization that collect the donated blood and are able to go to various areas with limited blood collection capacity. Finally, the last layer is the blood transfusion centers with numerous responsibilities, such as blood testing, processing, storage, transportation, and distribution. The proposed model is a multi-period one, which includes a one-day and a two-day period. The uncertainty in parameters and the decision variables of the model has also been considered according to different scenarios. The assumptions that govern the model are as follows:

- There can be permanent facilities for blood collection in different areas of Tehran.
- The capacity of blood collection facilities and blood transfusion facilities is limited.
- The blood derivatives do not exist in the stock at the onset of the crisis.
- The stock is not kept at the collection facilities; it is merely stored at the blood transfusion center.
- Demands for different blood types and derivatives include uncertainty and depend on diverse scenarios.
- The maximum amount of supply in all areas includes uncertainty.
- The post-crisis planning period may be divided into two distinct periods, which have been intended to respond up to 3 days following the crisis, based on the Crisis Management Organization the announcement. These two distinct periods include the first days and the two following days.
- Demand happens immediately following the crisis, and the unfulfilled demands in the first period are transferred to the second period.

- Each blood unit may be converted into a plasma unit and a platelet unit in a crisis situation.
- A portion of the kept blood is the pre-stock of the new blood and can be used for the next 72 hours.

Decisions of the problem

The proposed model makes the decisions in two stages. The most important decisions belong to the first stage and include the permanent facilities' locations and the amount of stored pre-stock. The rest of the decisions belong to the second stage.

- The Location of permanent facilities (first stage decision variable)
- The amount of pre-positioned blood unit of each blood type (first stage decision variable)
- The location of temporary facilities in the first period after the crisis occurred.
- Allocating facilities to various areas of Tehran in which the donation is made
- The quantity of blood that should be collected for meeting the demands
- The number of blood stocks required for each blood group at the end of each period
- The amount of transferred blood derivative for each blood group at the end of each period (includes plasma and platelet)
- The quantity of blood conversion to blood derivatives pertinent to demand
- The quantity of blood demand that should be supplied from outside the network at the end of each period (it can also be called a shortage)
- The amount of transferred demand to the next period that is not met at the end of the first period

Indices used in the modeling

I	Set of areas that are intended as blood donation locations that represented by i
J	Set of potential areas for location of facilities, represented by j or l
S	Set of scenarios related to the crisis occurrence, represented by s
T	Set of post-crisis periods, represented by t
K	Set of transfusion centers, represented by k

Parameters of the problem

f_j	The fixed cost required for the establishment of a blood collection facility in location *j*
Ce_{ijt}^{s}	The establishment and transportation cost for temporary facility between location *j* and *l*
Ca_{ijt}^{s}	The cost required for transportation of 1 blood unit from the collection point *j* to the center of blood transfusion *k*
h_k	Storing cost of one blood unit in the transfusion center *k*
hp_k	Storing cost of one platelet unit in the transfusion center *k*
hf_k	Storing cost of one plasma unit in the transfusion center *k*
hs_k	Storing cost of one blood unit in the transfusion center *k* as pre-stock
dr_k^s	Supplying cost of one blood unit from outside the network in scenario *s*
db_k^s	Blood demand in scenario *s* in the transfusion center *k*
dp_k^s	Platelet demand in scenario *s* in the transfusion center *k*
dfp_k^s	Plasma demand in scenario *s* in the transfusion center *k*
P_s	Probability of the occurrence of scenario *s*
M	A very big number
d_{ij}	The distance between area *j* and *i*
R	Maximum coverage distance by the collection points
Mb_i^s	The maximum quantity of suppliable blood from area *i* in scenario *s*

Cp_{jt}^{s}	The permanent facilities' capacity in point *j* in period *t* in scenario *s*
Ct_{jt}^{s}	The temporary facilities' capacity in point *j* in period *t* in scenario *s*
Cb_{k}	The capacity of the center of blood transfusion *k*
Cp_{k}	The capacity of the center of blood transfusion *k* for platelet
Cf_{k}	The capacity of the center of blood transfusion *k* for plasma

