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Pair difference cordial labeling of planar grid and mangolian tent

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ABSTRACT

Let $G = (V, E)$ be a (p, q) graph.
Define

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L = \{\pm 1, \pm 2, \pm 3, \cdots, \pm\rho\}$ called the set of labels. Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to p-1 elements of V and repeating a label for the remaining one vertex when p is odd.The labeling as defined above is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u) - f(v)|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \leq 1$,

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1 Abstract continued

where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 and number of edges not labeled with 1.A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph. In this paper we investigate pair difference cordial labeling behaviour of planar grid and mangolian tent graphs.

2 Introduction

In this paper we consider only finite, undirected and simple graphs. The concept of graceful labeling was introduced by Rosa in [11] and harmonious labeling by Graham and Sloane [3] . Similar line Cachit introduced the weaker version of graceful and harmonious labeling called cordial labeling in [1]. Motivated by these labeling technique, the pair difference cordial labeling of a graph was introduced in [5]. Also pair difference cordial labeling behavior of several graphs like path, cycle, star,ladder,wheel,helm,web,fan, umberalla,(n,t)-kite, mobius ladder, slanting ladder,triangular ladder have been investigated in [5,6,7,8,9,10].In this paper we investigate the pair difference cordial labeling behaviour of planar grid and mangolian tent graphs.

3 Preliminaries

Definition 1. [4]. *For any graph* $G_1 = G_1(V_1, E_1)$ *and* $G_2 = G_2(V_2, E_2)$ *their product* $G_1 * G_2$ *is defined as the graph with vertex set* $V_1 \times V_2$ *and two vertices* (u_1, v_1) *and* (u_2, v_2) *in* $G_1 \times G_2$ *are adjacent if* $u_1 = u_2$ *and* v_1 *is adjacent to* v_2 *(or) if* $v_1 = v_2$ *and* u_1 *is adjacent to* u_2.

Definition 2. [2]. *The graph* $P_n \times P_m, m, n \geq 2$ *is called the planar grid graph. Let* a_{ij} *be the vertex in the* i^{th} *row and* j^{th} *column.*

Definition 3. [2]. *The Mangolian tent graph* $M_{m,n}$ *is obtained from the grid* $P_m \times P_n$, n *is odd by joinging one extra vertex above the grid and joining the vertex a with the vertices* $a_{11}, a_{12}, a_{13}, \cdots, a_{1n}$.

4 Pair difference cordial labeling

Definition 4. [4]. *Let* $G = (V, E)$ *be a* (p, q) *graph. Define*

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L = \{\pm 1, \pm 2, \pm 3, \cdots, \pm \rho\}$ *called the set of labels.*

Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to p-1 elements of V and repeating a label for the remaining one vertex when p is odd.The labeling as defined above is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u) - f(v)|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \leq 1$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 and number of edges not labeled with 1.A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph.

5 Main Results

Theorem 5. *The plannar grid $P_n \times P_n$ is pair difference cordial for all values of $n \geq 2$.*

Proof. Take the vertex set and edge set from the definition 2.2 There are two cses arises.
Case 1. n is even.
Assign the labels $1, 2, 3, \cdots, n$ respectively to the vertices $a_{11}, a_{12}, a_{13}, \cdots, a_{1n}$ and assign the labels $n + 1, n + 2, n + 3, \cdots, 2n$ to the vertices $a_{21}, a_{22}, a_{23}, \cdots, a_{2n}$ respectively. Next assign the labels $2n + 1, 2n + 2, 2n + 3, \cdots, 3n$ respectively to the vertices $a_{31}, a_{32}, a_{33}, \cdots, a_{3n}$. Proceeding like this until we reach the vertex $a_{\frac{n}{2}n}$. Note that in this process the vertex $a_{\frac{n}{2}n}$ receive the label $\frac{n^2}{2}$.

Now we assign the labels $-1, -2, -3, \cdots, -n$ respectively to the vertices $a_{\frac{n+2}{2}1}, a_{\frac{n+2}{2}2}, a_{\frac{n+2}{2}3}, \cdots, a_{\frac{n+2}{2}n}$ and assign the labels $-(n + 1), -(n + 2), -(n + 3), \cdots, -2n$ to the vertices $a_{\frac{n+4}{2}1}, a_{\frac{n+4}{2}2}, a_{\frac{n+4}{2}3}, \cdots, a_{\frac{n+4}{2}n}$ respectively. Next assign the labels $-(2n + 1), -(2n + 2), -(2n + 3), \cdots, -3n$ respectively to the vertices $a_{\frac{n+6}{2}1}, a_{\frac{n+6}{2}2}, a_{\frac{n+6}{2}3}, \cdots, a_{\frac{n+6}{2}n}$. Proceeding like this until we reach the vertex a_{nn}. Clearly the vertex a_{nn} receive the label $-\frac{n^2}{2}$.

Case 2. n is odd.
Assign the labels $1, 2, 3, \cdots, n$ respectively to the vertices $a_{11}, a_{12}, a_{13}, \cdots, a_{1n}$ and assign the labels $n + 1, n + 2, n + 3, \cdots, 2n$ to the vertices $a_{21}, a_{22}, a_{23}, \cdots, a_{2n}$ respectively. Next assign the labels $2n + 1, 2n + 2, 2n + 3, \cdots, 3n$ respectively to the vertices $a_{31}, a_{32}, a_{33}, \cdots, a_{3n}$. Proceeding like this until we reach the vertex $a_{\frac{n-1}{2}n}$. In this process $\frac{n^2-n}{2}$ is the label of the vertex $a_{\frac{n-1}{2}n}$.

