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RESEARCH PAPER

Phase II Monitoring of the Ordinal Multivariate Categorical Processes

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Abstract

Statistical variables are divided into two categories: nominal and ordinal, both of which have many uses. In some statistical process monitoring applications, the quality of a process or product is described by multiple ordinal quality characteristics, which is called ordinal multivariate process. An ordinal contingency table is used to show the relationships between these variables and is modeled on an ordinal log-linear model. In our manuscript, two new statistics including simple ordinal categorical and Generalized-*p* are developed for Phase II monitoring the ordinal log-linear model-based processes. The performance of the proposed statistics will be evaluated using some simulation studies and real-world numerical examples. The results show the advantages of a simple ordinal category control card. In addition, the performance of these statistics is accessed through sensitivity analysis of the row and column sizes of the contingency table. Meanwhile, a sensitivity analysis with three and four categorical factors is performed and similar results are obtained.

Keywords:
Multivariate Processes;
Statistical Process Monitoring;
Ordinal Variables;
Phase II;
Contingency Table

Introduction

Some processes, known as multivariate ordinals, include multiple ordinal factors using a multivariate categorical chart based on an ordinal log-linear model. Two types of log-linear models, including the nominal log-linear model (NMLLM) and the ordered log-linear model (OLLM), are designed to correlate expected numbers of multivariate categorical characteristics with two or more control factors. In SPM, contingency tables are used for simultaneous monitoring of multivariate category processes [1]. In addition, OLLM is used to show the relationship between ordinal factors and their corresponding observations in contingency table (OCT) cells. For example, the independent hypothesis with 2×n OCT are tested and odds ratios for several conditions are calculated by Subramanyam and Rao [2]. The category Pearson Kai-square have been developed in a three-way contingency table under an orthogonal multinomial distribution [3]. Zafar [4] OLLM was used in conjunction with correspondence analysis in the pharmaceutical industry to predict opiates in the drug detection process. Yamamoto and Morakami [5] proposed a model for square OCT. In this research, the assumption of unbalanced

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normal distribution was also considered which was applied for caries of the teeth. Model for OCT analyzing by considering linear effect, rows, columns and concurrent consequence [6].

Soleymanian et al. [7] proposed Phase II monitoring of the binary logistic regression profiles based on four approaches including T^2, Multivariate Exponentially Weighted Moving Average (MEWMA), Likelihood Ratio Test (LRT) and LRT/EWMA. Atashgar [8] provided a review paper on the multivariate processes monitoring methods based on the artificial neural network. In this paper, analytical analyses based on some criteria including strength and weakness as well as the efficiency comparing of the existed methods is done. Zolfaghari and Amiri [9] presented two-stage multivariate attribute based process monitoring in Phase II. To this aim, three methods including Exponentially Weighted Moving Average (EWMA) and DA (EWMA-DA) and P-value-DA and MEWMA and T^2 based on DA are used. Ghashghaei and Amiri [10] proposed two new control charts for Phase II monitoring and diagnosing of the multivariate multiple regression profiles. They used Max-MEWMA and Max-MCUSUM statistics for simultaneous monitoring of mean vector and covariance matrix and then diagnosed the variation of the process responsible for the out-of-control condition.

Notice that, there is little research in OLLM/OCT-based processes monitoring in SPM. In this area, Zhen and Basawa [11] present a time-dependent contingency table called the categorical time series table. Ghoreishi and Alijani [12] proposed an approach to forecast the changing patterns of communication variables using a dynamic contingency table. Kieffer et al. [13] applied a generalized form of the contingency table proposed by Kijima and Matsui [14] to evaluate the effects of genetic attribute 10,000 patients on cancer occurrence.

A Multivariate categorical approach was proposed by Yashchin [15] to monitor the MNP with sudden parameters using the generalized likelihood ratio test statistic in Phase II. The performance of the proposed approach is evaluated by using a real case study in a semiconductor production system. Li et al. [16] proposed a generalized likelihood ratio test (GLRT) for monitoring multivariate categorical processes in Phase II by applying the log-linear model. They developed the EWMA-GLRT to enhance the GLRT chart performance for small shifts in the log-linear model parameters. Li et al. [17] proposed an integrated multivariate spatial-sign test and EWMA scheme to monitor the shape parameters of the multivariate nonparametric processes in Phase II.

A new multivariate categorical statistic based on binomial/multinomial was proposed by Li et al. [18] to monitor MNP by considering the correlation between categorical factors. Results showed that the proposed control scheme was robust to detect different shifts in Phase II. After that, Kamranrad et al. [1] proposed GLT statistics to monitor the MNP processes in Phase-II. Then, GLT is combined with an EWMA statistic to improve its performance in small and medium shifts. Kamranrad et al. [19] proposed Wald and Stuart score test methods to monitor the nominal contingency tables based processes in Phase-II. They presented EWMA-Wald and EWMA-Stuart score test statistics to better the performance of proposed control schemes in small and moderate shifts in the contingency table cells parameter. Kamranrad et al. developed two SLRT and F statistics for monitoring the log-linear based processes in Phase I. In this paper, the performance of the proposed control charts is compared with another *-2LRT* statistic [20]. In addition, they presented a change point estimation scheme to obtain the real time of the change in the process. Performance of the proposed schemes is evaluated under different shifts based on steps, drifts and the presence of outliers in the log-linear model parameters. Finally, to show the efficiency of the proposed control charts in the real world, a case study in health-care based on a kidney patient's data set is applied [21]. A simple ordinal categorical control chart for detecting location changes in univariate ordinal processes was proposed by Li et al. [22]. To put it another way, they presented a new control chart, in Phase-II, to monitor the ordinal logistic regression based processes.

A new EWMA control scheme to monitor the social network with multinomial categorical data has been developed by Perry [23]. This scheme could be useful to the organization's stakeholders when interest lies in monitoring for shifts in the general health of the organization. Li et al. [24] presented a nonparametric KNN-ECUSUM control chart to monitor the multivariate processes by using mixed IC and OC data. Note that, this scheme is the machine learning based black-box control chart and it is utilized for dimension reduction to transform multivariate data into univariate data. Xiang et al. [25] proposed a new control scheme to monitor the multivariate categorical process with a sparse contingency table. To this aim, they combined the LASSO and Ridge methods to estimate the contingency table distribution and propose the useful EWMA chart.

All the researches, as mentioned above, are related to the multivariate nominal process monitoring in both Phases I and II. However, there are little researches in multivariate processes monitoring with ordinal data in SPM. For instance, Wang et al. [26] proposed OLLM based methods monitoring statistics, including MOC and LMBM in Phase II. They showed that the MOC control chart outperforms the LMBM manage charts below one of a kind shifts in the parameters of OLLM. As it is clear from the literature, there is little research on monitoring the MOP. Besides, two control charts were proposed in Phase II called multivariate ordinal-normal statistic (MONS) and multivariate Generalize-p (MG-p), then two statistics were compared and some analysis was done in various methods [27]. Therefore, we develop some new monitoring schemes to monitor the OLLM/OCT based processes in Phase II, which is the main contribution of this paper.

The multivariate ordinal processes

MOP has at least two elements with multiple ordered levels called OCT. OLLM is used for OCT analysis and to suggest multivariate ordinal control charts because it shows the main and interaction effects between ordinal factors.

The ordinal log-linear model

As already mentioned, the OCT is used to show the concurrent relationship between two or more ordinal factors. These variables such as $y_1, y_2, \ldots, y_p$ each with h_i, $i = 1, 2, \ldots, p$ possible levels are considered. Thus, the table cells represent $h_1 \times h_2 \times \ldots \times h_p$ possible frequencies [1]. In order to model the relationship between the levels of ordinal factors and the associated count in each cell, the OLLM has been presented in the previous section. Suppose the contingency table with two ordinal factors (y_1, y_2) with h_1 and h_2 categories. Now, the OLLM is defined as the following equation:

$$\log \mu_{ij} = \mu + \alpha_i + \beta_j + \varphi(u_i - \bar{u})(v_j - \bar{v}), \tag{1}$$

where, $\mu_{ij} = N\pi_{ij}$ is the expected observation value for cell (*i*,*j*). u_i and v_j are the row and column scores, respectively, so that $u_i = i$ and $v_j = j$. In addition, μ is the constant effect, α_i and β_j are the main effects of the *i*th row and *j*th column, respectively. Note that, φ is defined as linear by linear interaction parameter in OLLM which can be estimated by the following equation:

$$\log\left(\frac{\mu_{ij}\ \mu_{i+1.j+1}}{\mu_{i.j+1}\mu_{i+1.j}}\right)=\varphi(u_i-u_{i+1})(v_j-v_{j+1}), \tag{2}$$

where, $(u_i-u_{i+1})=1$ and $(v_j-v_{j+1})=1$. Noted that, in Phase-I monitoring of OLLM (according to unknown parameters), parameters could be estimated using iterative Newton's single-dimensional algorithm [4]. The OLLM for two factors is defined as:

$$\log\boldsymbol{\mu}=\beta_0+\beta_1 y_1+\beta_2 y_2+\varphi(y_1-\bar{y}_1)(y_2-\bar{y}_2). \tag{3}$$

where, $\boldsymbol{\mu}$ is the expected counts vector for OCT and $\bar{y}_i$ ($i=1,2$) is the mean of the ith ordinal factor.