X_{j}	Binary variable; being equal to 1, if the permanent facility is located in area *j* and 0 is 1; otherwise.
Z_{jit}^{s}	Binary variable; being equal to 1, if the temporary facility is in area *l* in period *t*-1 and moves to area j by period *t* and 0 otherwise.
Y_{ijt}^{s}	Binary variable; being equal to 1, if area *i* is assigned to collection to area *j* and 0 otherwise.
o_{ijt}^{s}	Cost of blood collected from point *j* in area *i*, at period *t* in scenario *s*
Q_{ijkt}^{s}	The quantity of blood collection from point *j* in area *i*, at period *t*, and delivered to the transfusion center *k* in scenario *s*
$Conv_{kt}^{s}$	The quantity of blood conversion to blood derivatives in the transfusion center *k*, at period *t*, in scenario *s*.
Ib_{kt}^{s}	The quantity of bloodstock of blood group *g*, in the transfusion center *k*, in scenario *s*, at the end of period *t*.
Ip_{kt}^{s}	The quantity of platelet stock, in the transfusion center *k* in scenario *s*, at the end of period *t*.
If_{kt}^{s}	The quantity of plasma stock, in the transfusion center *k*, in scenario *s*, at the end of period *t*.
ST_{k}^{t}	The amount of blood stocked in the transfusion center *k* and period *t*, before the crisis.
Ub_{kt}^{t}	The blood demand for blood group *g* left unmet during the first period in the transfusion center *k* and moving to the next period in scenario *s*
Up_{kt}^{t}	The platelet demand for blood group *g* left unmet during the first period in the transfusion center *k*, moving to the next period in scenario *s*
Uf_{kt}^{t}	The amount of the plasma demand left unmet in the first period in the transfusion center *k*, moving to the next period in scenario *s*
Out_{k}^{s}	The blood supply amount in scenario *s* from external resources at the end of the planning period

First, different parts and different costs are explained, the total of which is aimed to be minimized:

The total costs of establishment of the permanent blood facilities in different areas:

$$FC = \sum_{j} f_{j} X_{j} \tag{1}$$

The cost of keeping pre-stock before the time of crisis:

$$SC = hs_{k} ST_{k} \tag{2}$$

The total cost of transportation of the blood collection temporary facilities:

$$VC_{s} = \sum_{j}\sum_{l}\sum_{t} Ce_{jlt}^{s} Z_{jlt}^{s} \tag{3}$$

The total blood collection operational costs in different areas:

$$OC_{s} = \sum_{i}\sum_{j}\sum_{k}\sum_{t\in T} o_{ijt}^{s} Q_{ijkt}^{s} \tag{4}$$

The total cost of keeping blood and blood derivatives in different periods:

$$IC_s = \sum_k \sum_t (h_k Ib_{kt}^s + hp_k Ip_{kt}^s + hf_k If_{kt}^s) \tag{5}$$

The total blood supply costs from outside the network in case of shortage:

$$DC_s = \sum_k dr_k^s * Out_k^s \tag{6}$$

With ξ_s representing the total costs and ξ_s^* denoting the optimal amount for each scenario in a deterministic state, the model and its components are as follows:

$$\min \xi_s = FC + SC + \sum_{s \in S} p_s (VC_s + OC_s + TC_s + IC_s + DC_s) \quad \forall s \in S \tag{7}$$

$$ST_k + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} Q_{ijkt}^s - Conv_{kt}^s - db_k^s + Ub_{kt}^s = Ib_{kt}^s \quad t=1; \forall s \in S \tag{8}$$

Blood's inventory balance at the end of the first period is shown in Eq. 8, in a way that the amount of the blood converted to its derivatives and the demand during the period are subtracted from the supplied and pre-stock blood inventory level with surplus or shortage being transferred to next period.

$$Ib_{kt-1}^s + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} Q_{ijkt}^s - Conv_{kt}^s + Out_{kt}^s - Ib_{kt}^s = Ub_{kt-1}^s \quad t=2; \forall s \in S \tag{9}$$

Eq. 9 shows the stock balance for blood at the end of the second period. Besides the input and output shown in Eq. 8, the amount of blood that enters the chain at the second period from outside the chain network is also considered, with the shortage being supplied from other provinces near Tehran.

$$\alpha_1 Conv_{kt}^s + Up_{kt}^s - dp_k^s = Ip_{kt}^s \quad t = 1; \forall k \in K \ \& \ \forall s \in S \tag{10}$$

Eq. 10 shows the stock balance for platelet, one of the blood derivatives, at the end of the first period. In this model, it is assumed that one platelet unit can be extracted from one blood unit, and the shortage or surplus at the end of the first period is transferred to the second period.

$$\alpha_1 Conv_{kt}^s + Up_{kt}^s = Ip_{kt}^s \quad t=2; \forall k \in K \ \& \ \forall s \in S \tag{11}$$

Eq. 11 expresses that all the platelet demand must be answered at the end of the second period, even if it has to be supplied from outside the chain.

$$\alpha_2 Conv_{kt}^s + Up_{kt}^s - df_k^s = If_{kt}^s \quad t = 1; \forall k \in K \ \forall s \in S \tag{12}$$

Eq. 12 shows the stock balance for plasma.

$$\alpha_2 Conv_{kt}^s + Uf_{kt}^s = If_{kt}^s \quad t=2; \forall k \in K \ \& \ \forall s \in S \tag{13}$$

Eq. 13 is for the second period when all the demand for plasma should be met.

$$\sum_{l \in j} Z^{s}_{ljt} \leq \sum_{l \in j} Z^{s}_{jlt-1} \quad \forall l \in J \,\&\, \forall j \in J \,\&\, \forall t \in T \,\&\, \forall s \in S \tag{14}$$

