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RESEARCH PAPER

A review on stress-driven nonlocal elasticity theory

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Abstract

The behavior of materials at the nanoscale cannot be studied by classical theories. Accordingly, new theories have been developed to predict the behavior of materials at the nanoscale; some of them are nonlocal elasticity, strain gradient theory, couple stress theory, and surface effect theory. In most articles, the authors use a differential form of nonlocal elasticity theory. Recently, many authors have used the integral form of this theory and obtained interesting results. Therefore, in the present research, the articles related to the integral form of non-local theory have been examined for small-scale tubes, beams, shells, and plates.

Keywords: Nonlocal elasticity theory, Stress-driven method, Strain-driven method, Strain gradient model, Functionally graded material, Scale parameter

Introduction

With the advancement of human knowledge in the field of various sciences and technology and the achievement of human beings in very small dimensions of matter, it is predicted that all human imaginations will one day enter the realm of human science. Utilizing the properties of matter at the nanoscale is promising benefits that cause fundamental changes in human life. The nanometer is a unit of measurement equal to 10^{-9} meters, and all objects and organisms that range in size from 1 to 100 nanometers are called nanoscale [1]. Material properties are divided into physical properties and chemical properties. A nanometer-sized material will have different properties from its larger particles. On the other hand, particle shrinkage is a physical change, and it is expected that the physical properties of matter will not change with this physical change. Due to their very small size, nanomaterials show unique and sometimes different properties from other conventional materials. Two main factors have caused nanostructured materials to behave differently from materials in normal dimensions: surface effects and quantum effects. These two factors affect the chemical reactivity of materials, mechanical, optical, electrical, magnetic properties, etc., and in general, all their physical and chemical properties [2, 3]. Chemical reactions take place at the surface of materials, where materials come into contact with each other. Therefore, more contact surfaces will make the reactions easier. As the particles of matter get small and smaller, the surface-to-volume ratio increases. Nano interdisciplinary science encompasses almost all engineering and medical sciences.

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The classical theories used to study the behavior of materials in large dimensions can no longer predict the behavior of materials in nanoscale. Therefore, new theories for studying the behavior of materials in nanoscale were proposed and developed. All of these theories include parameters for considering the effects of small-scale. Some of the applied theories in nanoscience are nonlocal elasticity, strain gradient theory, surface effect theory, and couple stress theory. Much research has been conducted based on these theories [4-59]. The nonlocal theory is the most widely used theory in the field of elasticity for nanostructures. Most researchers use the differential mode of this theory to simplify relationships. But recent research has been done using the integral mode of this theory so that the results are very significant. This article examines some of these results.

Application in the Analysis of Mechanical Structures

(A). Nano-rod

Barretta *et al*. [60] investigated the torsional behavior affected by the size of a nano-rod under bending moment. By selecting the non-local stress-driven model and combining it with the local model as well as the biphasic stress-driven model and considering a specific Bi-Helmholtz kernel function, they could derive the equation governing the torsion and the constitutive boundary conditions required to solve the equation of this nano-rod. Moreover, they could calculate the torsional angle of nano-rod with different boundary conditions by directly solving this equation, and also compared their results with biphasic strain-driven and strain gradient models and indicated that the biphasic stress-driven model, unlike strain-driven and biphasic strain-driven models, did not show singular behavior and led to the enhancement of solutions to elasto-static problems. Their results revealed that the use of a biphasic strain-driven model results in an increase in the torsional angle, while the use of biphasic stress-driven and strain gradient models leads to a decrease in torsional angle. Furthermore, the hardening effect of the biphasic stress-driven model is more significant compared to the strain gradient elasticity theory. In the meantime, the effect of the combined parameter for biphasic strain-driven and stress-driven models had decreasing and increasing effects on torsional-rotational behavior, respectively. These results can be useful in the design and optimization of nano-devices and provide new criteria in numerical analyses.

Barretta *et al.* [61] used non-local strain-driven and stress-driven models to study the functionally graded elastic annular nano-beams. The Bi-Helmholtz kernel function was used in their analysis, and they extracted the torsional-rotational elastic function of nano-beam with a clamped-clamped boundary condition under concentrated and uniform bending moment as an analytical solution. They also examined their findings with the results of nano-beam modeling based on the strain gradient model. Their results indicated that the rotation angle decreases with increasing the size parameter, showing the hardening effect of the stress-driven model. Moreover, it was demonstrated that the non-local strain-driven model could not be a suitable model for the analysis of structural mechanics. As an alternative, the stress-driven model is mathematically and mechanically appropriate for nanotechnology applications. The obtained results can be applied in the design and optimization of modern micro- and nano-devices.