The generalized ordinal log-linear model

For p factors, Eq. 3 can be expanded as follows:

$$\begin{aligned}\log\boldsymbol{\mu}&=\beta_0+\beta_1 y_1+\beta_2 y_2+\ldots+\beta_p y_p+\varphi_{12}(y_1-\bar{y}_1)(y_2-\bar{y}_2)+\ldots+\varphi_{1p}(y_1-\bar{y}_1)(y_p-\bar{y}_p)+\ldots\\&+\varphi_{2p}(y_2-\bar{y}_2)(y_p-\bar{y}_p)+\ldots+\varphi_{p-1,p}(y_{p-1}-\bar{y}_{p-1})(y_p-\bar{y}_p)+\varphi_{123}(y_1-\bar{y}_1)(y_2-\bar{y}_2)(y_3-\bar{y}_3)+\ldots\\&+\varphi_{p-2,p-1,p}(y_{p-2}-\bar{y}_{p-2})(y_{p-1}-\bar{y}_{p-1})(y_p-\bar{y}_p)+\ldots+\varphi_{1,\ldots,p-1,p}(y_1-\bar{y}_1)\ldots(y_{p-1}-\bar{y}_{p-1})(y_p-\bar{y}_p)\end{aligned} \tag{4}$$

where, $\boldsymbol{\mu}$ is the expected counts vector for OCT and $\bar{y}_i$ ($i=1,2,\ldots,p$) is the mean of the ith ordinal factor. As mentioned before, Li et al. [22] proposed two SOC and Generalize-p control charts to monitor the univariate ordinal processes (ordinal logistic processes) in Phase-II that are the basic statistics for our research. Hence, in this paper we overview these two control charts.

The univariate SOC and the Generalized-*p* control charts

Li et al. [22] proposed the SOC and Generalized-p control charts to monitor the univariate ordinal processes in Phase-II. In this subsection, an overview of both control charts is done.

SOC control chart

In particular, suppose that there are known IC probabilities $p_k^{(0)}$ (k=1,2,…,h) for each ordinal level of categorical factor. Hence, the known cumulative probabilities is $c_k=\sum_{j=1}^{k}p_j^{(0)}$ (k=1,2,…,h). In addition, the ordinal level count n_k (k=1,2,…,h) with total count as $N=\sum_{j=1}^{h}n_k$ follows $MN(N,\mathbf{p})$, where $\mathbf{p}=[p_1,\ldots,p_h]^T$. Let, $\mathbf{n}_i=[n_{i1},\ldots,n_{ih}]^T$ be the ith ordinal level count vector of size N which is subject to multinomial distribution $MN(N,\mathbf{p}^{(0)})$ with in-control $\mathbf{p}=[p_1^{(0)},\ldots,p_h^{(0)}]^T$; hence l_i statistic is defined as follows:

$$l_i=\sum_{k=1}^{h}(c_{k-1}^{(0)}+c_k^{(0)}-1)n_{ik} \tag{5}$$

To the better performance of the above statistic to detect the small and moderate shifts, they combined l_i with an EWMA and proposed the charting statistic as follows:

$$R_i = \left| \sum_{k=1}^{h} (c_{k-1}^{(0)} + c_k^{(0)} - 1) z_{ik} \right| \quad (6)$$

Note that, in R_i, n_{ik} is replaced by:

$$\mathbf{z}_i = a_{0,i,\lambda}^{-1} \sum_{j=1}^{i} (1-\lambda)^{i-j} \mathbf{n}_j , \quad (7)$$

where, $a_{t_0,t_1,\lambda}^{-1} = \sum_{j=t_0+1}^{t_1} (1-\lambda)^{t_1-j}$ is a sequence of constants put in place to ensure that all the weights sum up to 1 and $0 < \lambda < 1$ is the smoothing parameter and $\mathbf{z}_i = \left[z_{i1}, z_{i2}, \ldots, z_{ih} \right]$. The null hypothesis is rejected, if R_i is bigger than an ascertained limit. This limit is calculated by simulation in a way that a desired in-control (IC) average run length (ARL_0) is achieved.

Generalized-p control chart

Generalized-*p* control chart is developed using Pearson chi-squre statistic to detect various shifts in process parameters. Li et al. [22] combined this statistic with an EWMA scheme to a better performance of basic control chart to detect small and moderate shifts. Let $\mathbf{q}^{(0)} = [p_1^{(0)}, \ldots, p_{h-1}^{(0)}]^T$ and $\mathbf{w}_i = [z_{i1}^{(0)}, \ldots, z_{i\,h-1}^{(0)}]^T$. Where, $\mathbf{q}^{(0)}$ is known as estimated parameters vector from an IC contingency table; hence, the modified statistic for EWMA/Generalized-*p* is:

$$G_i = \frac{1}{N} (\mathbf{w}_i - N\mathbf{q}^{(0)})^T \, \mathbf{\Sigma}^{-1} (\mathbf{w}_i - N\mathbf{q}^{(0)}), \quad (8)$$

where, $\mathbf{\Sigma}$ is the covariance matrix and is defined as below:

$$\mathbf{\Sigma}_{rc} = \begin{cases} q_r^{(0)}(1-q_r^{(0)}) & if \;\; r = c \\ -q_r^{(0)} q_c^{(0)} & if \;\; r \neq c \end{cases} \quad r, c = 1, \ldots, h-1. \quad (9)$$

Note that, the diagonal elements (the variance of ordinal factors) is calculated by multiplying the r*th* IC probability and (1-r*th* IC probability). In addition, the non-diagonal elements of this matrix is also calculated by multiplying the r*th* IC probability and the c*th* IC probability presented in $\mathbf{q}^{(0)}$. For more information see [22,27].

Proposed methods

As mentioned before, the aim of this paper is to monitor the multivariate ordinal processes by developing univariate SOC and Generalized-*p* to the multivariate case. Hence, in this section, we propose multivariate ordinal categorical (MOC) and multivariate Generalized-*p* (MG-*p*) control charts.

Multivariate ordinal categorical control chart

Consider p-way OCT (p-ordinal factors) with h_1, h_2,…, h_p categories. Hence, the known in-control probabilities for the ordinal cell count is $\pi^{(0)}_{ijk\ldots p}=\frac{f(i,j,k,\ldots,p)}{\sum_{i=1}^{h_1}\sum_{j=1}^{h_2}\sum_{k=1}^{h_3}\cdots\sum_{p=1}^{h_p}f(i,j,k,\ldots,p)}$. Where, $f(i,j,k,\ldots,p)$ is the ordinal level count for cell (i,j,k,…,p). Now, the modified R_i charting statistic for MOC (MR_t) is defined as follows:

$$MR_t=\left|\sum_{i=1}^{h_1}\sum_{j=1}^{h_2}\sum_{k=1}^{h_3}\cdots\sum_{p=1}^{h_p}(F^{(0)}_{ijk\ldots p-1}+F^{(0)}_{ijk\ldots p}-1)z_{ijk\ldots p}\right|, \tag{10}$$

where, $F^{(0)}_{ijk\ldots p}=\sum_{i=1}^{h_1}\sum_{j=1}^{h_2}\sum_{k=1}^{h_3}\cdots\sum_{p=1}^{h_p}\pi^{(0)}_{ijk\ldots p}$. In addition, z_t is:

$$\mathbf{z}_t=a^{-1}_{0,t,\lambda}\sum_{s=1}^{t}(1-\lambda)^{t-s}\mathbf{n}_t, \tag{11}$$

where, $a^{-1}_{t_1,t_2,\lambda}=\sum_{t=t_1+1}^{t_2}(1-\lambda)^{t_2-t_1}$ and $\mathbf{n}_t=[n_{111\ldots1t}\ n_{112\ldots1t}\cdots n_{11h_3\ldots h_pt}\ n_{121\ldots1t}\cdots n_{12h_3\ldots h_pt}\cdots n_{h_1h_21\ldots1t}\cdots n_{h_1h_2h_3\ldots h_pt}]$

If MR_t> L at a particular monitoring time, the null hypothesis is rejected. The process is out of control and adjusts L to get the desired average run length in control (ARL_0).