Eq. 14 ensures that the transportation of a temporary facility is possible from area *j* to area *l* only if the temporary facility occurs in area *j* in period *t*.

$$Y^{s}_{ijt} \leq X_{j} + \sum_{l \in j} Z^{s}_{jlt} \quad \forall i \in I \,\&\, \forall j \in J \,\&\, \forall t \in T \,\&\, \forall s \in S \tag{15}$$

Eq. 15 ensures that the assignment of the donation areas to the collection area j happens only if either a temporary or a permanent collection facility is already present in the area j .

$$\sum_{j \in J} \sum_{k \in K} \sum_{t \in T} Q^{s}_{ijkt} \leq Mb^{s}_{i} \quad \forall i \in I \,\&\, \forall j \in J \,\&\, \forall t \in T \,\&\, \forall k \in K \,\&\, \forall s \in S \tag{16}$$

Eq. 16 ensures that the amount of blood collected from each area cannot surpass the maximum suppliable amount in the area.

$$d_{ij} Y^{s}_{ijt} \leq d \quad \forall i \in I \,\&\, \forall j \in J \,\&\, \forall t \in T \,\&\, \forall s \in S \tag{17}$$

Eq. 17 is related to the covering areas limitation by collection points such that the distance between areas does not exceed the maximum allowable covering distance.

$$Q^{s}_{ijkt} \leq MY^{s}_{ijt} \quad \forall i \in I \,\&\, \forall j \in J \,\&\, \forall t \in T \,\&\, \forall k \in K \,\&\, \forall s \in S \tag{18}$$

Eq. 18 ensures that blood collection from a point occurs only if in the case of assignment of the area to the collection facility.

$$\sum_{g \in G} \sum_{i \in I} \sum_{k \in K} Q^{s}_{ijkt} \leq Cp^{s}_{jt} X_{j} + Ct^{s}_{jt} * \sum_{l \in j} Z^{s}_{jlt} \quad \forall i \in I \,\&\, \forall j \in J \,\&\, \forall t \in T \,\&\, \forall k \in K \,\&\, \forall s \in S \tag{19}$$

Eq. 19 expresses that the amount of collection from various areas may not exceed the assigned temporary or permanent facility's capacity.

$$Ib^{s}_{kt} \leq Cb_{k} \quad \forall t \in T \,\&\, \forall k \in K \,\&\, \forall s \in S \tag{20}$$

Eq. 20 guarantees that the amount of blood stocked at the end of any period in the blood transfusion center may not surpass the center's capacity.

$$Ip^{s}_{kt} \leq Cp_{k} \quad \forall t \in T \,\&\, \forall k \in K \,\&\, \forall s \in S \tag{21}$$

$$If^{s}_{kt} \leq Cf_{k} \quad \forall t \in T \,\&\, \forall k \in K \,\&\, \forall s \in S \tag{22}$$

Eqs. 21 and 22 ensure that the amount of plasma and platelet stocked in the blood transfusion center may not exceed the center's capacity for these products.

$$ST^{s}_{kt} \leq Cb_{k} \quad \forall t \in T \,\&\, \forall k \in K \,\&\, \forall s \in S \tag{23}$$

Eq. 23 ensures that the amount of pre-stock inventory kept in the blood transfusion center may not exceed the center's capacity.

$$X_j \in \{0,1\} \qquad \forall j \in J \tag{24}$$

$$Z_{jlt}^{s} \in \{0,1\} \qquad \forall j \in J \ \& \ \forall l \in J \ \& \ \forall t \in T \ \& \ \forall s \in S \tag{25}$$

$$Y_{ijt}^{s} \in \{0,1\} \qquad \forall j \in J \ \& \ \forall l \in J \ \& \ \forall t \in T \ \& \ \forall s \in S \tag{26}$$

$$Q_{ijkt}^{s} \geq 0 \qquad \forall i \in I \ \& \ \forall j \in J \ \& \ \forall t \in T \ \& \ \forall k \in K \ \& \ \forall s \in S \tag{27}$$

$$Ib_{kt}^{s}, Ip_{kt}^{s}, If_{kt}^{s} \geq 0 \qquad \forall t \in T \ \& \ \forall k \in K \ \& \ \forall s \in S \tag{28}$$

$$Ub_{kt}^{s}, Up_{kt}^{s}, Uf_{kt}^{s}, Out_{kt}^{s} \geq 0 \qquad \forall t \in T \ \& \ \forall k \in K \ \& \ \forall s \in S \tag{29}$$

And finally, Eqs. 24-29 show the definition domain for decision variables.

