Viscoelastic effects on nonlinear dynamics of microplates with fluid interaction based on consistent couple stress theory

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Abstract

In this study, for the first time the forced vibration of a viscoelastic microplate in incompressible still fluid is investigated, simultaneously. The consistent couple stress theory which newly developed on the basis of the original couple stress theory is used in this study. The microplate is supposed simply supported in stationary fluid and made of viscoelastic material that follow the Leaderman viscoelastic model. The fluid inertial effects as well as fluid damping on microplate vibration are also studied by applying three dimensional aerodynamic theories. The added mass values are obtained for various aspect ratios of the microplate and compared with available models. The non-classic stress and couple-stress tensors are calculated based on consistent couple stress theory and Leaderman integral. The resultant virtual works inserted in Hamilton's principle and the governing equations of motion are derived. The results show that considering the added mass effects reduces the microsystem nonlinearity.

Keywords: Microplate, Fluid effects, Added mass, Consistent couple stress, Viscoelastic

Introduction

Micro/nano-electro-mechanical systems (MEMS/NEMS) due to less energy consumption and low cost are extensively used in sensors and actuators [1], micro-pumps [2-4] and micro-densitometers [5]. In this application, it is very important to study the vibration behavior of these micro and nano structure [6-9]. For example, Nejad *et al.* [10] and Hadi *et al.* [11]studied the free vibration behavior of two and three directional functionally graded elastic nano-beam by applying the nonlocal elasticity and strain gradient theories. Adeli *et al*.[12] the torsional vibration of a nano-sized elastic cone with nonlinear cross section in transverse direction. Moreover, She G.L. [13] investigated the force vibration and resonance of a curved elastic microbeam that made of nanoplates with incorporating the modified strain gradient theory.

However, in these applications as biosensors and chemical sensors these systems act inside the fluid environment. It should be noted that the system natural frequencies decrease when submerged in fluid media [14]. Therefore, understanding the fluid effects on microsystems dynamics is a crucial issue for their design. Generally, fluid surrounding has a damping effect due to its viscosity [15, 16], stiffness effects, and inertial effects [17, 18] on vibration and dynamic characteristics of the micro/nanostructure. The fluid inertial effect can be demonstrated as added mass which is characteristic of the fluid loading [19]. There are several researches that investigated the fluid effects on free vibration and dynamic of the macro and microstructures. For example, the added mass value for cantilever and simply supported plate

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presented by Yadykin [17]. Rezazadeh *et al.* [20] studied the fluid inertial effects on the dynamic response of an electrostatically actuated microbeam. In this study, the effect of the fluid loading was modeled as added mass on cantilever and double clamped microbeam. Sinha *et al*. [21] derived the added mass values for a plate vibrating in fluid media. Amiri *et al.* [22] studied the natural frequencies of a plate shape micro-pump made with magneto-electro-elastic materials. Jabbari *et al.* [23] investigated the nonlinear force and frequency responses of electro actuated plate shape micro-resonator that submerged in fluid based on modified couple stress. Besides the fluid surrounding effects which are investigated in MEMS structures. Besides fluid effects MEMS/NEMS structures, some researches results have shown that viscoelastic properties widely exist in MEMS materials such as silicon [24] and polysilicon [25]. In some researches, the viscoelasticity effects on the vibration behavior of the micro/nanoplates have been investigated. For example, Liu and Zhang [26] derived an analytical solution for vibration of double layer viscoelastic nanoplates with in-plane loads. The vibration frequency of a viscoelastic nanoplate including the viscoelastic foundation effects studied by Pouresmaeeli *et al*. [27]. Their investigation shows that nanoplate frequencies decrease as the viscoelastic coefficient increases. The size-dependent free vibration of viscoelastic multiple nanoplate structure embedded in viscoelastic surrounding studied by Karlicic *et al*.[28]. In this work the exact analytical solution was obtained for simply supported nanoplates natural frequencies. Moreover, Farokhi and Ghayesh [29] considered viscoelasticity effects on force and frequency response of a shear-deformable microbeam by employing the modified couple stress theory (MCST). Their study showed that

Recently, Ajri *et al.*[30-32] studied the free vibration and resonance analyses of a viscoelastic nanoplates based on MCST and strain gradient theory (MSGT) at different length-scale values, respectively. These studies showed that the viscoelastic model energy dissipation was amplitude dependent which results in more accurate outcomes compared to an elastic one. In this paper, for the first time the fluid inertial and damping effects on a microplate dynamic behavior that made of viscoelastic material is studied simultaneously.