Multivariate Generalized-*p* control chart

In this subsection, we develop the univariate EWMA/generalized-p proposed by Li et al. [22] to monitor the M-OLLM in Phase-II. Suppose p ordinal factors with h_1, h_2,…, h_p levels. Hence, the modified EWMA/MG-p charting statistic (MG_t) is developed as follows:

$$MG_t=\frac{1}{N}(\mathbf{w}_t-N\mathbf{q}^{(0)})^T\boldsymbol{\Sigma}^{-1}(\mathbf{w}_t-N\mathbf{q}^{(0)}), \tag{12}$$

where, $N=\sum_{i=1}^{h_1}\sum_{j=1}^{h_2}\sum_{k=1}^{h_3}\cdots\sum_{p=1}^{h_p}f(i,j,k,\ldots,p)$ is the total sample size and $\mathbf{w}_t=[z^{(0)}_{t111\ldots1},z^{(0)}_{t112\ldots1},z^{(0)}_{t11h_3\ldots h_p}\ldots,z^{(0)}_{t(h_1-1)(h_2-1)(h_3-1)\ldots(h_p-1)}]^T$. Covariance matrix for EWMA/MG-p is defined as follows:

$$\boldsymbol{\Sigma}=\begin{bmatrix}\boldsymbol{\Sigma}_{h_1} & \boldsymbol{\Sigma}_{h_1h_2} & \boldsymbol{\Sigma}_{h_1h_3} & \cdots & \boldsymbol{\Sigma}_{h_1h_p}\\ \boldsymbol{\Sigma}_{h_1h_2} & \boldsymbol{\Sigma}_{h_2} & \boldsymbol{\Sigma}_{h_2h_3} & \cdots & \boldsymbol{\Sigma}_{h_2h_p}\\ \boldsymbol{\Sigma}_{h_1h_3} & \boldsymbol{\Sigma}_{h_2h_3} & \boldsymbol{\Sigma}_{h_3} & \cdots & \boldsymbol{\Sigma}_{h_3h_p}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ \boldsymbol{\Sigma}_{h_1hp} & \boldsymbol{\Sigma}_{h_2h_p} & \boldsymbol{\Sigma}_{h_3h_p} & \cdots & \boldsymbol{\Sigma}_{h_p}\end{bmatrix} \tag{13}$$

where, $\Sigma_{h_i h_j}$ is the (h_i-1)×(h_j-1) matrix that is the covariance matrix between levels of the factors i and j. Note that, Σ elements could be calculated from Eq. 9, where, $\mathbf{q}^{(0)} = [p^{(0)}_{111\ldots1}, p^{(0)}_{112\ldots1}, p^{(0)}_{11h_3\ldots h_p}, \ldots, p^{(0)}_{(h_1-1)(h_2-1)(h_3-1)\ldots(h_p-1)}]^T$. If $MG_t > S$, the null hypothesis is rejected, that means the process is out-of-control, where S is set to obtain a desired in-control average run length (ARL_0).

Performance assessment

In this section, we evaluate the performances of the two proposed control charts through the OC *ARL* (ARL_1) criterion. Then, other evaluations are done based on a sensitivity analysis on the size of the rows and the columns as well as the number of ordinal factors of the contingency table. To this aim, the contingency tables with 4 and 5 rows and with 5 and 6 columns, and contingency tables with 3 and 4 ordinal factors are considered.

Performance of the proposed MR and MG-*p* control charts is assessed by using simulation runs through applying ARL_1 under various shifts in the OLLM parameters in units of the corresponding standard deviations. Consider a contingency table with 3 rows and 4 columns. Note that, the UCL of the MR and MG-*p* charts in the 3x4 contingency table is obtained equal to 37.10 and 0.93, respectively, to achieve a predetermined IC *ARL* equal to 200. The ARL_1 values for the two proposed charts are calculated using different shifts of the OLLM parameters and different smoothing parameters based on 5,000 simulation runs and are shown in Table 1-4. In addition, the standard deviation of mean run length (SDARL) is shown in parentheses below the ARL value.

Besides, the in-control parameter vector of the OLLM is assumed $\boldsymbol{\beta} = [1, -0.5, -0.5, 0.15]$. In addition, the IC standard deviations of the OLLM parameters are as follows:
$\hat{\boldsymbol{\sigma}}_{0\hat{\boldsymbol{\beta}}} = [2.14, 1.43, 1.28, 0.89]$,

Note that, the standard deviations of the parameters estimated in the OLLM are obtained by using the following covariance matrix:

$$\mathrm{cov}(\boldsymbol{\beta}) = \{\mathbf{X}'[\mathbf{diag}(\boldsymbol{\mu}) - \boldsymbol{\mu}\boldsymbol{\mu}'/N]\mathbf{X}\}^{-1}, \quad (14)$$

where $\mathbf{X}$ and $\boldsymbol{\mu}$ are the design matrix and the expected counts vector of the contingency table cells, respectively. Also, $\mathbf{diag}(\boldsymbol{\mu})$ is a diagonal matrix of the expected counts of the contingency table cells and N is the contingency table sample size [28].

Table 1. The ARL and SDARL values under the different shifts in the intercept ($\beta_0 + \gamma.\sigma_{0\beta}$)

λ	γ	-1.2	-0.8	-0.5	-0.2	-0.1	0	0.1	0.2	0.5	0.8	1.2
0.05	MR	98.39 (1.18)	130.18 (1.90)	153.41 (2.02)	173.75 (2.01)	188.89 (2.10)	200.15 (2.61)	185.08 (2.08)	175.89 (2.01)	151.02 (2.01)	132.35 (1.91)	105.68 (1.10)
	MG-*p*	93.57 (1.13)	130.25 (1.95)	156.84 (1.99)	175.69 (2.04)	190.05 (2.11)	201.05 (3.32)	189.98 (2.05)	180.39 (2.02)	155.61 (2.00)	131.58 (1.97)	99.48 (1.01)
0.1	MR	99.84 (1.32)	129.97 (1.99)	151.15 (2.00)	172.39 (2.00)	186.21 (2.00)	199.98 (3.01)	184.36 (2.02)	174.93 (2.08)	150.25 (1.98)	130.05 (1.97)	102.20 (1.02)
	MG-*p*	92.68 (1.27)	121.15 (1.90)	155.03 (1.97)	178.41 (2.01)	189.87 (2.14)	200.10 (3.05)	189.19 (2.10)	181.04 (2.06)	151.36 (1.84)	130.01 (2.00)	98.08 (1.05)

0.2	MR	90.30 (2.10)	128.64 (2.02)	150.02 (1.69)	175.63 (2.01)	182.96 (2.00)	200.03 (2.93)	181.25 (2.05)	173.68 (2.00)	148.31 (2.00)	125.36 (1.98)	100.98 (1.05)
	MG-*p*	91.05 (1.70)	124.87 (1.95)	152.97 (1.87)	178.84 (1.94)	189.31 (1.99)	199.96 (2.68)	185.37 (1.95)	179.65 (1.92)	150.33 (2.01)	120.05 (2.01)	99.21 (1.06)

Table 2. The ARL and SDARL values under the different shifts in the first slope ($\beta_1 + \gamma.\sigma_{1\beta}$)