Barretta *et al*. [62] also investigated the size-dependent free vibrations of a nano-rod. By selecting the non-local stress-driven model and the Helmholtz kernel function, they could derive the equation of motion governing this nano-rod. Afterward, they calculated the natural frequencies and mode shapes of the nano-rod by analytically solving the free vibrations problem. Moreover, in order to evaluate the efficiency of this model, longitudinal vibrations of nano-rod with the non-local strain gradient model were derived, and also natural frequencies and vibration mode shapes were re-derived and compared with the results obtained from the analyses of the stress-driven model. They indicated that the size parameter significantly affects the longitudinal vibrational behavior of the nano-rod, so that the natural frequencies increase with raising this parameter. It was also demonstrated that both non-local stress-driven and strain gradient models have hardening behaviors with increasing the size parameter and this effect is for the stress-driven model is higher than that for the strain gradient model. Meanwhile, the difference between the natural frequencies and the mode shapes increases for both models with raising the vibration mode and the value of the length scale parameter. As the size parameter approaches zero, this difference also approaches zero. It was also indicated that the difference between the predicted natural frequency with the strain gradient and the non-local stress-driven theories decreases as the aspect ratio of nano-rod increases. They also concluded that the natural frequency of nano-rod with clamped-clamped boundary conditions is two times higher than the frequency obtained for the cantilever boundary conditions.

Apuzzo *et al*. [63] investigated the size effect on the tensile and torsional vibrations of a nano-rod with different boundary conditions using a biphasic stress-driven integral model. They indicated that the biphasic stress-driven integral model could be easily replaced by the constitutive boundary conditions by applying an equivalent differential equation. They also derived the natural frequencies of nano-rod using exact solutions and compared them with the results of the strain-driven and strain gradient models. The results demonstrate that the value of natural frequency of a nano-rod modeled based on biphasic stress-driven and strain gradient models has increased, while for a strain-driven model, this value was decreased with raising the size parameter. Accordingly, it was concluded that the stress-driven and strain gradient models resulted in more hardening of nano-rod, while the strain-driven model had the opposite results. Moreover, the rate of increase in natural frequency in the stress-driven model was higher compared to the strain gradient model. It is noteworthy that this model can provide an effective method for dynamic analysis of nano-devices, such as the modern beam used in nanotechnology.

Bian *et al*. [64] using the one-dimensional stress-driven integral model with bi-Helmholtz kernel function, investigated the effect of non-local parameters on the static tensile behavior and free vibrations of a micro-rod with different boundary conditions. They extracted precise tensile displacements of a micro-rod under different boundary and loading conditions through applying boundary and constitutive equations and solving the equations by Laplace transform. The vibration frequencies of a micro-rod with cantilever and clamped-clamped boundary conditions were also obtained. The results indicated that the non-local microrod model is reduced to a local rod model as the non-local parameters approach zero. The exact solutions extracted in this article can be applied as a criterion for validating approximate or numerical methods for solving the complex mechanical problem.

(B). Nano-beam

In a non-linear strain-driven elasticity model proposed by Eringen, the stress at one point of the material under loading is the result of the strain of all points in the material. This relationship is determined using a Fredholm integral equation, in which the stress is the convolution output between the elastic strain and a kernel function dependent on a non-local parameter [65]. However, in the stress-driven non-linear elasticity model, the strain at one point of the material under loading is the result of the stress of all points in the material. This relationship is determined using a Fredholm integral equation, in which the strain is the convolution output between the stress and a kernel function dependent on a non-local parameter[67 ,66]. Romano and Barretta [67 ,66] indicated that in the Euler-Bernoulli beam model, the elastic curvature field is a linear point function of the bending interaction domain. In the non-local stress-driven law of elasticity, the elastic curvature field is expressed to the bending interaction and an averaging kernel by an integral convolution between the local elastic response. It was also indicated that this integral relationship could be easily transformed into a differential equation by applying constitutive boundary conditions compatible with the physics of the problem. However, the constitutive conditions for the integral relationship of the strain-driven model are not compatible with the physics of the problem and will lead to some challenges [68].

Apuzzo *et al*. [69] applied a non-local stress-driven integral model to investigate and estimate the size effect on the free vibrational behavior of a nano-beam. The equations of motion of the nano-beam were calculated using the Euler-Bernoulli model and solved for different boundary conditions by employing numerical integration methods and using MATLAB software, and the first natural frequency of the nano-beam was obtained. Moreover, in order to compare the results, the first frequency of the nano-beam was calculated with two Eringen's non-local differential models and strain gradient theory. It was also indicated that the stress-driven model, similar to the strain gradient model for all boundary conditions, results in the hardening of the structure, and the natural frequency increases with raising the size parameter. Moreover, the highest value of frequency is achieved for the stress-driven model. However, with increasing the size parameter, the Eringen differential model has led to softening of the structure and a decrease in the value of natural frequency except for the cantilever boundary conditions.

Barretta *et al.* [70] used a non-local stress-driven model with a bi-Helmholtz kernel function to investigate and estimate the size effect on the bending behavior of a nano-beam. The equations of motion of the nano-beam were calculated using the Euler-Bernoulli model, and the deflection function was computed for different boundary conditions. The results indicated that the use of stress-driven models results in the hardening of the structure and reduces the beam deflection. In the meantime, this phenomenon is independent of special loading conditions and kinematic boundary conditions.

Faraji Oskouie *et al.* [71] investigated the bending behavior of a nano-beam according to a Euler-Bernoulli model with different boundary conditions. The non-local integral stress-driven and strain-driven models were used for this purpose. Moreover, the governing equation based on non-local stress-driven and strain-driven models was obtained by reducing the total potential energy into two weak and strong forms. Furthermore, finite difference and trapezoidal integration methods were employed to solve the governing equations of beams. This numerical method was proposed for a direct solution to overcome the challenges of converting the governing integral equation to the differential equation. The results show that this numerical method can be used effectively for non-local strain-driven and stress-driven models. It was also indicated that increasing the non-local parameter based on strain-driven and stress-driven models has softening and hardening effects, respectively. Moreover, it was demonstrated that the paradox related to the bending of a cantilever nano-beam is solved by providing an integral strain-driven model.