Mousavi Khoram *et al.* [33] reviewed the recent works that studied the nanoplates mechanical behavior. The researchers used the MSGT, nonlocal elasticity and surface theory with incorporating Hamilton's principle to derive the governing equation. Recently, Ajri *et al*. [34] studied the viscoelastic damping effects on the frequency and force response of a nanoplate in the frame work of the consistent couple stress theory (CCST). It is worth to mention that, the CCST is developed on the basis of the original couple stress theory using the true continuum kinematical displacement and rotation [35-37]. In this theory the skew-symmetric couple-stress and curvature tensors are coupled to each other in the virtual work relation [35]. This theory was also used to study the buckling and free vibration of elastic nanobeams by some authors [38, 39].

However current study, for the first time the authors investigate the nonlinear dynamics of a viscoelastic microplate in incompressible still viscous fluid, including the frequency and force responses. The microplate is simply supported and follows the Leaderman viscoelastic model. The fluid loading and damping are applied as added mass and added damping to governing equation of motion. The non-classic stress and couple-stress tensors are calculated based on CCST and Kirchhoff plate theory with nonlinear von Karman strains. The harmonic balance analytical method (HBM) is used to obtain analytical solution for frequency and force responses. A parametric study has been done to investigate the effect of the fluid inertial load and viscous damping on frequency and force responses.

Problem Formulation

This section includes the formulation required to obtain and solve the governing equations of the viscoelastic microplate that vibrate in fluid surroundings.

Viscoelastic microplate governing equation based on CCST

The Hamilton's principle is employed to derive the governing equation

$$\delta \int_{t_1}^{t_2} \left[-K + U + W \right] dt = 0 \tag{1}$$

where δ, K, U, and W are the first variation operator, the kinetic energy, strain energy elastic portion, and external loads works, respectively. The microplate is excited by the out of plane loads in this study. In viscoelastic structures the work of the viscous dissipation forces is added to the work of the external forces [34].

Based on the CCST in elastic structures, the strain energy can be expressed as [35-37]

$$U = \int (\sigma_{ij}^e . \varepsilon_{ij} + \mu_{ij}^e . \kappa_{ij}) dV \tag{2}$$

Where σ_{ij}^e and μ_{ij}^e are the elastic force-stress and elastic couple-stress tensors, respectively. Moreover, ε_{ij} and κ_{ij} are the strain second order tensor and the rotation vector rotations.

Similarly, the viscous loads works can be written as following form

$$W_{vis} = U_{vis} = \int (\sigma_{ij}^v . \varepsilon_{ij} + \mu_{ij}^v . \kappa_{ij}) dV \tag{3}$$

Where σ_{ij}^v and μ_{ij}^v are the viscous force-stress and viscous couple-stress tensors, respectively.

By applying the Leaderman viscoelastic relation to the CCST the viscoelastic stress and couple-stress second order tensors can be obtained as [34].

$$\sigma_{ij} = \sigma_{ij}^e + \sigma_{ij}^v = \left[\lambda_0 \delta_{ij} \varepsilon_{pp} + 2\mu_0 \varepsilon_{ij}(t) \right] \tag{4}$$
$$+ \int_0^t \left[\dot{\lambda}(t - \tau) \delta_{ij} \varepsilon_{kk} + 2\dot{\mu}(t - \tau) \varepsilon_{ij}(\tau) \right] d\tau$$

$$\mu_{ij} = \mu_{ij}^e + \mu_{ij}^v = -8\mu_0 l^2 \kappa_{ij}(t) - 8 \int_0^t \left[\dot{\mu}(t-\tau) \kappa_{ij}(\tau) \right] d\tau \tag{5}$$

where λ_0 and μ_0 are the initial Lame's constants. Additionally, l is the material length-scale parameter. The over dot $(\cdot)$ denotes the first derivation with respect to the time.

The skew-symmetric curvature tensor κ_{ij} relates to the rotation tensor ω_{ij} as [35]

$$\kappa_{ij} = \frac{1}{2} \left[\omega_{i,j} - \omega_{j,i} \right] \tag{6}$$

where ω_{ij} can be written as

$$\omega_{ij} = \frac{1}{2} \left[u_{i,j} - u_{j,i} \right] \tag{7}$$

where $u_{j,i}$ is the gradient of the displacement field.

The displacement field assumed as below relation [40]

$$u_x = u - z\frac{\partial w}{\partial x} \quad v_y = v - z\frac{\partial w}{\partial y} \quad w_z = w \tag{8}$$

where $u,v,$ and w represent the time-dependent displacements of a point on the mid-plane along the (x,y,z) coordinate. The nonlinear von-Karman strain components can be written as [41].