λ	γ	-1.2	-0.8	-0.5	-0.2	-0.1	0	0.1	0.2	0.5	0.8	1.2
0.05	MR	1.00 (0.00)	8.72 (0.63)	25.51 (1.02)	111.98 (1.88)	177.39 (2.04)	199.48 (3.01)	175.05 (2.00)	113.45 (1.94)	28.67 (1.48)	9.91 (0.87)	1.00 (0.02)
	MG-p	1.00 (0.00)	6.92 (0.40)	26.29 (1.00)	116.49 (1.93)	183.67 (2.01)	201.30 (2.99)	180.69 (2.02)	116.39 (1.56)	26.75 (1.27)	7.63 (0.59)	1.00 (0.00)
0.1	MR	1.00 (0.00)	8.00 (0.51)	23.67 (1.00)	109.94 (1.84)	175.63 (2.01)	200.08 (2.65)	172.59 (2.05)	111.42 (1.79)	24.38 (1.21)	8.24 (0.94)	1.00 (0.00)
	MG-p	1.00 (0.00)	6.74 (0.39)	23.05 (0.91)	110.63 (1.67)	182.35 (2.09)	200.18 (2.97)	183.54 (2.01)	119.67 (1.91)	25.97 (1.30)	6.35 (0.86)	1.00 (0.00)
0.2	MR	1.00 (0.00)	6.01 (0.58)	25.45 (0.98)	101.25 (1.89)	172.54 (2.00)	199.05 (2.35)	170.05 (1.58)	105.89 (1.75)	21.05 (1.24)	21.05 (1.24)	1.00 (0.00)
	MG-p	1.00 (0.00)	3.89 (0.45)	28.41 (1.29)	100.14 (2.05)	180.25 (2.06)	200.04 (2.93)	181.02 (1.89)	109.54 (1.12)	25.20 (1.84)	25.20 (1.84)	1.00 (0.00)

Table 3. The ARL and SDARL values under the different shifts in the second slope ($\beta_2 + \gamma.\sigma_{2\beta}$)

λ	γ	-1.2	-0.8	-0.5	-0.2	-0.1	0	0.1	0.2	0.5	0.8	1.2
0.05	MR	1.00 (0.00)	5.99 (0.58)	22.17 (1.01)	113.54 (1.81)	170.05 (2.00)	200.11 (3.00)	173.18 (2.08)	122.30 (1.89)	23.67 (1.01)	5.51 (0.43)	1.00 (0.00)
	MG-p	1.00 (0.00)	5.02 (0.31)	22.69 (0.92)	115.69 (1.25)	171.69 (2.02)	200.35 (2.98)	182.39 (2.09)	130.10 (2.00)	22.89 (0.91)	4.81 (0.19)	1.00 (0.00)
0.1	MR	1.00 (0.00)	4.69 (0.34)	20.19 (0.75)	111.97 (1.68)	166.35 (1.98)	200.01 (2.91)	170.05 (2.00)	120.94 (1.95)	22.24 (1.00)	4.67 (0.61)	1.00 (0.00)
	MG-p	1.00 (0.00)	3.70 (0.21)	21.57 (0.89)	119.67 (1.09)	169.27 (2.01)	200.10 (3.01)	181.29 (2.11)	130.04 (1.69)	21.69 (0.98)	3.98 (0.32)	1.00 (0.00)
0.2	MR	1.00 (0.00)	3.25 (0.78)	17.75 (0.84)	109.68 (1.08)	139.05 (1.75)	199.69 (2.94)	169.08 (2.05)	121.87 (2.01)	20.14 (1.08)	3.34 (0.50)	1.00 (0.00)
	MG-p	1.00 (0.00)	3.05 (0.88)	15.69 (0.98)	115.52 (1.00)	161.24 (1.89)	200.17 (2.99)	180.05 (2.09)	135.59 (2.00)	19.97 (1.30)	3.09 (0.29)	1.00 (0.00)

Table 4. The ARL and SDARL values under the different shifts in φ ($\varphi + \gamma.\sigma_{\varphi}$)

λ	γ	-1.2	-0.8	-0.5	-0.2	-0.1	0	0.1	0.2	0.5	0.8	1.2
0.05	MR	7.20 (0.97)	43.31 (0.96)	90.54 (1.12)	110.15 (1.90)	161.47 (2.01)	201.39 (2.96)	160.81 (2.00)	111.68 (1.87)	82.28 (1.61)	48.91 (1.05)	6.61 (0.24)
	MG-p	7.01 (0.39)	41.47 (0.82)	91.00 (1.80)	112.39 (1.91)	166.30 (2.05)	200.57 (2.99)	162.98 (2.02)	114.15 (1.99)	85.39 (1.53)	45.60 (0.99)	6.04 (0.30)
0.1	MR	6.71 (0.29)	41.48 (0.74)	79.96 (1.21)	119.69 (1.94)	160.24 (2.04)	200.10 (2.62)	159.00 (2.01)	110.05 (1.58)	80.12 (1.48)	44.26 (0.95)	6.00 (0.15)
	MG-p	6.18 (0.52)	39.05 (0.92)	79.07 (1.69)	111.34 (1.87)	163.69 (2.02)	200.18 (3.14)	161.98 (2.00)	112.34 (1.79)	81.25 (1.78)	40.15 (1.05)	5.59 (0.19)

0.2	MR	6.09 (0.72)	35.67 (0.87)	80.05 (1.45)	107.68 (1.85)	158.68 (1.98)	199.95 (2.96)	152.5 (1.56)	109.67 (1.50)	82.35 (1.00)	39.45 (1.75)	5.87 (0.63)
	MG-p	5.88 (0.17)	32.89 (1.01)	80.95 (2.04)	110.05 (1.64)	161.98 (2.05)	200.42 (2.90)	154.14 (1.75)	111.97 (1.78)	83.05 (1.36)	35.05 (1.41)	5.31 (0.21)

As it is clear from Tables 1-4, the ARL_1 values of the MR chart are smaller than the ARL_1 values of MG-*p* under small and moderate shifts in all intercept and slope parameters of OLLM. Hence, the MR chart outperforms compared to the MG-*p* in these mentioned shifts. In addition, the performance of the proposed charts is assessed under different λ and results show the superiority of the mentioned control charts under λ equal to 0.2. In other words, proposed control charts have more efficiency in detecting OC conditions under λ=0.2 than the other λ. For this reason, this λ is selected for other calculations in this paper.

Note that, other simulation studies under different simultaneous shifts in the OLLM parameters based on λ=0.2 are done and they are shown in Figs. 1-6.

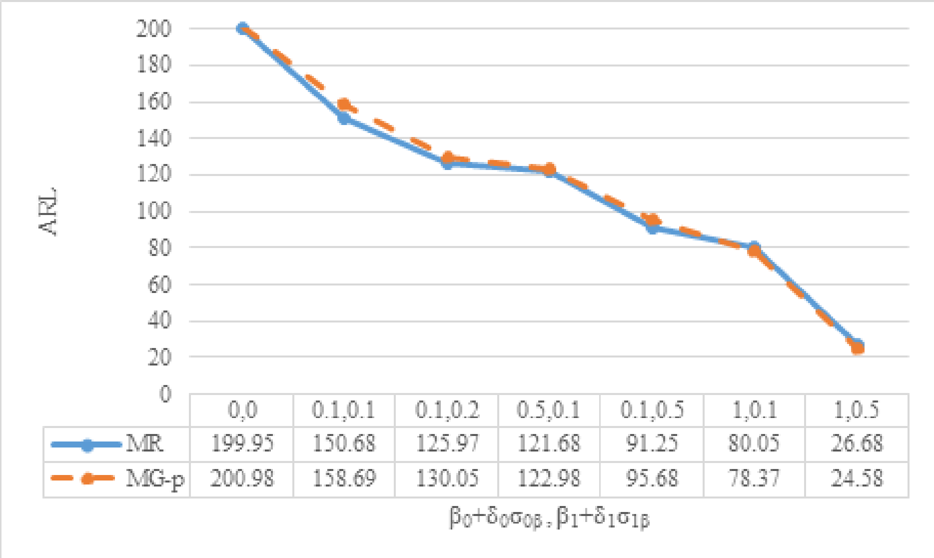


Fig. 1. Compare the performance of control charts for concurrent shifts in intercept and the first slope