Faraji Oskouie *et al.* [72] investigated the vibrational behavior and buckling of an Euler-Bernoulli nano-beam with different boundary conditions. The non-local integral stress-driven and strain-driven models were used for this purpose. In addition, finite difference and trapezoidal integration methods were employed to solve the governing equations of beams. By applying this numerical method, the governing integral equation is directly solved, and the challenges related to the conversion of the governing integral equation to a differential equation are eliminated. Moreover, a comparison between the results of strain-driven and stress-driven models regarding the vibration response and buckling of nano-beam was performed in different final conditions. The results indicate that according to the stress-driven model, the frequency and critical buckling load increase with raising the non-local parameter; however, these parameters decrease when using the integral strain-driven model. This conclusion holds true for higher modes as well. Furthermore, the use of the integral strain-driven model, unlike the differential strain-driven model, does not lead to contradictory results in the vibration analysis of cantilever nano-beams.

Faraji Oskouie *et al.* [73] investigated the vibrational and bending behaviors of a Timoshenko nano-beam with different boundary conditions. They obtained the governing equations of the Timoshenko nano-beam based on the strain gradient model, the integral stress-driven model, and the stress-driven differential model with constitutive boundary conditions. The finite difference method and trapezoidal integration were also applied to solve the free vibrations and static bending problems. It was indicated that there is a good agreement between the results obtained from solving differential and integral equations of non-local stress-driven models. Several comparative studies have also been performed to analyze the vibrational behavior and bending of the beam based on strain gradient and non-local stress-driven models.

The results indicated that in all three models, increasing the non-local scale parameter led to an increase in the natural frequency and a decrease in the deflection caused by the bending. It was also concluded that the use of these three models resulted in the hardening of the beam, and the rate of beam hardening related to the strain gradient model is higher compared to that related to the non-local stress-driven model.

Barretta *et al.* [74] using a combination of local and non-local biphasic stress-driven and stress-driven models, were able to investigate the bending behavior of a nano-beam made of FGMs based on the Euler-Bernoulli geometric model with different boundary and loading conditions. According to their investigation, the non-local strain-driven model cannot be used in constitutive mechanics, and the combination of local and non-local models based on the Eringen integral model can partially overcome the weakness of the model. The singular behavior of continuous nano-structures appears if the local fraction tends to disappear; therefore, the weakness of the Eringen integral model is not eliminated. In contrast, the combinations of local and non-local models based on the stress-driven theory are mathematically and mechanically suitable for nano-systems. It was also shown that the deflection for all loadings and boundary conditions decreases with increasing the value of size parameter, indicating that this model causes more hardening of the structure; however, the deflection for all loadings and boundary conditions increases with raising the value of the combined parameter, demonstrating that this model causes more softening of the structure. Finally, the results indicated that the bending moment resulted from the biphasic strain-driven model is smaller than the local moment, and also, the bending moment resulted from the biphasic stress-driven model is greater than the local moment. Therefore, the results presented in this study can be applied in the design and optimization of nano-devices.

Barretta *et al.* [75] studied the vibrational behavior of nano-beam made of FGMs with different boundary conditions based on non-local stress-driven and strain gradient models. They also applied the Euler-Bernoulli and Timoshenko beam hypotheses. It was indicated that for both non-local stress-driven and strain gradient models, the natural frequencies increase with raising the size parameter; this increase is greater for the stress-driven model. Moreover, as the small-scale parameter approaches zero, the natural frequencies calculated by both the non-local strain gradient and stress-driven models become consistent with local models. The difference between the vibrational results of the non-local stress-driven and strain gradient theories increases with raising the size parameter. Meanwhile, the difference between the vibrational responses of Timoshenko and Euler-Bernoulli nano-beams is also eliminated with increasing the values of aspect ratio. In addition, the effect of rotational inertia on the principal natural frequencies of the Euler-Bernoulli nano-beams, which was predicted by each of the size-dependent models, is more significant in lower rations. It was concluded that the non-local stress-driven integral model provides an effective approach to describe the vibrational behavior of nano-beams made of FGMs based on the Timoshenko beam geometric model. The relevant results can also be easily applied in the design of modern rod components such as nano-electromechanical systems.

Barretta *et al.* [76] investigated the bending behavior of the size-dependent elastic nano-beams exposed to bending. In this analysis, the integral stress-driven and strain-driven models with different kernel functions, error function of probability theory, and bi-Helmholtz kernel function were used, and the results were compared. The geometric Timoshenko beam hypothesis was also employed to make a relationship between the displacement and strain fields. It was indicated that integral equations could be replaced by suitable second-order differential equations and constitutive boundary conditions through selecting the bi-Helmholtz kernel function. The performance of the non-local stress-driven model was also evaluated by assessing the size-dependent behavior of the elastic Timoshenko nano-beams with cantilever, simple support, and doubly-clamped boundary conditions under concentrated and uniform distribution loads, and the accuracy of this model was demonstrated. The results indicated that the deflection of nano-beam decreased with increasing the value of the non-local parameter, which indicates that the stress-driven model results in more hardening of the structure.