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \qquad \varepsilon_{yy} = \frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial y^2} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right) \tag{9}$$

The microsystem kinetic energy is expressed as

$$K = \frac{1}{2}\int \rho\left(\dot{u}_x^2 + \dot{v}_y^2 + \dot{w}_z^2\right)dV \tag{10}$$

where ρ is mass density of the microplate.
The work done by the external forces on the microplate can be calculated as [42]

$$\delta W_{ext} = -\int_\Omega f_z \delta w \, dA \tag{11}$$

where f_z is the resultant force in z direction. The external load is assumed to be harmonic with amplitude f and frequency Ω. Additionally, the fluid damping force effects on the microplate can be written as [43, 44].

$$f_{fluid} = -c\frac{\partial w}{\partial t} \tag{12}$$

In which c is a viscous damping coefficient.
Therefore, the equation of motion can be written as [34].

$$\delta u: \quad \frac{E_0 h}{(1-\upsilon^2)}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} + \upsilon\left(\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x \partial y}\right)\right)$$

$$+ \int_0^t \frac{h\dot{E}(t-\tau)}{(1-\upsilon^2)}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} + \upsilon\left(\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x \partial y}\right)\right)d\tau$$

$$+ \mu_0 h\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x \partial y}\frac{\partial w}{\partial y}\right)$$

$$+ \int_0^t h\dot{\mu}(t-\tau)\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x \partial y}\frac{\partial w}{\partial y}\right)d\tau$$

$$+ \frac{l^2 h\mu_0}{2}\left(\left(\frac{\partial^4 v}{\partial x^3 \partial y} - \frac{\partial^4 u}{\partial x^2 \partial y^2}\right) + \left(\frac{\partial^4 v}{\partial x \partial y^3} - \frac{\partial^4 u}{\partial y^4}\right)\right)$$

$$\frac{l^2 h}{2}\int_0^t \dot{\mu}(t-\tau)\left(\left(\frac{\partial^4 v}{\partial x^3 \partial y} - \frac{\partial^4 u}{\partial x^2 \partial y^2}\right) + \left(\frac{\partial^4 v}{\partial x \partial y^3} - \frac{\partial^4 u}{\partial y^4}\right)\right)d\tau$$

$$= \rho h\ddot{u} \tag{13-a}$$

$$\delta v: \frac{E_0 h}{(1-\upsilon^2)}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2}+\upsilon\left(\frac{\partial^2 u}{\partial x \partial y}+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x \partial y}\right)\right)$$

$$+\int_0^t \frac{h\dot{E}(t-\tau)}{(1-\upsilon^2)}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2}+\upsilon\left(\frac{\partial^2 u}{\partial x \partial y}+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x \partial y}\right)\right)d\tau$$

$$+\mu_0 h\left(\frac{\partial^2 u}{\partial x \partial y}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 w}{\partial x^2}\frac{\partial w}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x \partial y}\right)$$

$$+\int_0^t h\dot{\mu}(t-\tau)\left(\frac{\partial^2 u}{\partial x \partial y}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 w}{\partial x^2}\frac{\partial w}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x \partial y}\right)d\tau$$

$$+\frac{l^2 h \mu_0}{2}\left(\left(\frac{\partial^4 u}{\partial x \partial y^3}-\frac{\partial^4 v}{\partial x^2 \partial y^2}\right)+\left(\frac{\partial^4 u}{\partial x^3 \partial y}-\frac{\partial^4 v}{\partial x^4}\right)\right)$$

$$\frac{l^2 h}{2}\int_0^t \dot{\mu}(t-\tau)\left(\left(\frac{\partial^4 u}{\partial x \partial y^3}-\frac{\partial^4 v}{\partial x^2 \partial y^2}\right)+\left(\frac{\partial^4 u}{\partial x^3 \partial y}-\frac{\partial^4 v}{\partial x^4}\right)\right)d\tau \qquad \text{(13-b)}$$

$$= \rho h \ddot{v}$$

$$\delta w: -\frac{E_0 h^3}{12(1-\upsilon^2)}\left(\frac{\partial^4 w}{\partial x^4}+\upsilon\frac{\partial^4 w}{\partial x^2 \partial y^2}\right)-\int_0^t \frac{h^3 \dot{E}(t-\tau)}{12(1-\upsilon^2)}\left(\frac{\partial^4 w}{\partial x^4}+\upsilon\frac{\partial^4 w}{\partial x^2 \partial y^2}\right)d\tau \qquad \text{(13-c)}$$

$$-\frac{E_0 h^3}{12(1-\upsilon^2)}\left(\frac{\partial^4 w}{\partial y^4}+\upsilon\frac{\partial^4 w}{\partial x^2 \partial y^2}\right)-\int_0^t \frac{h^3 \dot{E}(t-\tau)}{12(1-\upsilon^2)}\left(\frac{\partial^4 w}{\partial y^4}+\upsilon\frac{\partial^4 w}{\partial x^2 \partial y^2}\right)d\tau$$

$$-\frac{\mu_0 h^3}{3}\left(\frac{\partial^4 w}{\partial x^2 \partial y^2}\right)-\int_0^t \frac{h^3 \dot{\mu}(t-\tau)}{3}\left(\frac{\partial^4 w}{\partial x^2 \partial y^2}\right)d\tau-2l^2 h\left(\mu_0 \nabla^4 w+\int_0^t \dot{\mu}(t-\tau)\nabla^4 w d\tau\right)$$