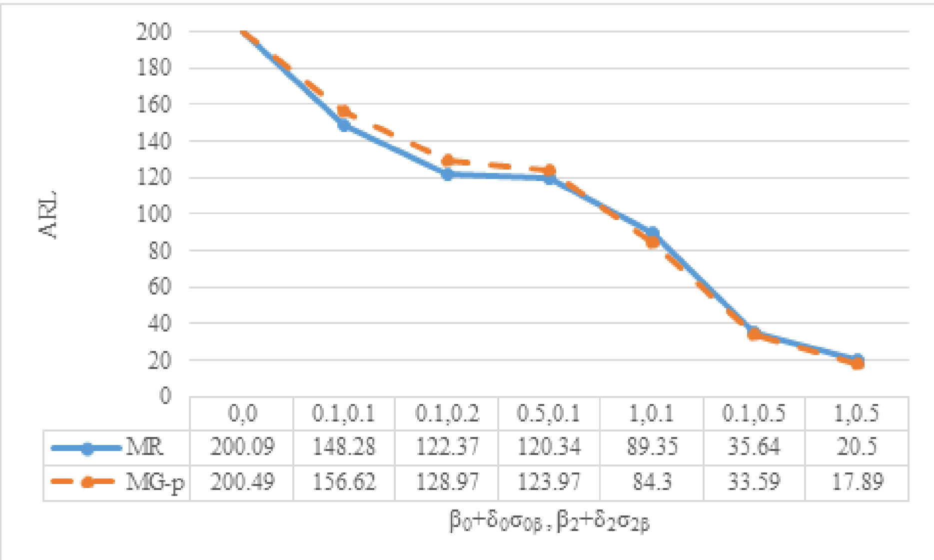
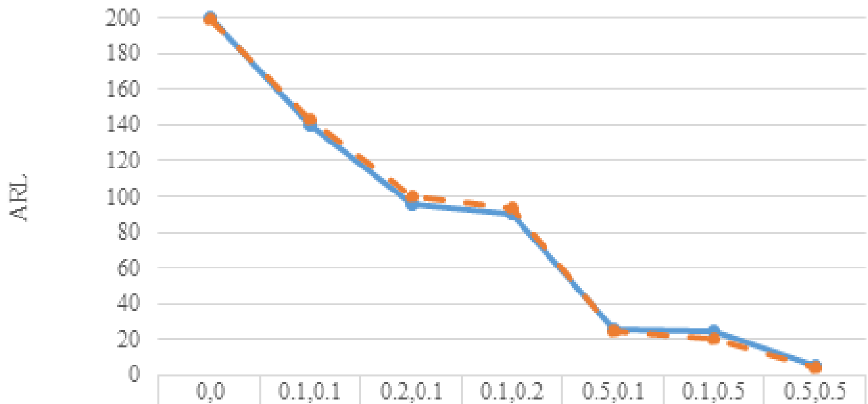


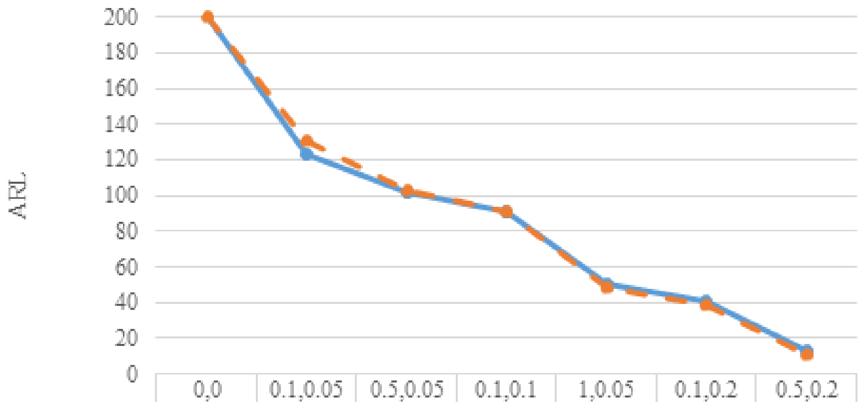
Fig. 2. Compare the performance of control charts for concurrent shifts in the intercept and the second slope



	0,0	0.1,0.1	0.2,0.1	0.1,0.2	0.5,0.1	0.1,0.5	0.5,0.5
MR	199.69	140.36	95.47	90.58	25.69	24.68	5.39
MG-p	199.36	142.98	99.84	93.68	24.38	20.57	4.62

$\beta_1+\delta_1\sigma_{1\beta}, \beta_2+\delta_2\sigma_{2\beta}$

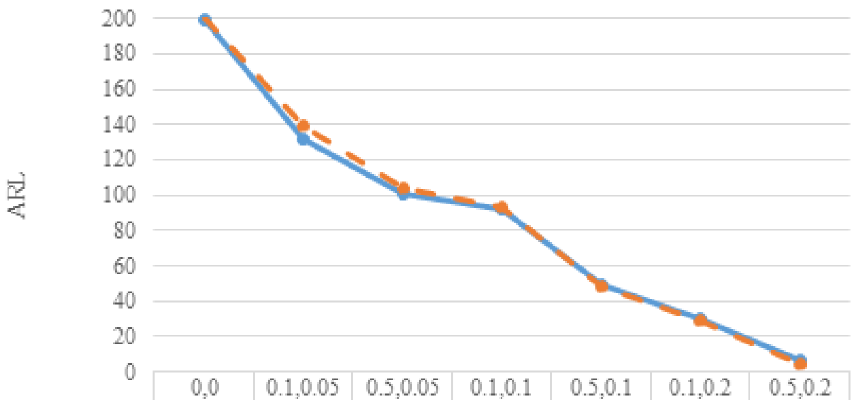
Fig. 3. Compare the performance of control charts for concurrent shifts in the first and the second slopes



	0,0	0.1,0.05	0.5,0.05	0.1,0.1	1,0.05	0.1,0.2	0.5,0.2
MR	200.59	123.58	101.97	90.56	50.68	40.94	12.54
MG-p	199.91	130.49	103.14	91.37	48.31	38.2	10.93

$\beta_0+\delta_0\sigma_{0\beta}, \varphi+\delta_3\sigma_\varphi$

Fig. 4. Compare the performance of control charts for concurrent shifts in the intercept and φ



	0,0	0.1,0.05	0.5,0.05	0.1,0.1	0.5,0.1	0.1,0.2	0.5,0.2
MR	199.05	131.98	100.25	92.36	49.67	30.57	5.98
MG-p	200.08	138.98	103.36	92.79	48.68	28.96	4.39

$\beta_1+\delta_1\sigma_{1\beta}, \varphi+\delta_3\sigma_\varphi$

Fig. 5. Compare the performance of control charts for concurrent shifts in the first slope and φ

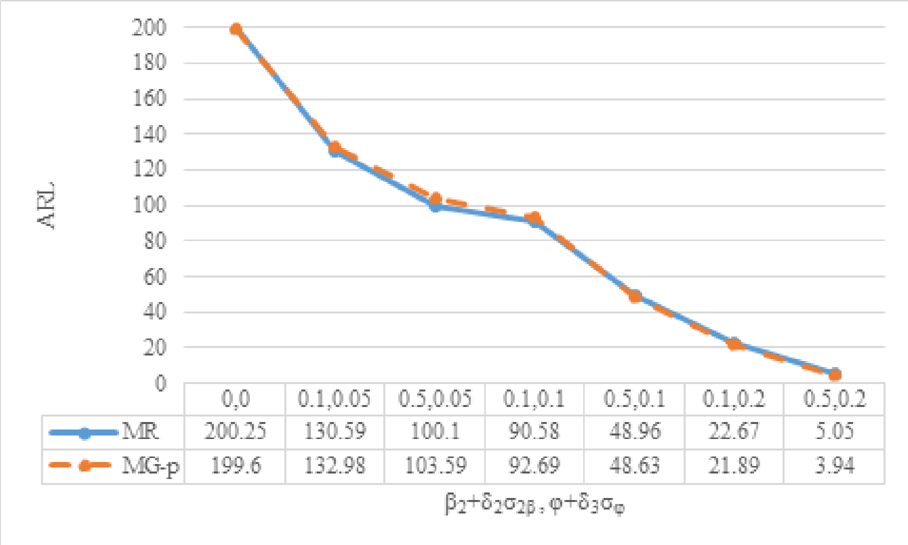


Fig. 6. Compare the performance of control charts for concurrent shifts in the second slope and φ

As it is clear from Figs. 1-6, the simultaneous shifts in two slope parameters result in a better out-of-control performance of the proposed control charts. These results are expected because the mean changes under simultaneous shifts larger the individual changes. In addition, in this study, sensitivity analyses based on 3x4 and 5x6 contingency tables, as well as simultaneous increases in the dimensional of the OCT, are done. Furthermore, higher contingency tables with three and four ordinal factors are performed to evaluate the performance of proposed control schemes, which mentioned OCTs based data set are presented in Appendix A section. Note that, the UCLs of proposed control charts are 39.90 and 101.91 respectively, for three-way and four-way OCTs. Tables 5-8 and Figs. 7-8 show the sensitivity analysis results as follows.