Solving procedure and Algorithm explanation (Step by Step)

According to the explanation of the L-Shaped model methodology in the appendix.1, the algorithm used in this model is described as follows:

1. Initialization; $s = 0, k1 = 0, k2 = 0, \phi = -$ □
2. The main model's solving and obtaining the objective functions values and ϕ and the first stage variables
3. Inserting the first-stage variables as parameters in the feasibility problem
4. If $s = S$, proceed to step 7, otherwise set $s = s + 1$ and solve the model for scenarios
5. If the objective function value of the feasibility problem is zero, proceed to step 4; otherwise, go to step 6
6. Setting $k1 = k1 + 1$, $s = 0$, calculating the feasibility cut parameters and applying feasibility cut k1 to the main problem, and returning to step 2
7. Setting $s = 0$, putting the first stage variables in the optimality problem
8. If $s = S$, go to step 10; otherwise, go to step 9
9. Setting $s = s + 1$, solving the optimality problem for scenario s and saving the value of its objective function, and returning to step 8
10. If $\sum_{s} p(s) \times Zopt(s) = \varphi$ the answer is optimal, otherwise go to step 11
11. Setting $k2 = k2 + 1$, $s = 0$, calculating the feasibility cut parameters and applying feasibility cut $k2$ to the main problem returning to step 2.

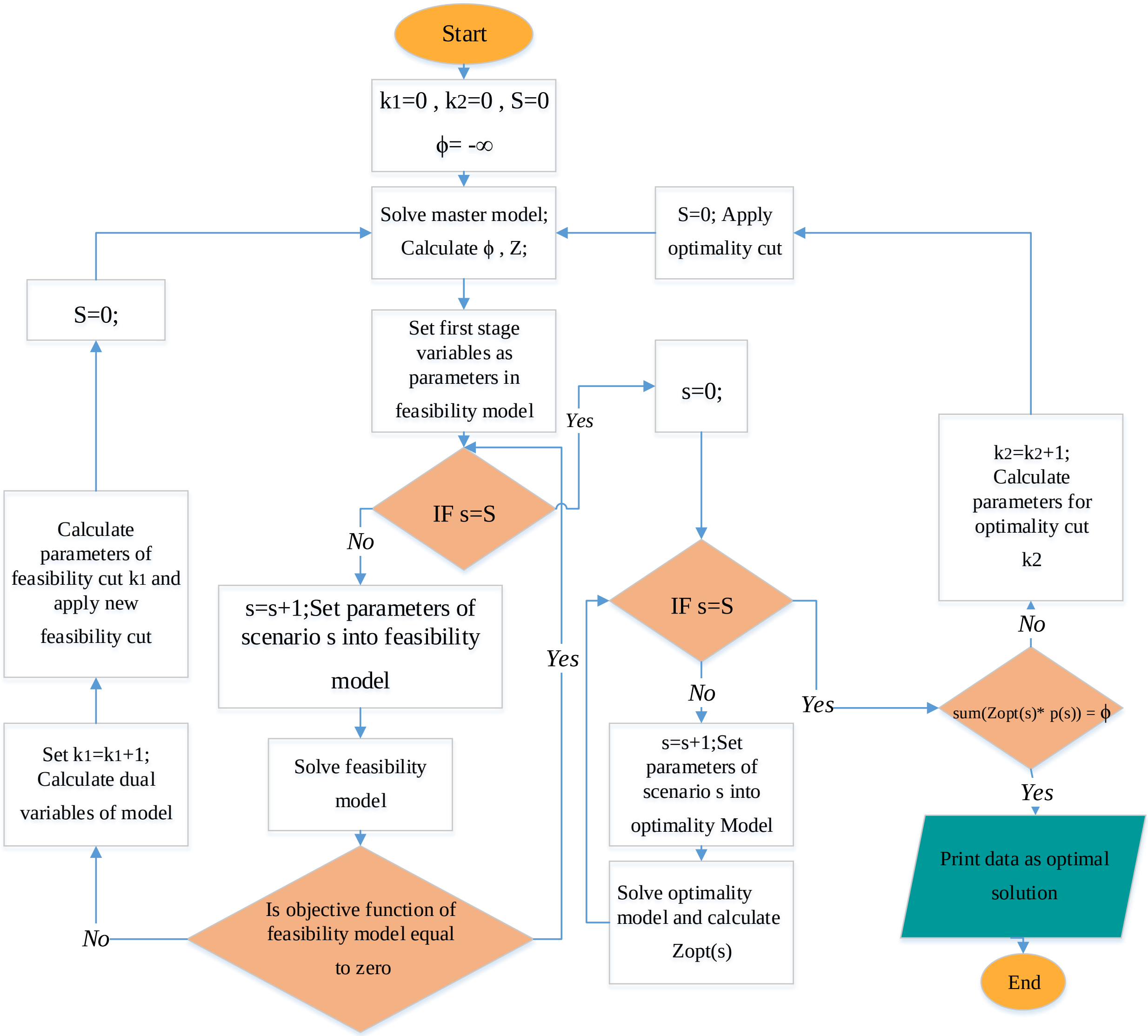

Fig 2. Flowchart of the proposed L-shaped method.

Applying the model

The suggested model was solved using a laptop of model Asus X450 and “GAMS” software using the L-Shape method. As for the data, real data from Tehran Seismic Research Center was considered in the model, which was first presented by Golmohamadi in his thesis. Scenarios were designed considering the active faults of Tehran based on which the possibility and the damages following each one was calculated.