$$\frac{h}{1-\upsilon^2}\left[\begin{array}{c}\left(\dfrac{\partial^2 w}{\partial x^2}\dfrac{\partial u}{\partial x}+\dfrac{1}{2}\dfrac{\partial^2 w}{\partial x^2}\dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial x}+\upsilon\dfrac{\partial^2 w}{\partial x^2}\dfrac{\partial v}{\partial y}+\dfrac{\upsilon}{2}\dfrac{\partial^2 w}{\partial x^2}\dfrac{\partial w}{\partial y}\dfrac{\partial w}{\partial y}\right) \\ +\left(\dfrac{\partial^2 w}{\partial y^2}\dfrac{\partial v}{\partial y}+\dfrac{1}{2}\dfrac{\partial^2 w}{\partial y^2}\dfrac{\partial w}{\partial y}\dfrac{\partial w}{\partial y}+\upsilon\dfrac{\partial^2 w}{\partial y^2}\dfrac{\partial u}{\partial x}+\dfrac{\upsilon}{2}\dfrac{\partial^2 w}{\partial y^2}\dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial x}\right) \\ +(1-\upsilon)\left(\dfrac{\partial^2 w}{\partial x \partial y}\dfrac{\partial u}{\partial y}+\dfrac{\partial^2 w}{\partial x \partial y}\dfrac{\partial v}{\partial x}+\dfrac{\partial^2 w}{\partial x \partial y}\dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y}\right)\end{array}\right]+f_z=\rho h \ddot{w}$$

In the above equations υ is the passion ratio and h is the microplate thickness.

Fluid-solid interaction

As discussed, in order to take into account, the fluid inertial effect on the microstructure that submerged in fluid the added mass concept is extensively used. In this section the added mass value will be obtained for vibration microplate and added to microplate mass, ρh, in governing equation of motion.

The rate of the change in kinetic energy at any portion of the incompressible still fluid is equal to work done by the pressures on its surface [45].

$$\frac{dE_k}{dt} = -\iint_s V_z \Delta P dS \tag{14}$$

where E_k, V_z and S are kinetic energy of the fluid, fluid particle vertical velocity, and the surface of the fluid-solid interaction, respectively. Furthermore, ΔP is the pressure applied from outside an element dS of the boundary and is doing work.

In the current study, the microplate is assumed to be the fluid-solid interface [46]. So Eq. (14) can be written as

$$\frac{dE_k}{dt} = -\int_0^b \int_0^a \frac{\partial w}{\partial t} \Delta P dx dy \tag{15}$$

where $\frac{\partial w}{\partial t}$ is the transverse velocity of the microplate.

According to Minami study [47] the rate of change in the kinetic energy of the vibrating microplate having a mass M per unit area equivalent to the added mass, is written as:

$$\frac{dE_k}{dt} = M \int_0^b \int_0^a \frac{\partial w}{\partial t} \frac{\partial^2 w}{\partial t^2} dx dy \tag{16}$$

So the added mass value can be found as

$$M = -\frac{\int_0^b \int_0^a \frac{\partial w}{\partial t} \Delta P dx dy}{\int_0^b \int_0^a \frac{\partial w}{\partial t} \frac{\partial^2 w}{\partial t^2} dx dy} \tag{17}$$

According to three dimensional aerodynamic theory proposed by Lucy and Carpenter [48] ΔP can be expressed as

$$\Delta P = \frac{\rho_F a}{\pi} \frac{1}{\psi} \int_0^1 \int_0^1 \frac{1}{R} \frac{\partial^2 w}{\partial t^2}\Big|_{(\bar{x},\bar{y})=(\xi,\eta)} d\xi d\eta \tag{18}$$

where ξ and η are dummy parameters. Moreover, $\bar{x}$, $\bar{y}$ and ψ are non-dimensional parameters and defined as

$$\bar{x} = \frac{x}{a} \qquad \bar{y} = \frac{y}{b} \qquad \psi = \frac{a}{b} \tag{19}$$

and

$$R = \sqrt{(\bar{x}-\xi)^2 + \frac{1}{\psi^2}(\bar{y}-\eta)^2} \tag{20}$$

where a and b are the microplate length and width, respectively. In addition, ρ_F is the fluid density. The non-dimensional form of the added mass is equal to $M_{non\dim} = \frac{M\pi}{\rho_F a}$

Solution procedure

The relaxation function of young's modulus is defined as Eq. (21) for the viscoelastic structure.

$$E(t) = C + De^{-\gamma t} \tag{21}$$

where γ is the relaxation coefficient.