Table 5. ARL and SDARL values of charts in other dimensions for different shifts in the intercept ($\beta_0+\gamma.\sigma_{0\beta}$)

Dimension	γ	-1.2	-0.8	-0.5	-0.2	-0.1	0	0.1	0.2	0.5	0.8	1.2
3×5	MR	90.04 (1.24)	121.18 (1.58)	140.21 (1.84)	171.57 (2.61)	182.07 (2.84)	201.05 (2.69)	180.19 (2.32)	170.81 (2.04)	138.20 (2.01)	118.04 (1.64)	88.24 (1.90)
	MG-*p*	85.61 (1.04)	120.48 (1.69)	141.99 (2.00)	173.45 (2.21)	190.34 (2.68)	199.64 (2.99)	183.92 (2.58)	174.18 (2.00)	141.87 (2.01)	117.08 (1.87)	84.33 (1.21)
3×6	MR	81.34 (1.44)	109.31 (1.91)	140.10 (1.51)	171.39 (2.04)	181.64 (2.27)	199.09 (2.67)	180.08 (2.42)	169.98 (2.11)	131.51 (1.99)	110.35 (1.96)	80.97 (1.17)
	MG-*p*	80.01 (1.30)	110.64 (1.47)	141.19 (1.81)	173.12 (2.10)	182.34 (2.44)	201.30 (3.01)	181.11 (2.51)	171.12 (2.15)	133.54 (1.94)	110.41 (1.97)	79.72 (1.00)
4×4	MR	82.34 (1.05)	105.30 (1.24)	145.61 (1.97)	170.18 (2.00)	181.00 (2.73)	200.28 (2.66)	180.97 (2.15)	170.34 (2.16)	132.39 (2.00)	113.67 (1.64)	84.31 (1.43)
	MG-*p*	76.97 (1.61)	107.33 (1.34)	142.36 (1.61)	172.75 (2.12)	184.67 (2.37)	199.38 (3.00)	182.24 (2.24)	172.94 (2.30)	133.04 (1.90)	112.99 (1.41)	80.68 (1.39)
5×4	MR	80.62 (1.04)	100.39 (1.27)	140.56 (1.74)	168.91 (2.05)	180.08 (2.46)	200.91 (2.69)	180.20 (2.00)	169.05 (2.05)	130.59 (2.01)	108.34 (1.67)	80.69 (1.41)
	MG-*p*	75.61 (1.01)	106.36 (1.34)	142.36 (1.91)	170.38 (2.14)	182.95 (2.40)	199.39 (2.94)	182.00 (2.10)	170.98 (2.21)	132.94 (1.99)	109.25 (1.91)	77.89 (1.05)
4×5	MR	78.94 (1.61)	105.68 (1.05)	133.64 (1.25)	165.39 (2.08)	180.10 (2.95)	200.56 (2.34)	177.35 (2.61)	162.96 (2.03)	126.48 (1.98)	102.60 (1.38)	75.64 (1.15)
	MG-*p*	73.05 (1.08)	101.67 (1.42)	140.97 (1.49)	169.94 (2.43)	180.93 (3.05)	199.67 (2.97)	181.39 (2.59)	169.37 (2.00)	130.32 (1.90)	100.49 (1.31)	71.89 (1.00)

Table 6. ARL and SDARL values of charts in other dimensions for different shifts in the first slope ($\beta_1 + \gamma.\sigma_{1\beta}$)

Dimension	γ	-1.2	-0.8	-0.5	-0.2	-0.1	0	0.1	0.2	0.5	0.8	1.2
3×5	MR	1.00 (0.00)	7.87 (0.67)	25.05 (1.00)	112.57 (1.98)	175.51 (2.84)	200.26 (2.89)	174.33 (2.68)	110.54 (1.36)	25.67 (1.64)	8.05 (0.82)	1.00 (0.00)
	MG-*p*	1.00 (0.00)	6.04 (0.62)	24.09 (1.02)	113.49 (1.97)	180.69 (2.56)	200.94 (3.02)	180.01 (2.94)	112.49 (1.84)	25.03 (1.24)	7.00 (0.92)	1.00 (0.00)
3×6	MR	1.00 (0.00)	6.65 (0.25)	24.51 (1.24)	108.64 (1.92)	172.94 (2.94)	199.60 (2.90)	172.49 (2.75)	105.61 (1.67)	22.15 (1.30)	7.36 (0.62)	1.00 (0.00)
	MG-*p*	1.00 (0.00)	5.92 (0.32)	23.04 (1.13)	109.34 (1.99)	180.02 (3.00)	200.91 (2.48)	179.67 (2.63)	106.97 (1.34)	23.35 (1.25)	6.93 (0.45)	1.00 (0.00)
4×4	MR	1.00 (0.00)	8.64 (0.93)	26.91 (1.02)	110.23 (1.68)	173.34 (2.63)	200.58 (2.53)	172.52 (2.37)	109.63 (1.95)	25.93 (1.01)	9.02 (0.98)	1.00 (0.00)
	MG-*p*	1.00 (0.00)	6.61 (0.49)	24.41 (1.14)	110.99 (1.95)	181.36 (2.67)	201.07 (3.01)	179.04 (2.91)	110.38 (1.98)	24.39 (1.07)	8.00 (0.67)	1.00 (0.00)
5×4	MR	1.00 (0.00)	6.31 (0.29)	24.45 (1.00)	106.32 (1.98)	171.62 (2.95)	200.06 (2.75)	170.08 (2.92)	105.31 (1.55)	24.00 (1.05)	7.09 (0.59)	1.00 (0.00)
	MG-*p*	1.00 (0.00)	5.04 (0.20)	22.36 (1.09)	108.64 (2.00)	177.69 (2.06)	199.00 (2.95)	173.15 (2.57)	108.79 (1.31)	23.02 (1.11)	5.94 (0.38)	1.00 (0.00)
4×5	MR	1.00 (0.00)	7.80 (0.90)	26.87 (1.67)	109.61 (1.63)	172.68 (2.38)	200.81 (2.87)	170.85 (2.05)	106.96 (2.00)	27.03 (1.54)	9.60 (0.79)	1.00 (0.00)
	MG-*p*	1.00 (0.00)	6.09 (0.68)	26.01 (1.00)	110.99 (2.08)	178.73 (2.90)	199.47 (2.53)	172.94 (2.34)	110.31 (1.97)	25.36 (1.09)	7.94 (0.69)	1.00 (0.00)

Table 7. ARL and SDARL values of the charts in other dimensions for different shifts in the second slope ($\beta_2 + \gamma.\sigma_{2\beta}$)