Case study and scenarios

According to both historical and recorded earthquakes in Iran, the Alborz region is an earthquake-prone region in Iran, where Tehran is located. Tehran is surrounded by some active faults [28]. Among several active faults in this region, the ones with the highest risk and probability are as follows:

- Masha fault (with an approximate length of 200 km)
- Tehran fault in the north (with an approximate length of 90 km)
- Rey fault in the south (with an approximate length of 20 km)
- Floating fault

There could be hidden faults under Tehran's margins. If so, their precise location is very difficult, and their probable existence does not differ throughout Tehran.

For considering the mentioned issue in the calculations, we considered the Floating model**.** Thus, besides the three faults mentioned above, we may consider the floating fault as the fourth fault.

Seismic intensity

The modified Mercalli intensity (MMI) scale is based on quake damages and perceptions endured by human beings when experiencing an earthquake. Numerous studies have investigated the relationship between seismic waves' physical parameters and the seismic intensity, including velocity, acceleration, and spectral intensity. In the current study, the theories of Trifonic and Brady (1975) were accepted based on which the result for the aforementioned faults are as follows:

1) Rey fault model: Base on the model, the south of Tehran will experience a seismic intensity of 9, while the northern part will experience a seismic intensity between 7 and 8.
2) North of Tehran fault model: Based on this model, it is expected that the seismic intensity may go up to 9 in the north of Tehran and down to 7 in the south of Tehran. A seismic intensity of 8 will occur in most parts of Tehran.
3) Masha fault model: Based on the model, the seismic intensity of 7 will occur in most parts of Tehran.
4) The floating model: A seismic intensity of 8 may occur in most parts of Tehran, while some parts of Tehran may experience a seismic intensity of 9.

Scenario-making and determination of their probability

As mentioned, JAICA has suggested 4 seismic models for Tehran's active faults. Earthquake scenarios can be calculated according to these models. In the present study, 8 different scenarios were suggested considering these four models, as well as the daytime of the occurrence of an earthquake.

Given the fact that time of the day when the earthquake happens has been taken into account in the JAICA report, as well as the Mete and Zabinsky [29] study, which also employes the daytime division for making scenarios, the hours of the day are classified as:

1. Relaxing hours, considered to be 8 hours
2. Regular hours, considered as the remaining 16 hours

This analysis is rational because since people are relaxing at home, their reacting speed to carry out security measures is decreased, which increases the casualties. On the other hand, in ordinary hours, when people are usually at work, in stronger buildings, and at higher alertness levels, casualties decrease rewardingly.

Fault's rate of activity, length, and movement mechanism are the considered features in danger assessment for the earthquakes of a fault. Different ideas may be used to calculate how probable the activity of a fault is. One of these ideas is that the risk of an earthquake increases with the length of a fault (http://www.iiees.ac.ir/). Therefore, the activity probability of any fault may be considered proportionate to the length. Table 1 shows these faults' lengths and activity probability.

According to day hours division, a total of 8 scenarios may be imagined in the present study, which is presented in Table 2. Since the ordinary hours are twice as long as resting hours, the probability of an earthquake happening in ordinary hours is two times higher than ordinary hours.

Table 2. Information about the faults and their probability of activity

Faults model	Floating fault	Rey fault	North of Tehran fault	Masha fault
Length (km)	10	20	90	200
Probability of occurrence	0.0312	0.0625	0.2812	0.625
Percentage (%)	3.12	6.25	28.12	62.5

Table 2. Information about faults

Model	Masha fault		North of Tehran fault		Rey fault		Floating fault	
Classification	Day	Night	Day	Night	Day	Night	Day	Night
Probability of the model	0.625		0.2812		0.0625		0.0312	
Probability of model, separated by hours	0.4167	0.2083	0.1875	0.0937	0.0417	0.0208	0.0208	0.0104

Table 3. Scenarios

Scenario number	Scenario name	Probability of occurrence
1	Rey fault (day)	0.0417
2	Rey fault (night)	0.0208
3	North of Tehran fault (day)	0.1875
4	North of Tehran fault (night)	0.0937
5	Masha fault (day)	0.4167
6	Masha fault (day)	0.2083
7	Floating fault (day)	0.0208
8	Floating fault (night)	0.0104

Calculating human casualties and the need for blood

In this study, all areas of Tehran were considered as case studies. The map of Tehran can be seen in Fig. 3.