With introducing the following non-dimensional parameters, Eq. (21) can be rewritten as Eq. (23)

$$\bar{C} = \frac{C}{C+D}, \quad \bar{D} = \frac{D}{C+D}, \quad n(t) = \frac{E(t)}{E_0}, \quad \bar{\gamma} = \gamma T \tag{22}$$

where T is defined later

$$n(t) = \bar{C} + \bar{D}e^{-\bar{\gamma}t} \tag{23}$$

Similarly, the following non-dimensional parameters are defined

$$\bar{x} = \frac{x}{a}, \bar{y} = \frac{y}{b}, \bar{w} = \frac{w}{h}, \bar{t} = \frac{t}{T}, \psi = \frac{a}{b}, \tag{24}$$

$$l_0 = \frac{l}{h}, \bar{f} = \frac{fa^4}{E_0 h^4}, T = \sqrt{\frac{(\rho h + M)a^4}{E_0 h^3}}, \bar{c} = \frac{ca^2}{\sqrt{E_0(\rho h + M)h^3}}$$

The Galerkin method is employed to convert the partial differential equation of motion to an ordinary differential equation. The simply supported rectangular microplates displacement field based on this theory can be obtained as [49],:

$$u(\bar{x}, \bar{y}, \bar{t}) = \frac{1}{16} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \alpha \frac{h}{a} \Phi_{mn}^2(\bar{t}) \sin 2\alpha\bar{x}(\cos 2\beta\bar{y} - 1 + \upsilon\psi^2 \frac{\beta^2}{\alpha^2}) \tag{25}$$

$$v(\bar{x}, \bar{y}, \bar{t}) = \frac{1}{16} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \beta \frac{h}{b} \Phi_{mn}^2(\bar{t}) \sin 2\beta\bar{y}(\cos 2\alpha\bar{x} - 1 + \upsilon\frac{1}{\psi^2}\frac{\alpha^2}{\beta^2})$$

$$w(\bar{x}, \bar{y}, \bar{t}) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{mn}(\bar{t}) \sin \alpha\bar{x} \sin \beta\bar{y}$$

where $\alpha = m\pi$ and $\beta = n\pi$.
By using the Galerkin approach the transverse motion equation with fluid interaction is given as:

$$\frac{1}{16(1-\upsilon^2)}\left(4\upsilon\psi^2\alpha^2\beta^2 + (3-\upsilon^2)(\alpha^4 + \psi^4\beta^4)\right)\left(\Phi^3(t) - D\gamma\int_0^t e^{-\gamma(t-\tau)}\Phi^3(\tau)d\tau\right) + \tag{26}$$

$$\left(\frac{1}{12(1-\upsilon^2)} + \frac{1}{(1+\upsilon)}l_0^2\right)(\alpha^2 + \psi^2\beta^2)^2\left(\Phi(t) - D\gamma\int_0^t e^{-\gamma(t-\tau)}\Phi(\tau)d\tau\right) + c\dot{\Phi}(t) + \ddot{\Phi}(t) = f$$

The fourth order Runge-Kutta is used to solve this equation. In order to inspect the primary resonance of the microsystem, the harmonic balance method is used to solve this equation[50]. Taking into account the periodic solution first-order approximation, $\Phi = X \cos(\Omega t + \alpha)$, and replacing it in Eq. (26) gets.

$$\left(-X\Omega^2 + \frac{3}{4}\Pi_1 X^3 + \Pi_2 X\right)\cos(\Omega t + \alpha) + \left(\frac{1}{4}\Pi_1 X^3\right)\cos(3\Omega t + 3\alpha) - cX\Omega\sin(\Omega t + \alpha) - \tag{27}$$

$$\left(\frac{3}{4}\Pi_3 X^3 + \Pi_4 X\right)\int_0^t e^{-\gamma(t-\tau)}\cos(\Omega t + \alpha)d\tau - \frac{1}{4}\Pi_3 X^3\int_0^t e^{-\gamma(t-\tau)}\cos(3\Omega t + 3\alpha)d\tau = f \cos\Omega t$$

where

$$\Pi_1 = \frac{1}{16(1-\upsilon^2)}\left(4\upsilon\psi^2\alpha^2\beta^2 + (3-\upsilon^2)(\alpha^4 + \psi^4\beta^4)\right) \tag{28}$$

$$\Pi_2 = \left(\frac{1}{12(1-\upsilon^2)} + \frac{1}{(1+\upsilon)}l_0^2\right)(\alpha^2 + \psi^2\beta^2)^2$$

$$\Pi_3 = D\gamma\,\Pi_1 \qquad\qquad \Pi_4 = D\gamma\,\Pi_2$$

Neglecting the terms with higher frequency, 3Ω, and considering the the steady-state phase:

$$\left(a_4^2 + a_2^2\right)Z^3 + 2\left(a_1a_2 + a_3a_4\right)Z^2 + \left(a_1^2 + a_3^2\right)Z^2 - f^2 = 0 \tag{29}$$

where $Z = X^2$, and

$$a_1 = \Pi_2 - \Omega^2 - \frac{\Pi_4\gamma}{\gamma^2 + \Omega^2}, \; a_2 = \frac{3}{4}\left(\Pi_1 - \frac{\Pi_3\gamma}{\gamma^2 + \Omega^2}\right) \tag{30}$$

$$a_3 = \left(\frac{\Omega\Pi_4}{\gamma^2 + \Omega^2}\right) + c\Omega, a_3 = \left(\frac{3\Omega\Pi_4}{4(\gamma^2 + \Omega^2)}\right)$$