Barretta *et al*. [77] studied the non-local thermoelastic behavior of nano-beams. By solving several cases, they indicated that the new integral thermo-elastic stress-driven model with the Helmholtz kernel function is equivalent to a higher-order differential equation with constitutive boundary conditions.

Mahmoudpour *et al*. [78] investigated the effect of size parameters and the increase of ambient temperature on the non-linear vibrations of a nano-beam made of FGMs. The size effect, which plays an important role in the vibrational behavior of nano-beams, was modeled using two non-local stress-driven and strain-driven models. In this study, the vibrational equation governing the structure was extracted using the Euler-Bernoulli beam and von Kármán strain-displacement relation. Meanwhile, this equation was reduced to a non-linear ordinary differential equation using the Galerkin method for the first mode of vibration, and an analytical solution for different boundary conditions was derived using the Homotopy analysis method. The results indicate that as the size parameter increases, the non-linear natural frequencies resulted from the stress-driven model increase as well; while, for the strain-driven model, in addition to the cantilever beam boundary condition, increasing the size parameter reduces the non-linear natural frequencies. Moreover, the increase in the temperature for both stress-driven and strain-driven models has resulted in decreasing and increasing the non-linear frequency, respectively.

Barretta *et al*. [79] investigated the bending behavior of a nano-beam based on the Timoshenko geometric model with different boundary conditions and different loadings by applying a combination of local and non-local stress-driven models and biphasic stress-driven model. The non-local part of this biphasic model is in the form of a convolution integral between the stress field and the biexponential averaging kernel function, which is specified by a size parameter. They have indicated that the biphasic stress-driven model is equivalent to a differential problem with constitutive boundary conditions involving bending and shear fields. The governing equations on Timoshenko nano-beam were also analyzed for different boundary and loading conditions. The results demonstrate that as the size parameter increases, the deflection for all loadings and boundary conditions decreases, which shows that this model causes more hardening of the structure.

Pinnola *et al*. [80] studied stochastic flexural vibrations of small-scale Bernoulli–Euler nano-beam with external damping based on a non-local stress-driven model. This investigation can be useful for designing and optimizing the constitutive components of modern small-scale devices, such as micro- and nano-electromechanical systems. They modeled both the size effect and damping phenomenon with the non-local stress-driven model and external viscosity interactions, respectively. In this analysis, the accidental input assumption in loading was applied to simulate accidental external load. The exact solutions of power spectral density, correlation function, and variance of displacement fields have also been investigated by differential eigen-analysis. The results show that increasing the non-local scale parameter leads to a significant reduction in stationary variances and transition of the processing time in the responses, as well as an increase in the system's natural frequencies.

Roghani and Rouhi [81] investigated the non-linear bending behavior of a nano-beam made of FGMs with different boundary conditions using a non-local stress-driven model. These results can help to understand the non-local phenomena of nano-electromechanical systems. For this purpose, they obtained the governing equations of beams using the integral and differential forms of the stress-driven model by the Timoshenko beam hypothesis. The GDQ method was also applied to solve the equations. In this analysis, the effect of non-local parameters, the heterogeneity index of the FGMs, length to thickness ratio, and non-linearity of the bending deformation for different boundary conditions were evaluated. It was indicated that the inconsistency related to the cantilever boundary conditions is resolved by the present method. Their results show that the deflection of the beam decreases for a given loading with increasing the heterogeneity index of the FGMs. Moreover, the deflection value of the beam decreased with increasing the non-local parameter. Finally, it was concluded that the decrease in the length to thickness ratio resulted in a reduction of beam deflection.

Apuzzo *et al*. [82] studied the vibrational behavior of a nano-beam with different boundary conditions and in several modes of vibration according to the Euler-Bernoulli assumption and using a biphasic local/non-local combined stress-driven model. They indicated that the combination of local and non-local stress-driven models with the definition of a combined parameter could be used as a suitable model to investigate the vibrational behavior of nano-beams. Moreover, they were able to extract the first four natural frequencies of the structure as a function of non-local combined parameters by analytically solving the governing equation of the nano-beam. Besides, in order to evaluate the efficiency of the presented model, the results were compared and validated with the strain gradient model. It was shown that natural frequencies increase with increasing the size parameter. The results indicate that the highest increase is related to clamped-clamped boundary conditions, and the lowest increase is related to simply supported boundary conditions. Meanwhile, the rate of increase raises as the mode of vibration increases. In addition, increasing the combined parameter has led to a decrease in the natural frequency. Finally, it was shown that the proposed biphasic local/non-local combined model is consistent with the results available in the literature, and the mechanical evaluation also provides an adaptation of size effects in nano-beams under free vibrations for higher modes of vibration.

Barretta *et al*. [83] examined the buckling behavior dependent on the size of a nano-beam with different boundary conditions and Euler-Bernoulli beam geometric relations using the stress-driven model and the bi-Helmholtz kernel function. They also compared and validated the extracted results with the findings obtained from the non-local strain gradient model. It was also shown that the non-local stress-driven model predicted critical loads better than the strain gradient model and can provide an effective guide for the analysis and design of modern devices for nanotechnology applications.