Dimension	γ	-1.2	-0.8	-0.5	-0.2	-0.1	0	0.1	0.2	0.5	0.8	1.2
3×5	MR	1.00 (0.00)	3.06 (0.30)	19.62 (0.65)	110.59 (1.84)	172.36 (3.02)	200.29 (2.56)	170.15 (2.35)	116.31 (2.00)	19.05 (1.00)	3.00 (0.35)	1.00 (0.00)
	MG-*p*	1.00 (0.00)	2.94 (0.23)	16.97 (0.69)	119.63 (1.92)	173.49 (2.09)	201.00 (3.05)	178.52 (2.96)	122.67 (1.97)	18.86 (0.96)	2.26 (0.20)	1.00 (0.00)
3×6	MR	1.00 (0.00)	2.64 (0.38)	18.52 (0.53)	105.37 (1.40)	170.37 (2.08)	200.54 (2.97)	170.12 (2.03)	111.32 (2.02)	18.01 (0.94)	2.67 (0.28)	1.00 (0.00)
	MG-*p*	1.00 (0.00)	2.09 (0.31)	14.67 (0.34)	111.82 (1.89)	172.57 (2.47)	200.02 (3.01)	176.96 (2.67)	120.34 (1.56)	17.08 (0.85)	2.02 (0.19)	1.00 (0.00)
4×4	MR	1.00 (0.00)	2.96 (0.26)	19.00 (0.45)	112.09 (1.95)	170.98 (3.00)	200.52 (2.92)	171.25 (2.65)	113.51 (2.55)	18.90 (0.62)	3.08 (0.59)	1.00 (0.00)
	MG-*p*	1.00 (0.00)	2.36 (0.12)	15.93 (0.49)	114.38 (1.56)	176.81 (2.69)	200.09 (2.68)	175.39 (2.28)	119.97 (2.00)	18.61 (0.97)	2.68 (0.27)	1.00 (0.00)
5×4	MR	1.00 (0.00)	2.63 (0.20)	18.06 (0.28)	110.57 (1.49)	169.37 (2.84)	200.18 (2.99)	169.30 (2.21)	108.79 (1.69)	17.04 (0.54)	2.62 (0.29)	1.00 (0.00)
	MG-*p*	1.00 (0.00)	2.00 (0.10)	15.00 (0.28)	114.97 (1.94)	172.39 (2.49)	200.29 (2.47)	174.08 (2.87)	111.58 (2.01)	16.82 (0.67)	2.08 (0.18)	1.00 (0.00)
4×5	MR	1.00 (0.00)	2.05 (0.11)	16.91 (0.21)	109.57 (1.68)	169.02 (2.60)	200.36 (2.32)	168.24 (2.48)	106.43 (1.41)	15.59 (0.40)	2.00 (0.22)	1.00 (0.00)
	MG-*p*	1.00 (0.00)	1.92 (0.08)	14.97 (0.19)	111.36 (1.80)	171.29 (2.15)	199.94 (2.97)	173.95 (2.63)	109.48 (1.69)	14.89 (0.26)	1.33 (0.09)	1.00 (0.00)

Table 8. ARL and SDARL values of the charts in other dimensions for different shifts in the φ ($\varphi+\gamma.\sigma_{\varphi}$)

Dimension	γ	-1.2	-0.8	-0.5	-0.2	-0.1	0	0.1	0.2	0.5	0.8	1.2
3×5	MR	4.52 (0.36)	33.46 (0.49)	79.24 (1.15)	105.89 (1.81)	151.63 (2.01)	200.50 (2.63)	150.62 (1.97)	105.93 (1.68)	79.35 (1.41)	32.26 (1.04)	3.65 (0.63)
	MG-*p*	4.08 (0.16)	31.97 (0.67)	79.67 (1.90)	108.06 (1.67)	155.07 (2.02)	200.01 (3.00)	153.98 (1.80)	108.25 (1.38)	82.64 (1.48)	32.05 (1.12)	3.14 (0.31)
3×6	MR	3.02 (0.11)	29.69 (0.34)	75.56 (1.00)	103.64 (1.40)	149.92 (1.79)	200.11 (2.76)	150.05 (1.69)	103.61 (1.42)	76.49 (1.32)	30.08 (1.15)	3.23 (0.43)
	MG-*p*	2.90 (0.08)	29.37 (0.15)	78.79 (1.08)	107.31 (1.33)	153.68 (1.63)	200.99 (3.14)	153.67 (1.82)	107.90 (1.49)	79.96 (1.09)	32.00 (1.00)	2.96 (0.35)
4×4	MR	3.22 (0.08)	28.37 (0.41)	76.88 (1.20)	104.32 (1.54)	150.65 (1.87)	200.03 (2.35)	151.64 (1.99)	104.32 (1.77)	78.81 (1.17)	31.55 (1.00)	3.98 (0.05)
	MG-*p*	3.00 (0.09)	27.74 (0.26)	79.93 (1.14)	105.68 (1.42)	154.19 (1.48)	200.94 (2.66)	155.30 (1.93)	108.89 (1.68)	80.06 (1.19)	30.57 (0.68)	2.84 (0.23)
5×4	MR	3.02 (0.05)	27.02 (0.66)	75.51 (1.01)	101.55 (1.26)	150.07 (1.89)	200.22 (2.61)	150.11 (2.00)	102.21 (1.60)	76.78 (1.04)	30.62 (0.89)	3.25 (0.12)
	MG-*p*	2.67 (0.08)	26.00 (0.20)	77.84 (1.08)	103.97 (1.32)	154.04 (1.99)	200.45 (2.97)	154.01 (1.97)	107.75 (1.49)	79.33 (1.24)	29.47 (0.58)	2.08 (0.06)
4×5	MR	2.35 (0.10)	25.92 (0.31)	73.19 (0.55)	101.38 (1.09)	149.92 (1.95)	199.09 (2.57)	148.52 (2.01)	101.15 (1.46)	74.41 (1.00)	28.87 (0.87)	2.21 (0.24)
	MG-*p*	1.69 (0.04)	23.50 (0.34)	72.26 (0.94)	104.50 (1.29)	154.00 (2.02)	200.53 (2.73)	153.34 (2.53)	105.60 (1.89)	75.49 (1.03)	27.04 (0.67)	2.00 (0.10)

As shown in Tables 5-8, the MR control chart outperforms the MG-*p* chart in detecting small and moderate OC conditions in all parameters of the OLLM. In other words, the results in the mentioned tables show that the MR chart outperforms the MG-*p* chart in monitoring the OCT-based processes with 4 and 5 rows and 5 and 6 columns under small and moderate shifts. Furthermore, it is clear from the results of Tables 1-8, the sensitivity of the proposed control charts under negative shifts to detect the OC condition is more than corresponding positive shifts in all OLLM parameters.

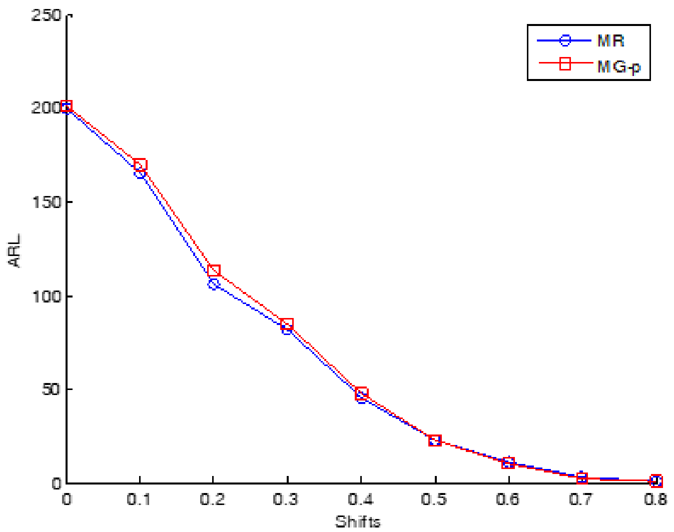


Fig. 7. Performance comparison of the control charts with three ordinal factors under different shifts in the second slope

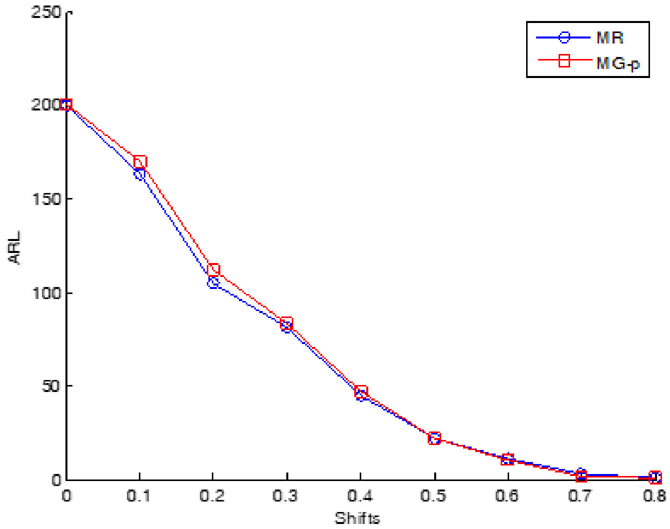


Fig. 8. Performance comparison of the control charts with four ordinal factors under different shifts in the second slope

Figs. 7–8 also show similar results such that the MR control chart outperform the MG-*p* chart to detect an OC condition under small and moderate shifts in the second slope parameter of the OLLM with 3 and 4 ordinal factors. Totally, it can be noted that the MR control chart has shown better performance than the MG-*p* chart to detect small and moderate OC conditions.

The numerical example

In this study, a real data set based on an OCT is presented to show the application of the proposed charts and compare their performances in monitoring OLLM based processes based on a real case extracted from [29].