The force and frequency response of the viscoelastic microplate that vibrates in fluid with considering fluid inertial and damping effects can be predicted by solving the Eq. (29)

Dynamic response analysis

In order to demonstrate the analytical solution achieved in the previous section, a viscoelastic microplate with simply supported properties is considered with the following properties:

$$a = 400\,\mu m \;\; b = 200\,\mu m \;\; h = 35.2\,\mu m \;\; l_0 = 17.6\,\mu m \;\; E = 1.44\,GPa \;\; \rho = 1220\frac{kg}{m^3}$$

$$\upsilon = 0.38 \;\;\; C = 0.7 \;\; D = 0.3 \;\; \gamma = 1 \;\; c = 0.5$$

It is supposed that the microplate vibrates in fluid and vacuum media at first natural mode. In this section at first step, the added mass values are calculated. Next, the microsystem frequency response predicted by analytical method is validated with numerical results. Then the effects of fluid media, which take into account with added mass and added damping, on frequency and force responses are studied.

The non-dimensional added mass values, for various aspect ratio, $M_{non\dim}$, of the simply supported microplate are calculated based on Lucy and Carpenter [48] and Wu [51] models are plotted in Fig.1. As seen, the Wu model predicts linear approximation unlike the Lucy and Carpenter model. It can be seen that as the aspect ratio of the microplate increases, the added mass values increase for both models. For example, in a square microplate the added mass value is equal to 2.6 and in microplate with the aspect ratio of 0.5 is equal to 1.66.
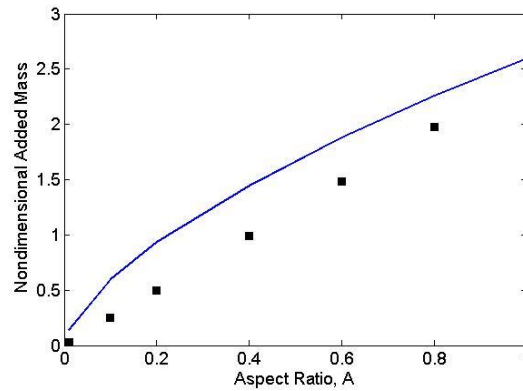
Figure 1. The non-dimensional added mass versus the microplate aspect ratio.

Lucy and Carpenter: *continues line*,Wu: *dotted symbol*

For verification of our results, the frequency responses of the viscoelastic microplate that vibrates in water with $\rho_f = 1000 \, Kg/m^3$ predicted by the analytical method and Rung-Kutta technique are compared to each other in Fig.2. It is shown that the numerical and analytical results are close to each other.
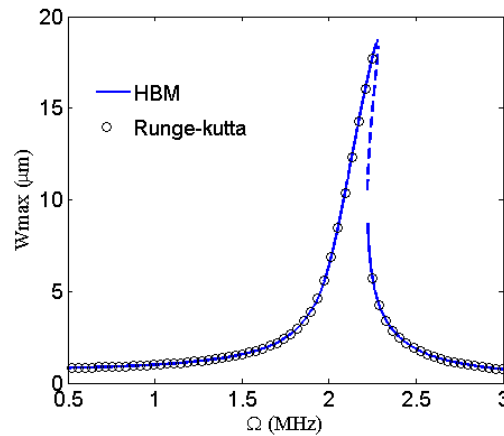

Figure 2. The frequency responses predicted by numerical and analytical methods.