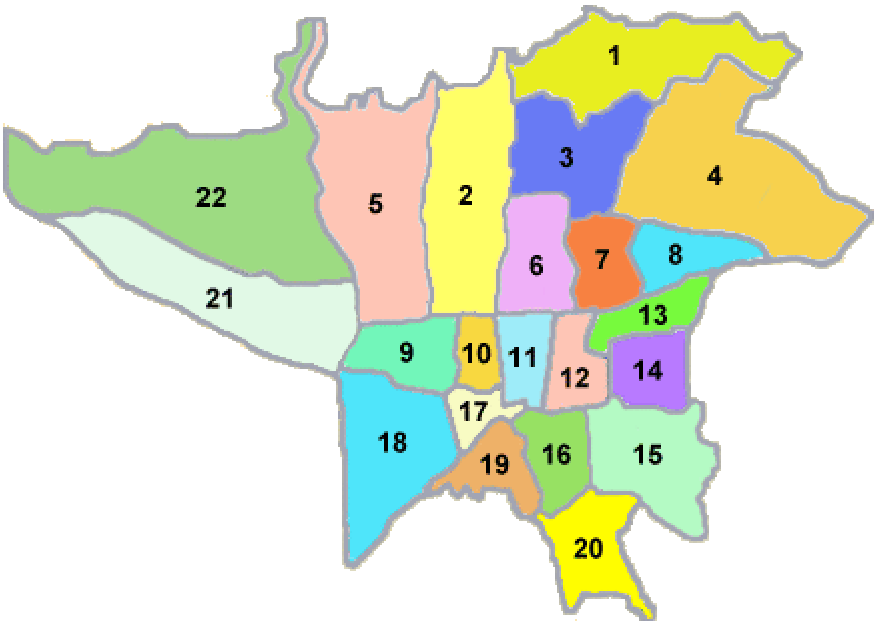

Fig 3. The map of Tehran

The casualty statistics in Iran's previous earthquakes

According to historical data, the casualties in an earthquake with a seismic intensity of 10 (MMI) is one-fourth (%25) if it occurs during the day compared with at night. These results are derived from official damage reports of previous earthquakes in Iran. In the present study, the same rate of casualties of humans is considered for the earthquakes during the day or at night.

The amount of demand for blood is calculated based on a study by Tabatabaee et al. [30], which focuses on earthquakes and the calculation of blood demand for casualties. According to what is given in the study, the injured to the killed ratio in an earthquake is 2.08 on average, where 30% of the injured are treated in hospitals, and 7% of total injuries are in need of blood. Therefore, the number of casualties has been obtained from the research by Golmohamadi.

Results

Based on the presented model and the process of developing scenarios to solve it, the findings of this research are explained. For solving the model using the L-shaped method, the GAMS software, in the Core i5, Asus X450 laptop was used. Solving the model took 2 minutes and 21 seconds. The first stage decisions were the most important decisions in this model since these decisions could and should be implemented prior to the crisis, while the second stage decisions are dependent on scenarios. Therefore, the first stage decisions, such as the amount of pre-inventory blood and the location of the fixed facilities, are explained first. The decision variable relating to the location of fixed facilities for blood collection is presented in Fig. 4.

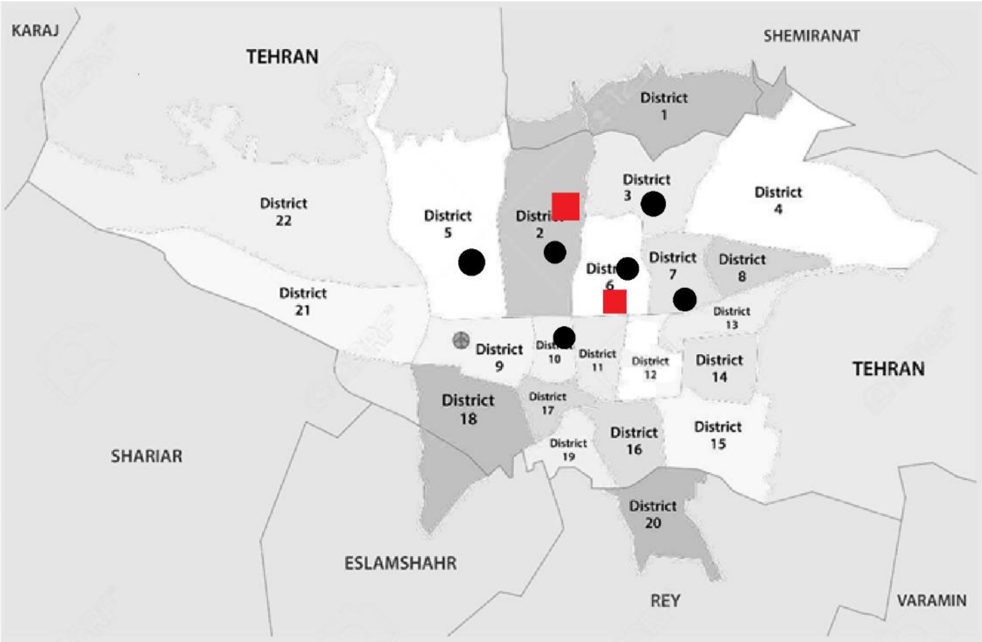

Fig 4. location of fixed facilities for blood collection

As shown in Fig. 4, blood centers located in districts 2 and 6 of Tehran have caused the high cost of building fixed blood collection centers in six regions of Tehran, i.e., districts 2, 3, 5, 6, 7, and 10, and the rest of the blood collection takes place using the moving facility after the crisis. The value of the objective function was equal to 1342755.281 $, which was obtained after ten iterations. The pre-inventory, which was difficult to preserve according to the data and conditions of blood storage, will be kept only at the transfusion center, two amounting to 1082 units, and at the transfusion center 6 in 1313 units.