Darban *et al*. [84] studied the buckling behavior of nano-beam on a two-parameter elastic substrate under different boundary conditions. They used the Euler-Bernoulli beam theory and the non-local stress-driven model for their analysis. Moreover, in this study, the governing buckling equation of nano-beam was numerically solved, and the buckling loads of nano-beam were extracted for higher modes with different boundary conditions, non-local parameters, and surrounding elastic medium. The results of the proposed model are consistent with the results available in the section of nano-beams without elastic foundation. It was indicated that the buckling load increased as the non-local parameters, Winkler's parameters, and Pasternak's parameters increased. It was also demonstrated that the effect of non-local parameters on buckling loads was greater for higher buckling modes. While the relationship of buckling load with the non-local parameter and the Winkler modulus is non-linear, buckling load increases linearly with the increase of the Pasternak modulus. The highest and lowest impacts of elastic foundation properties were on buckling loads of the nano-beam with cantilever and clamped-clamped boundary conditions, respectively.

Luciano *et al*. [85] using the non-local biphasic stress-driven model and the Timoshenko beam hypothesis, investigated the bending and buckling behaviors of a nano-beam made of FGMs applied in micro-electromechanical or nano-electromechanical systems. They also provided some variable formulas for the numerical solution of bending and buckling problems and validated their results by comparing numerical solutions with exact solutions of bending problems.

Luciano *et al*. [86] by taking advantage of a non-local stress-driven model, investigated the free flexural vibrations of a nano-beam with non-classical supports and elastic support with transversal and flexural springs. These supports simulate the free, simply supported, and clamped boundary conditions in extreme conditions when the springs' stiffnesses approach zero or infinite. In this study, the Euler-Bernoulli beam hypothesis was applied. Moreover, an analytical solution method was used to calculate the natural frequencies of nano-beam. In this investigation, the effects of the elastic support and non-local parameter on natural frequencies at higher vibrational modes were studied. Besides, the comparison between the extracted solutions and the solutions available in the literature demonstrated an extremely good agreement between them. According to the results, it was observed that natural frequencies were dependent on the mentioned parameters. In other words, as the spring's stiffness and the non-local parameter increases, the stiffness of the nano-beam increases, which results in an increase in the natural frequencies. In addition, the second and third vibrational modes were studied, and a higher dependence of natural frequencies on non-local parameters was observed in higher vibration modes. The results of this article indicated the important effect of elastic deformations of elastic supports on natural frequencies of the nano-beam.

He *et al*. [87] investigated the effect of non-local parameter and boundary conditions on the free vibrational behavior of a micro-beam. In this paper, equations of motion of the micro-beam were derived based on the geometric relations of Euler-Bernoulli and Timoshenko beams and using a non-local stress-driven integral model. In addition, the characteristic frequency equation was derived using the Laplace transform method, and finally, the natural frequency of the structure was obtained in terms of non-local parameters and boundary conditions. Afterward, the effects of the non-local parameter on the natural frequency of Euler-Bernoulli and Timoshenko beams were numerically evaluated and validated with the results of different studies. The numerical results indicated that as the non-local parameter approaches zero, the non-local stress-driven integral model is reduced to the corresponding classical beam model. Moreover, a size-dependent hardening effect was observed under different boundary conditions; it means that the natural frequencies of the structure increase as the non-local parameter raises. Furthermore, due to the reduction in beam strength resulted from shear deformation of the Timoshenko beam, the vibrational frequencies of the Timoshenko beam model are always lower than the frequencies of the Euler-Bernoulli model. Meanwhile, the frequency predicted by the Timoshenko beam model increases as the length to thickness ratio of the beam increases, which is due to a decrease in the share of shear deformation in the beam's deformation.

Jiang *et al*. [88] using a non-local stress-driven integral model, investigated the effect of non-local parameter on buckling behavior and the value of the critical load of Euler-Bernoulli and Timoshenko beams with different boundary conditions. In this paper, the Laplace transform was applied to solve integro-differential equations. Taking into consideration of the boundary conditions and additional Constitutive boundary conditions, they explicitly obtained the characteristic equations for the Euler-Bernoulli and Timoshenko beams under different boundary conditions, which can be applied to calculate the critical buckling load. Moreover, the effect of non-local parameters on the buckling behavior of Euler-Bernoulli and Timoshenko beams and the value of critical load these beams were evaluated numerically. The results indicated that the integral stress-driven model of the beam could be reduced to a local classical model as the non-local parameter approaches zero. Furthermore, it was shown that the value of the critical load is increased for all boundary conditions with increasing the non-local parameter, which indicates the hardening effect of the non-local parameter as well as the stress-driven model. Also, the lowest and greatest effect of non-local parameter were for simply support-guided support and clamped-clamped boundary conditions, respectively. Moreover, the critical load for the Timoshenko beam model increased as the length to thickness ratio of the beam raised.