The transverse and in-plane motions time history are plotted in Fig. 3 for the viscoelastic microsystem that oscillates in water. The initial conditions are set as $\Phi(0)=1 \; \dot{\Phi}(0)=0$. This figure shows the microsystem oscillation frequency decreases by considering the added mass effect. This is an important issue in designing the microsystem that vibrates in fluid surroundings.
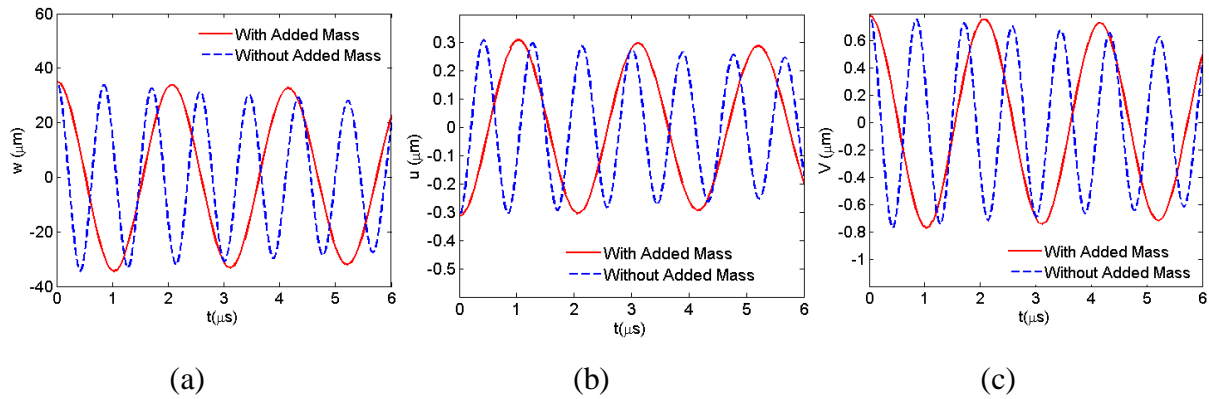
Figure 3. Time history: (a) transverse displacement at mid-plane (b), (c) in-plane displacements at x=300, y=150.

The effect of the fluid density on the microsystem natural frequency is plotted in Fig.4. The dotted symbol is predicted for the model without added mass effects. The fluid density is considered between 556 (butane) to 1584 kg/m³ (Carbon Tetrachloride). The figure shows that in fluid with higher densities the microsystem natural frequency is decreased. For example, the microsystem frequency is 0.45 MHz and 0.27 MHz for butane and Carbon Tetrachloride, respectively.
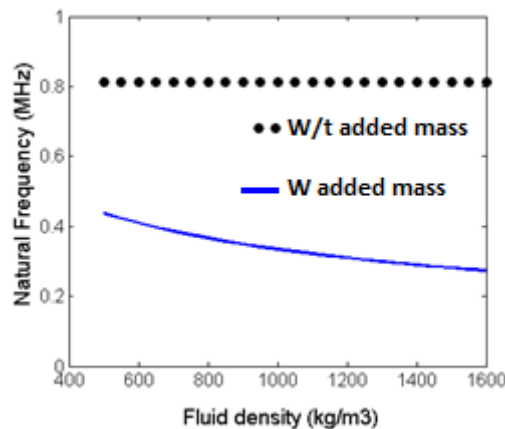

Figure 4. The natural frequency vibration vs. surrounding fluid media density.

The frequency responses of the microsystem in water and vacuum are shown in Fig.5. It can be seen that the fluid damping reduces the response amplitude with respect to the amplitude of the microsystem that oscillates in vacuum. In general, it can be expressed the fluid media reduces the nonlinearity and response amplitudes of the microsystem. In addition, it is shown that the resonance frequency in added mass model is smaller than the model without added mass effects. Furthermore, the bending of the response curves to right decreased in added mass model and hence the predicted nonlinearity becomes weaker with respect to the model without added mass effects.
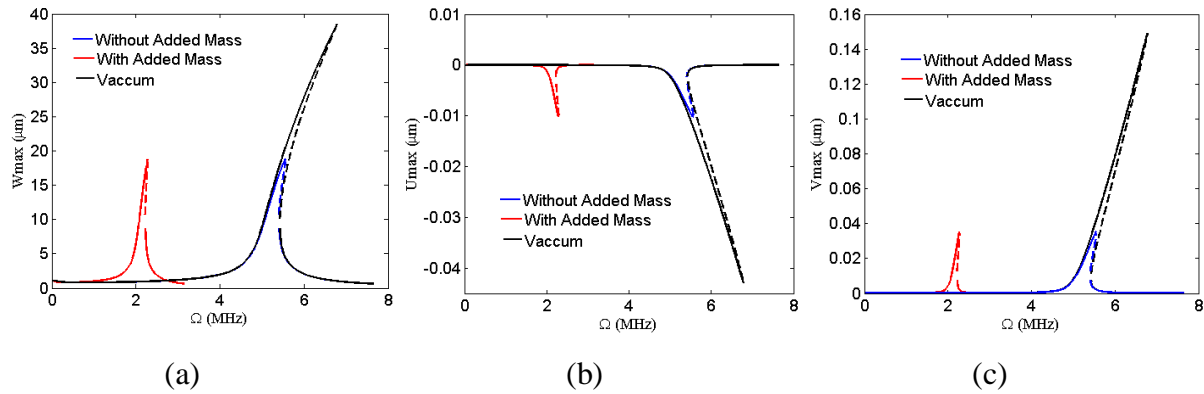
(a)   (b)   (c)

Figure 5. Frequency response, dashed line unstable solution solid line stable solution: (a) transverse motion amplitude at mid-plane (b), (c) in-plane motion amplitudes at x=300, y=150.