Conclusions and suggestions for furthure studies

In this study, the former works were extended, and a novel two-stage multi-period stochastic model was proposed for designing a blood supply network considering a disaster in Tehran. For solving the proposed two-stage stochastic model, an L-shaped algorithm, an efficient algorithm to solve scenario-based stochastic models, has been used. The structure of the network of this supply chain comprises three layers. The first one is developed for all city areas in which different blood groups are donated. The second one made up the blood collection facilities chain consisting of both temporary and permanent types. In the third one, all the blood units that are donated are collected. A donated blood unit in this model with specific blood groups may be transfused to a patient of the same blood group or other blood groups if matching each other. Moreover, the blood derivatives, including platelet and plasma, are also taken into account—the issue of matching blood groups and derivatives handled by a possibility matrix. The decision-making occurs in two stages: 1) Pre-disaster stage: The permanent facilities that are supposed to be implemented for blood collections, and the quantity of pre-stocked blood units each blood and derivatives for different blood groups, which are required for supplying the demands in the first days following the crisis, are determined. 2) Post-Disaster: This stage is related to the time when the crisis occurred and the scenario-dependent decisions made at this stage. In the proposed stochastic model, for each scenario, the demand for blood units and their derivatives in different types are uncertain variables. The unsatisfied demands in the first period could be transferred to the next period. Shortage penalized in this model it might be provided by some sources outside the network—L-shaped algorithm based on the idea of solving the problem under 8 aforementioned different scenarios.

In the proposed mathematical optimization model, two main decisions about locating fixed facilities and preposition of each blood type are made, as explained in previous sections in the two-stage stochastic mathematical programming; the first-stage decisions are the major decisions that should implement the second stage decisions depend on the scenarios. Blood groups are also taken into account in the proposed model that helps the emergency system to preposition blood units based on groups and tackle the shortage of special blood types like O. We also implement our model and solution approach Japanese International Cooperation Agency (JICA) that reported in 1999, and we updated the data based on the suggestion of Iran Blood Transfusion Organization (IBTO) and National Disaster Management Organization (NDMO). In Our proposed model, two periods are considered; the first 24 hours responses the urgent patients and following 72 hours responses the rest of the injured.

For future studies, Given the Coronavirus (COVID 19) pandemic all over the world, this virus has recently become one of the most dangerous causes of death in the world, posing a big threat to global health currently. It has been proven that plasma donated by persons who have recovered from COVID-19 can help infected patients and boost their immune systems' function against it. It is the first time that all countries, cities, and communities have joined in the same struggle. Hence, the model proposed in this research can be used for designing a plasma supply chain network in critical situations like the COVID-19 pandemic at the moment and other crises that the international community may face in the future. Moreover, considering the spread of some diseases following natural disasters such as infectious diseases or any other simultaneous crisis that might occur after the crisis can be a new interest field for researchers.

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Appendix

L-shape method

The L-shape method is used for solving a linear two-stage stochastic programming problem whose block structure and formulation are similar to Model 1.

Model 1:

$$
\begin{aligned}
& Min\, z = c^T x + \sum_{s \in S} p^s b^{s^T} y^s \\
& s.t. \\
& \qquad Ax \geq d, \\
& \qquad B^s x + D^s y^s = h^s, \qquad s \in S \\
& \qquad x \geq 0, \\
& \qquad y^s \geq 0, \qquad\qquad s \in S
\end{aligned}
$$

The L-shape algorithm

First step: Solving a version of the main problem with released restraints and using feasibility and optimality cuts as shown in Model 2.

Model 2:

$$
\begin{aligned}
& Min\, z = c^T x + \varphi \\
& s.t. \\
& \qquad Ax \leq d, && && (1) \\
& \qquad (\pi_1^{(k_1)})^T Bx \geq (\pi_1^{(k_1)})^T h, && k_1 = 1,2,\ldots,K_1 && (2) \\
& \qquad (\pi_2^{(k_2)})^T Bx + \varphi \geq (\pi_2^{(k_2)})^T h, && k_2 = 1,2,\ldots,K_2 && (3) \\
& \qquad x \geq 0, \ \ \varphi \in R
\end{aligned}
$$

Limitation (2) shows feasibility cuts, and limitation (3) shows optimality cuts. In the beginning, these cuts do not apply; so, $K_1 = K_2 = 0$. In solving this model, as long as there are no limitations of type 2 and 3, $\varphi = -\infty$, and the model is solved regardless of φ. If the model becomes infeasible, then Model 1 is concluded to be infeasible too, and the algorithm would end. Otherwise, by assuming that $(\hat{x} = \hat{\varphi})$ is the optimal solution, the second step proceeds.