Zhang and Qing [89] studied the buckling behavior of a micro-beam made of FGMs using an integral stress-driven model and the geometric relations of the Timoshenko and Euler-Bernoulli beams. They also applied the minimum total potential energy method to derive the governing equations and relevant boundary conditions. In this study, the Fredholm integral constitutive equations are transformed into the Volterra type of the first kind; then, it was solved analytically using the Laplace transformation and its inversion under different boundary conditions. Moreover, the results obtained from the analytical solution were compared and validated with the available results in the literature. The effects of the non-local parameter, the thickness to shell ratio of kernel layers, the heterogeneity index of FGMs, and the length to height ratio of the beam on the buckling loads were investigated. The results indicated that with increasing the non-local parameter, the value of critical load increased from the lowest to the highest for Simply -Simply, Simply -clamped, and clamped-clamped boundary conditions in respective order. Furthermore, due to the effect of shear deformation on the Timoshenko beam, the value of critical load was less than that for the Euler-Bernoulli beam. Besides, by taking into consideration of the non-local stress-driven model, the difference between these two theories is significantly small for the beams with supported boundary conditions, while for support-clamped and clamped-clamped boundary conditions, this difference increases as the non-local parameter increases. Meanwhile, by increasing the heterogeneity index of FGMs, the buckling loads of the sandwich micro-beam first decrease and then become stable.

Zhang and Qing [90] investigated the flexural behavior of a functionally graded materials micro-beam using a two-phase nonlocal strain-driven and stress-driven model. In this paper, the equations of motion are derived based on Timoshenko beams models and solved them using analytically based on using bi-Helmholtz kernel and the Laplace transform method for different boundary and loading conditions. For all boundary conditions and loading, increasing the nonlocal length scale parameter shows consistently softening or stiffening effect on the beam behavior, according to the strain- or the stress-driven model, respectively. In addition, they showed that, by increasing the heterogeneous parameter of the functionally graded materials, the non-dimensional deflection increases, while the softening effect caused by the strain-driven nonlocal phase seems to be more significant.

Zhang *et al*. [91] investigated the effect of size parameter on the static bending behavior of a functionally graded material curved nano-beam based on the Timoshenko beam model as well as the stress-driven model. They solved the equations are analytically for different boundary conditions and loading using Laplace transform technique and its inversion. According to the numerical results, it can be seen that for a Timoshenko nano-beam with different boundary conditions and loading, increasing the non-local parameter has led to a decrease in the non-dimensional deflections and an increase in the stiffness of the beam. Also, non-dimensional deflections have decreased with increasing heterogeneity index for all boundary conditions. In addition, comparing the results of the Timoshenko beam with the results of the Euler-Bernoulli beam, it is observed that in simply-clamped, clamped-clamped, and free-clamped nano-beam, the effect of shear deformation increases with increasing non-local parameter; which cannot be ignored.

Penna *et al*. [92] investigated the effect of size parameter on the nonlinear vibration behavior of a nano-beam with an initial curvature, an initial tensile force, and made of functionally graded material with different boundary conditions. Also, they investigated the effect of displacement and initial tensile force on the nonlinear vibrations of the structure. In order to derive the equation of motion, the geometric relations of the Euler-Bernoulli beam based on von Karmen strain and two non-local stress-driven and strain-driven models have been used, and the effect of these two models has been compared with each other. Their results show that for the two models of strain-driven and stress-driven, for each boundary condition and the initial displacement, the increase of the parameter size has led to the decrease and increase of the natural nonlinear frequency, respectively. Also, the effect of initial displacement for low-amplitude vibrations was negligible, while this effect had a significant effect on high-amplitude vibrations. In addition, for each value of the size parameter and boundary condition, increasing the amplitude of the initial tensile force leads to an increase in the nonlinear natural frequency.

Barretta *et al*. [93] investigated the effect of non-local parameter and boundary conditions on the dynamic behavior of a micro-beam under axial loading. The equations of motion are derived based on the geometric relationships of Euler-Bernoulli beams using the non-local stress-driven integral model and the two-phase strain-driven model. In this study, the natural frequencies calculated using the two models are compared with each other. The effects of non-local thermoelastic and initial axial force (tension and compression) on dynamic responses are also analyzed and discussed.

Vaccaro *et al*. [94] studied the bending behavior of a thick nano-beam with an initial curvature under different boundary conditions. They obtained the governing equations of nano-beam based on geometric relations of the Timoshenko beam and the biphasic stress-driven integral model with constitutive boundary conditions. Moreover, several comparative studies were performed in this article for analyzing the bending behavior of the beam based on a biphasic non-local stress-driven model. The results show that increasing the non-local scale parameter leads to a decrease in the deflection caused by the bending, which indicates that the use of this model resulted in beam hardening.

(C). Nano-plate

Barretta *et al.* [95] investigated the bending behavior of circular and annular nano-plates with different boundary and loadings conditions and based on non-local stress-driven and strain gradient models. They also employed the classical plate hypothesis. This analysis can be useful in designing and optimizing plate-like components of ground-breaking Nanoelectron-Mechanical-Systems (NEMS). The results indicate that except for the plate with simply supported boundary conditions and uniform boundary couples, the deflection of the plate decreased for a given load with increasing the non-local parameter for both stress-driven and strain gradient models compared to the deflection obtained from the local model. Moreover, for the mentioned boundary and loading conditions, the deflection of the plate based on the strain gradient model was equal to the deflection of the local model, while for the stress-driven model, the deflection decreases as the non-local parameter increases. Therefore, it was indicated that the strain gradient model for the simply supported boundary condition with bending couple on the boundaries cannot simulate the size effect and is therefore not a suitable model.