The effect of fluid density on the frequency responses at three different fluids, acetone, water, and Carbon tetrachloride with density equal to 785, 1000, and 1590 kg/m^3 are shown in Fig. 6. It is shown that the resonance frequency and bifurcation points shift to higher frequency at the fluid with lower density.
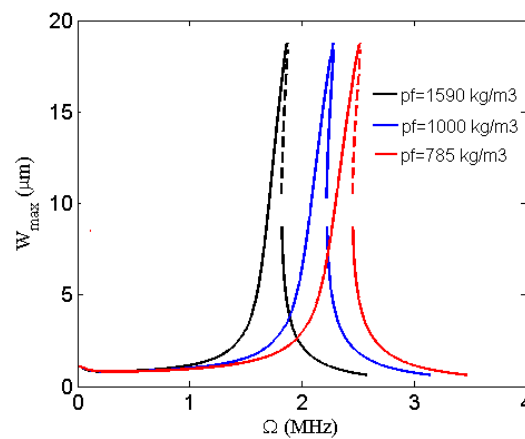
Figure 6. Frequency response of transverse motion at different fluid densities.

The force responses of the viscoelastic microsystem in water and vacuum at Ω=6.45 MHz are plotted in Fig.7. This figure shows that there are jumps and instabilities at response for both surroundings. Also it can be seen that the first and second instabilities shift to greater forcing amplitude in water media for all transverse and in-plane motions. Additionally, the response amplitudes at fluid are smaller than vacuum.
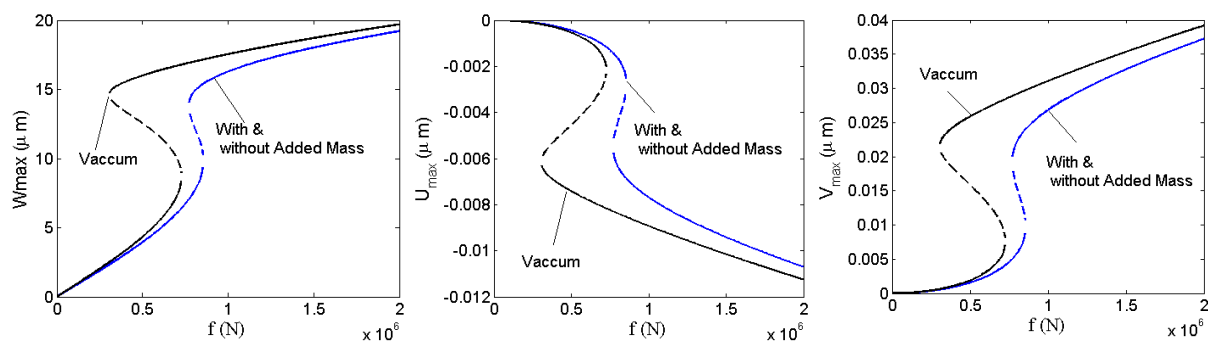
(a)   (b)   (c)

Figure 7. Force response, dashed line unstable solution solid line stable solution: (a) transverse motion amplitude at mid-plane
(b), (c) in-plane motion amplitudes at x=300, y=150.

Conclusion

In this paper, the nonlinear dynamics of a viscoelastic microplate was studied in incompressible still viscous fluid by using added mass and added damping to governing equation of motion. The non-classic stress and couple-stress tensors were calculated based on CCST and Kirchhoff plate theory with nonlinear von Karman strains. A parametric study had been done and the following results were obtained:

The fluid surroundings reduce the microsystem natural frequencies. In addition, as the fluid density increases the microsystem natural and resonance frequencies reduce to smaller values. It was observed that the nonlinearly of the microsystem is reduced in the fluid with considering the added mass effects. Also the resonance frequency shifts to smaller values in the added mass model. Moreover, in force response the occurrence instabilities are shifted to higher forcing amplified in fluid with added mass effects.

Nomenclature

a,b	Plate length and width	E	Relaxation function	c	Viscous damping coefficient
f	Body force	G	Modulus of rigidity	h	Plate thickness
K	Kinetic energy	t	Time	U	Elastic strain energy
V	Volume	W	Non-conseravtive forces virtual work		

Greek symbols

σ_{ij}	Force stress tensors	ε_{ij}	Strain tensor	l	length-scale parameter
μ_{ij}	Couple-stress tensors	κ_{ij}	Curvature tensor	λ	Lame constants
μ	Lame constants	γ	Relaxation coefficient	l_0	Length-scale ratio
υ	Poisson ratio	Ω	External load frequency		
ρ	Microplate Density	ρ_F	Fluid Density		

Subscript

| E | Elastic | V | Plastic | vis | Viscous forces |

ext   External forces

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