Second step: Feasibility evaluation and feasible cut generation

Model 3:

$$
\begin{aligned}
& Min\ w = e^T v^+ + e^T v^- \\
& s.t. \\
& \qquad Dy + Iv^+ - Iv^- = h - B\hat{x}, \\
& \qquad y \geq 0, v^+ \geq 0, v^- \geq 0.
\end{aligned}
$$

Model 3 is solved for $x = \hat{x}$. If the optimal value in the objective function becomes zero, the third step proceeds; otherwise, $K_1 = K_1 + 1$, and following feasibility cut is added to Model 2 where $\pi_1^{(K_1)}$ is the dual optimum for Model 3. Then, it returns to the first step.

$$
\left(\pi_1^{(K_1)}\right)^T Bx \geq \left(\pi_1^{(K_1)}\right)^T h
$$

Third step: Optimality evaluation and optimal cut generation

Model 4:

$$\begin{aligned}&Min\ \phi = b^T y\\&s.t.\\&\qquad Dy = h - Bx,\\&\qquad y \geq 0.\end{aligned}$$

Model 4 is solved for $x = \hat{x}$. If the problem is unbounded, then the main problem is unbounded as well, and the algorithm will end; otherwise, by assuming $\hat{y}$ as the optimum solution for the problem and $\hat{\pi}_2$ as the optimum solution for the dual problem, the optimality criterion is checked. If the following phrase is true $(\hat{x}, \hat{y})$ is the optimal solution for Model 1.

$$\hat{\varphi} = (\hat{\pi}_2)^T (h - B\hat{x})$$

Otherwise, $K_2 = K_2 + 1$ and $\hat{\pi}_2{}^{(K_2)} = \hat{\pi}_2$. After adding the following optimality cut to Model 2, it returns to the first step.

$$\left(\pi_2^{(k_2)}\right)^T Bx + \varphi \geq \left(\pi_2^{(k_2)}\right)^T h$$

The L-shape algorithm in solving two-stage stochastic programs
First step: Solving problem with feasibility and optimality cuts

Model 5:

$$\begin{aligned}&Min\, z = c^T x + \varphi\\&s.t.\\&\qquad Ax \geq d, && && (4)\\&\qquad G^{(k_1)} x \geq g^{(k_1)}, && k_1 = 1,2,\ldots,\mathrm{K}_1 && (5)\\&\qquad L^{(k_2)} x + \varphi \geq l^{(k_2)}, && k_2 = 1,2,\ldots,\mathrm{K}_2 && (6)\\&\qquad x \geq 0, \varphi \in R.\end{aligned}$$

Model 5 should be solved considering constraints 5 and 6, which imply feasibility and optimality cuts. Since these cuts are not applied at the beginning, $K_1 = K_2 = 0$. While solving this model, as long as there are no limitations of type 6, $\varphi = -\infty$, and the model is solved regardless of φ. If this model is found to be infeasible, then Model 1 is infeasible, and the algorithm will end; otherwise, by assuming $\hat{x} = \hat{\varphi}$ as the optimal solution, the second step proceeds.

Second step: Feasibility criterion evaluation and feasible cut generation

For each $s \in S$, Model 6 is solved for $x = \hat{x}$. If the optimum value of the objective function for each $s \in S$ becomes zero, the third step proceeds; otherwise, it is assumed that $s \in S$ is the first index in S that $w^{*\tilde{s}} \geq 0$ and $\hat{\pi}_1^{\tilde{s}}$ is the optimal point of its dual problem; so, the following feasibility cut $\left(\hat{\pi}_1^{\tilde{s}}\right)^T B^{\tilde{s}} x \geq \left(\hat{\pi}_1^{\tilde{s}}\right)^T h^{\tilde{s}}$ is added to Model 5, and $K_1 = K_1 + 1$ ، $G^{(K_1)} = (\hat{\pi}_1^{\tilde{s}})^T B^{\tilde{s}}$ ، ,$g^{(K_1)} = (\hat{\pi}_1^{\tilde{s}})^T h^{\tilde{s}}$

$G^{(K_1)} x \geq g^{(K_1)}$ is added to Model 5, and then it returns to the first step.

Model 6:

$$
\begin{aligned}
& Min\, w^s = ev^+ + ev^- \\
& s.t. \\
& \qquad D^s y^s + Iv^+ - Iv^- = h^s - B^s \hat{x}, \\
& \qquad y^s, v^+, v^- \geq 0.
\end{aligned}
$$

Third step: Optimality criterion evaluation and optimal cut generation
For each $s \in S$, Model 7 is solved.

Model 7:

$$
\begin{aligned}
& Min\ b^{s^T} y^s \\
& s.t. \\
& \qquad D^s y^s = h^s - B^s \hat{x}, \\
& \qquad y^s \geq 0.
\end{aligned}
$$