Farajpour *et al.* [96] studied the vibrational and bending behaviors of a rectangular nano-plate with quad-simply supported and quad-clamped boundary conditions under different loadings and based on non-local stress-driven and strain gradient models. They derived the governing equations of the structure by applying the classical plate hypothesis and applying Hamilton's principle. Moreover, they solved the equations governing vibrational behavior and nano-plate bending based on the two non-local models using the GDQ numerical method, extracted the natural frequencies and maximum deflection of the structure for different boundary conditions and various loads, and compared the results of these two models. The results indicate that for a specific loading and boundary condition, with increasing non-local parameter, the deflection of the plate and natural frequency decreased and increased respectively for both stress-driven and strain gradient models compared to the local model. Furthermore, this effect for the stress-driven model was greater than the strain gradient model. In addition, in both non-local models, the clamped boundary conditions are more affected compared to simply supported boundary conditions.

Shariati *et al.* [97] studied the vibrational behavior of circular nano-plate and calibrated small-scale parameters of non-classical continuum theories such as nonlocal strain gradient theory, strain gradient theory, stress-driven nonlocal elasticity, and strain-driven nonlocal elasticity. They solved the equation of motion using the general differential quadrature rule (GDQR) and determined the first natural frequencies for different radiuses, and different size parameters were obtained. On the other hand, the first natural frequencies of circular nanoplates are calculated using molecular dynamics simulation based on AIREBO and Tersoff potentials for different radiuses. Fast Fourier transform (FFT) was utilized to calculate natural frequencies based on the molecular dynamic simulation (MD). Using the accurate size parameter is an important point in the application of non-classical continuum theories. To obtain the size parameters related to different non-classical methods, the results of molecular dynamics compared to those of nan-classical methods and simulated annealing (SA) algorithm optimization technique was utilized. Results show that stress-driven nonlocal, strain-driven nonlocal, and strain gradient methods cannot predict the behavior predicted by molecular dynamics for all ranges of radius. In other words, the responses of these three methods for any value of the size parameters (in some interval radius) and results of the molecular dynamics method are not equal for a few numbers of studied radii. In contrast to these three methods, the nonlocal strain gradient method predicts the results obtained by molecular dynamics well for all radii. The results of this paper are very useful for researchers in the field of non-classical continuum mechanics.

Shishehsaz *et al*. [98] analyzed the vibrational behavior of functionally graded size-dependent annular nano-plate using the stress-driven nonlocal integral elasticity, as well the strain gradient theory was used in conjunction with the classical plate theory. The resulting equilibrium equations were solved using the generalized differential quadrature rule (GDQR). They showed the influences of various parameters such as; size-effect parameter, material heterogeneity index, the aspect ratio of the inner to outer radii, and the effects of different boundary conditions on the vibrational behavior of the nano-plate, based on different types of boundary conditions. They indicated that the natural frequencies of the FGM nanoplate increase with an increase in the heterogeneity index n, and the increase in size-effect parameter show a similar effect in both models.

Shariati *et al*. [99] modeled the axisymmetric-vibrational behavior of a size-dependent circular nano-plate with functionally graded material with different types of boundary conditions using the Stress-driven model (SDM) and Strain-gradient theory (SGT) in conjunction with classical plate theory. They used Hamilton's principle to obtained the governing equations of motion and their corresponding equations for boundary conditions and solved them using the generalized differential quadrature rule. Their results indicate that the natural frequencies of the mentioned structure increase with an increase in the heterogeneity index n, and size-effect parameter Lc. Additionally, these parameters appear to have a stiffening effect on the nano-plate vibrational behavior. However, for a nano-plate resting on a knife or simply supported edge, in the first mode, the SDM shows a more stiffening effect on the plate behavior as compared with the SGT. Nonetheless, for the clamped and free edge boundary conditions, both models predicted the same behavior. The SGT showed a higher-stiffening effect only in the fourth mode for all types of considered boundary conditions.

(D). Nano-frame

Russillo *et al*. [100] evaluated the dynamic responses dependent on the size parameter of nano-frame using the non-local stress-driven model and the finite element method. They applied Wittrick-William's algorithm to calculate frequencies and natural modes and claimed that this two-dimensional formulation could be generalized to three-dimensional structures. Their proposed method is a presentation of the first example of a non-local element with two nodes, the dynamics of which are precisely related to the dynamic stiffness matrix method. It means that the frame can be modeled by a single, exact element without any internal mesh. The stress-driven approach offers a consistent nonlocal description of size effects that do overcome the theoretical flaws of alternative nonlocal formulations. Finally, the use of the Wittrick–Williams algorithm ensures that all natural frequencies are calculated precisely. It seems that this method provides an effective and robust tool for investigating size effects on nano-frames in a general framework of non-local mechanics.