Consider the OCT from the 2006 general social survey. One of the questions from the respondents is, "Overall, do you think you are very happy, quite happy, or not very happy?" They are also asked about their family's income, "Do you think your family's income is below average, average, or above average compared to a typical American family?" These two questions. Based on this, the OCT is formed [29] and shown in Table 9.

Table 9. The ordinal cross-classified table for happiness and relative family income [29]

	Happiness			
Family income	Very happy	Pretty happy	Not too happy	Total
Above average	272	294	49	615
Average	454	835	131	1420
Below Average	185	527	208	920
Total	911	1656	388	2955

Now, the performance of the proposed schemes under two different shifts in the first slope and φ parameters of the OLLM. For that, shifts are done in $-0.1\sigma_{1\beta}$ in β_1 and $0.1\sigma_\varphi$ in φ and results are shown in Figs. 9-12. The OLLM for the above OCT with two ordinal factors including happiness (*H*) and family income (*FI*) under an in-control state is defined as follows:

$$\log \boldsymbol{\mu} = 1 - 0.5H - 0.5FI + 0.15(H - \bar{H})(FI - \overline{FI});\ H = 1,2,3\ \textit{and}\ \ FI = 1,2,3. \qquad (15)$$

Note that, the UCLs of the MR and MG-*p* charts based on the model in Eq. 15 are set through 10,000 simulation runs to determine an IC *ARL* of 200.

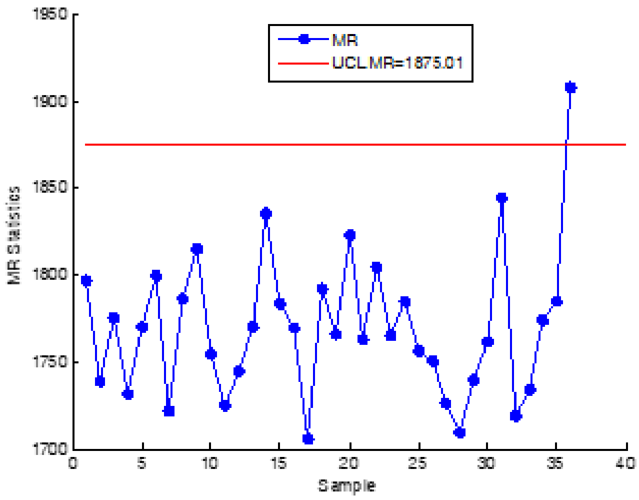


Fig. 9. MR control chart performance under $-0.1\sigma_{1\beta}$ shift in β_1

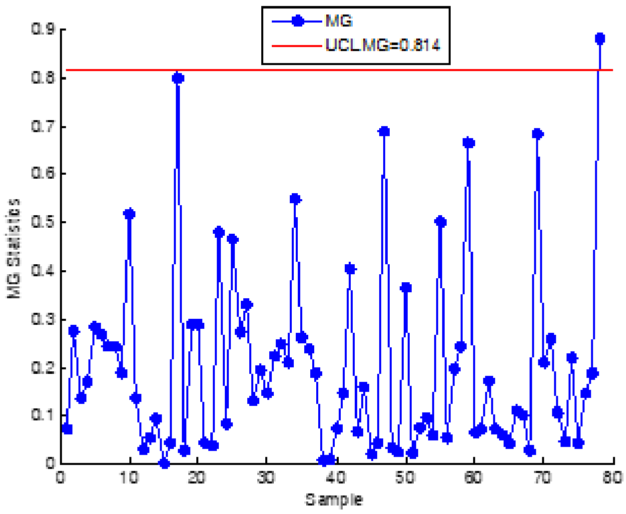


Fig. 10. MG-*p* control chart performance under $-0.1\sigma_{1\beta}$ shift in β_1

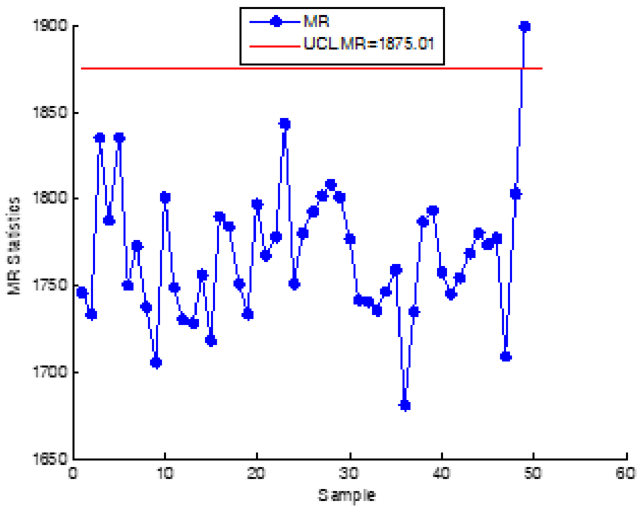


Fig. 11. MR control chart performance under $0.1\sigma_{\varphi}$ shift in φ

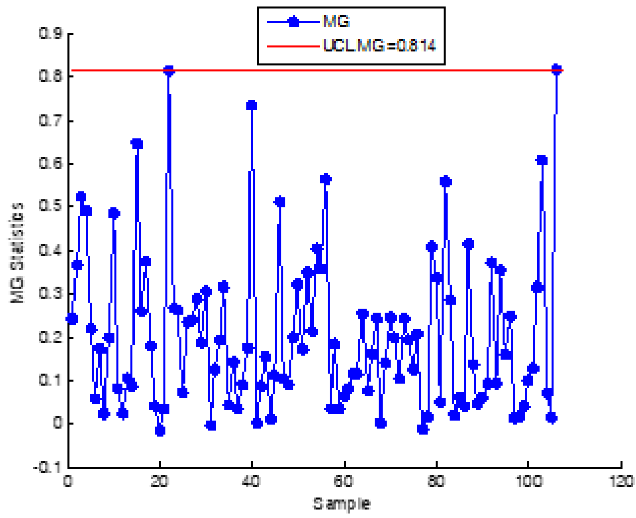


Fig. 12. MG-*p* control chart performance under $0.1\sigma_{\varphi}$ shift in φ

Figs. 9-12 demonstrate the performance of the MR and MG-*p* control charts under the mentioned shifts in β_1 and φ. As shown in figures, the OC signals by the MR and MG-*p* control charts occur at the 36th and 79th sample under $-0.1\sigma_{1\beta}$ shift in β_1 and 51st and 107th sample under $0.1\sigma_{\varphi}$ shift in φ, respectively. These results show that under the mentioned shifts in β_1 and φ of the OLLM, the MR control chart detects the OC condition sooner than the MG-p control chart.

Conclusion and future studies

Two new control schemes including MR and MG-*p* were proposed for Phase II monitoring the M-OLLM based processes. The results show that the MR control chart performs better than MG-p control charts, where all OLLM parameter shifts are small and medium. In addition, several sensitivity analyzes were performed to evaluate the efficiency of the proposed control charts based on the various values of the smoothing parameters, increasing the number of rows and columns as well as simultaneous increasing of rows and columns of the OCT. Furthermore, another analysis was done to evaluate the effect of the increasing number of ordinal factors on the performance of the proposed control charts. The results showed that the proposed control charts have better performance in detecting the out-of-control condition under $\lambda = 0.2$ rather than the other smoothing parameters considered. Moreover, increasing the number of rows and columns, as well as the simultaneous increasing in rows and columns, improve the performance of the proposed control charts. Furthermore, increasing the ordinal factors in OLLM led to relatively better performance of the proposed control charts. Also, a real data set based on an OCT was performed to illustrate the applicability of the proposed charts and similar results were obtained. For future study, Phase I monitoring of the OCT could be a fruitful area for other researchers.

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Appendix

Table A.1. Data set for the three-way OCT

		y_3			
y_1	y_2	1	2	3	4
1	1	2	5	1	8
	2	5	3	0	7
2	1	1	3	9	6
	2	10	7	11	8
3	1	4	5	8	1
	2	3	4	8	1

Table A.2. Data set for the four-way OCT

		y_3							
		1		2		3		4	
		y_4							
y_1	y_2	1	2	1	2	1	2	1	2
1	1	2	5	1	8	4	9	10	12
	2	5	3	0	7	10	12	8	11
2	1	1	3	9	6	9	9	11	5
	2	10	7	11	8	9	11	4	6
3	1	4	5	8	1	7	13	7	8
	2	3	4	8	1	5	14	5	6