(E). Nano-tube

Sedighi and Malikan [101] investigated the vibrational behavior of a carbon/boron-nitride hetero-nanotube under a magneto -thermal environment based on a non-local stress-driven differential model and using the finite element method. They discussed the effect of length ratio, thermal gradient, and size dependence on non-linear vibrations of composite carbon/boron-nitride hetero-nanotube. In order to deal with the dynamic behavior of size-dependent nano-tubes in more detail, a bar element with two nodes and six degrees of freedom was introduced, which included the node values of bending deformation, slope, and curvature. In comparison with the normal beam element, the node displacement vector of the proposed element has an additional component that indicates the curvature of the node to be matched with the non-local stress-driven beam model. The non-linear term related to the von Kármán strain was added to the equation of motion. The nano-tube structure was also subject to temperature changes and was surrounded by a magnetic field. By matching the amplitude of non-linear frequencies, the obtained results in this study were in line with the reports presented in the literature, and a detailed investigation was performed to evaluate the effect of different parameters on the vibrational behavior of the considered nano-hetero structure. The results indicate that the increase in initial deformation significantly has a significant effect on the non-linear frequency. Moreover, altering the maximum amplitude and length ratio leads to the change of the node location and the maximum amplitude of the mode shape. This change is more sensitive to the values of length ratio compared to the value of the initial increase. Meanwhile, at room temperature, increasing the temperature reduces the natural frequencies; on the other hand, at high temperatures, its effect is in the form of increasing the values of non-linear frequencies. Finally, it was indicated that the effect of the non-local parameter in the stress-driven method leads to an increase in the non-linear frequency of the system and its effect is more significant in lower modes of vibration.

Sedighi *et al*. [102] conducted a comprehensive study on the vibrational properties and critical divergence rate of a hybrid carbon/boron-nitride nanotube, which is used as a transmitter of magnetic fluids. The effects of size-dependence, magnetic field, and thermal environment on the dynamic behavior of systems were considered in this model. In this paper, the effect of length ratio, magnetic-thermal field, and size-dependence on divergence rate, natural frequency, and mode shapes of a fluid transmitted hybrid nano-tube were investigated. Moreover, the governing equations of the nano-tube were solved by applying the finite element method. It was indicated that the non-local strain-driven differential model showed inconsistency when used in bounded continua of applicative interest; however, the non-local stress-driven theory resulted in the establishment of appropriate non-local elastic formulas. It was also shown that using hetero-nanotube caused higher natural frequencies and also higher critical divergence rate compared to a uniform tube made of carbon atoms. Moreover, in a low-temperature environment, the divergence rate raised with each increase in temperature. While in a high-temperature environment, the temperature gradient resulted in a decrease in the critical rate. Finally, it was concluded that in higher modes, hetero-nanotubes are more sensitive to changes of the non-local parameter. The results of this study are of great importance for the use of hybrid nano-tubes in the design of new nano-tubes and will be significantly effective in tuning the vibrational properties of biological devices composed of nano-tubes.

Ouakad *et al*. [103] evaluated the effect of material properties, non-local parameter, and Lorentz and electrical forces on the bending and vibrational behaviors of a hybrid carbon/boron-nitride hetero-nanotube under thermal loads. Moreover, the Euler-Bernoulli beam model, the Green-Lagrange small strains, and average rotations in non-linear geometry of nano-tube were used in this study. The non-linear equations of motion for the sections of clamped carbon/boron-nitride nano-tube were also extracted using D'Alembert's principle and the non-local stress-driven model. The solutions of the equations were also extracted using the Galerkin modal expansion method, and the validation of results and the parametric investigation were presented in general. The results show the effect of temperature change, non-local parameter, the length ratio of carbon and boron-nitride components, and direct current and alternating current voltages on the changes of the main frequency and maximum deflection of the hybrid nanotube. According to the reported results, all these parameters can affect the dynamics of composite nano-tube. However, the effect of each parameter is not as same as the effect of the other parameter.

(F). Nano-shell

Yang *et al*. [104] using the non-local strain gradient model, investigated the size-dependent non-linear buckling and the post-buckling behaviors of a micro-shell made of FGMs under hydrostatic pressures. In this study, a combination of two non-local strain gradient and strain-driven models and a non-local strain gradient model were applied to model the size effect. Moreover, after deriving the governing differential equations and by applying the boundary-layer theory, the shell buckling and solution procedure based on the perturbation method were used to obtain the stability paths of the micro-shell under hydrostatic pressures. It was indicated that the effect of increasing the size parameter in the strain-driven model leads to a decrease in the critical hydrostatic pressure and critical shortening of FGM micro-shells, while this increase for the size parameter of the strain gradient model results in an increase of these parameters. Furthermore, it can be concluded that the effect of the length scale parameter of strain gradient on the non-linear instability characteristics of micro-shell under hydrostatic pressures is slightly greater than that of the strain-driven model. The most important points regarding this research are the use of the strain-driven model and not employing the stress-driven model, while it has been mentioned in the title that the stress-driven model has been used.

Conclusion

In the last two years, researchers have discovered deficiencies in the use of Eringen's nonlocal elasticity theory, especially its differential form. The differential form presented by Eringen is partially obtained from simplifying the integral form using the Green's function properties is incomplete. This model applies only to the infinite domain. Therefore, in real matters, a series of additional constraints must be included in the issue. These constraints are known as structural constraints. In most cases, these constraints are in conflict with the physics of the problem and sometimes lead to incorrect solutions. Therefore, to address this problem, a new model is introduced as stress-driven, which in its differential state, has compatible boundary conditions with the physics of the problem and can provide appropriate responses. In this paper, the stress-driven model was investigated in modeling mechanical structures such as beams, columns, shells, and frames. As indicated in the literature presented, the model predicts a more rigid behavior for the structure, as well as the strain gradient theory. In this way, dynamic problems are accompanied by increasing frequency. As well as in static and flexural issues, it leads to a reduction in the deflection caused by external forces and buckling problems, leading to increased critical loads.

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