Secondly assign the labels $-1, -2, -3, \cdots, -n$ respectively to the vertices $a_{\frac{n+1}{2}1}$, $a_{\frac{n+1}{2}2}, a_{\frac{n+1}{2}3}, \cdots, a_{\frac{n+1}{2}n}$ and assign the labels $-(n + 1), -(n + 2), -(n + 3), \cdots, -2n$ to the vertices $a_{\frac{n+3}{2}1}, a_{\frac{n+3}{2}2}, a_{\frac{n+3}{2}3}, \cdots, a_{\frac{n+3}{2}n}$ respectively. Next assign the labels $-(2n + 1), -(2n+2), -(2n+3), \cdots, -3n$ respectively to the vertices $a_{\frac{n+5}{2}1}, a_{\frac{n+5}{2}2}, a_{\frac{n+5}{2}3}, \cdots, a_{\frac{n+5}{2}n}$. Proceeding like this until we reach the vertex a_{n-1n}. Clearly the vertex a_{n-1n} receive the label $-\frac{n^2-n}{2}$.

Next assign the labels $-\frac{n^2-n+2}{2}, -\frac{n^2-n+4}{2}, -\frac{n^2-n+6}{2}, \cdots, -\frac{n^2-1}{2}$ to the vertices $a_{nn}, a_{n(n-1)}, a_{n(n-2)}, \cdots, a_{n\frac{n+1}{2}}$ and assign the labels $\frac{n^2-n+2}{2}, \frac{n^2-n+4}{2}, \frac{n^2-n+6}{2}, \cdots, \frac{n^2-1}{2}$ to the vertices $a_{n\frac{n-1}{2}}, a_{n\frac{n-3}{2}}, a_{n\frac{n-5}{2}}, \cdots, a_{n2}$. Finally assign the label $\frac{n^2-3}{2}$ to the vertex a_{n1}.

□

Theorem 6. *The grid $P_n \times P_m$ is pair difference cordial for all values of $n \geq 2$ and $m \geq 2$.*

Proof. There are four cases arises.

Case 1. $n \equiv 0 \pmod 4$.
First we assign the labels $1, 2, 3, \cdots, m$ respectively to the vertices $a_{11}, a_{12}, a_{13}, \cdots, a_{1m}$ and assign the labels $2m, 2m-1, 2m-2, \cdots, (m+1)$ to the vertices $a_{21}, a_{22}, a_{23}, \cdots, a_{2m}$ respectively. Next assign the labels $2m+1, 2m+2, 2m+3, \cdots, 3m$ respectively to the vertices $a_{31}, a_{32}, a_{33}, \cdots, a_{3m}$ and assign the labels $4m, 4m-1, 4m-2, \cdots, (3m+1)$ to the vertices $a_{41}, a_{42}, a_{43}, \cdots, a_{4m}$ respectively. Proceeding like this until we reach the $\frac{n}{4}^{th}$ row. Next assign the labels $4m+1, 4m+2, 4m+3, \cdots, 5m$ respectively to the vertices $a_{51}, a_{52}, a_{53}, \cdots, a_{5m}$ and assign the labels $5m+1, 5m+2, 5m+3, \cdots, 6m$ respectively to the vertices $a_{61}, a_{62}, a_{63}, \cdots, a_{6m}$. Proceed this process until we reach the $\frac{n}{2}^{th}$ row. Next assign the remaining vertices.

Next assign the labels $-1, -2, -3, \cdots, -m$ respectively to the vertices $a_{\frac{n+2}{2}1}, a_{\frac{n+2}{2}2}, a_{\frac{n+2}{2}3}, \cdots, a_{\frac{n+2}{2}m}$ and assign the labels $-(m+1), -(m+2), -(m+3), \cdots, -2m$ to the vertices $a_{\frac{n+4}{2}1}, a_{\frac{n+4}{2}2}, a_{\frac{n+4}{2}3}, \cdots, a_{\frac{n+4}{2}m}$ respectively. Next assign the labels $-(2m+1), -(2m+2), -(2m+3), \cdots, -3m$ respectively to the vertices $a_{\frac{n+6}{2}1}, a_{\frac{n+6}{2}2}, a_{\frac{n+6}{2}3}, \cdots, a_{\frac{n+6}{2}m}$. Proceeding this process upto the $(n-1)^{th}$ row.

Subcase 1. $m \equiv 0, 1, 3 \pmod 4$.
Now we assign the labels $-(\frac{mn-2m}{2})-1, -(\frac{mn-2m}{2})-3, -(\frac{mn-2m}{2})-2, -(\frac{mn-2m}{2})-4$ respectively to the vertices $a_{n1}, a_{n2}, a_{n3}, a_{n4}$ and assign the labels $-(\frac{mn-2m}{2})-5, -(\frac{mn-2m}{2})-6, -(\frac{mn-2m}{2})-7, \cdots, -\frac{mn}{2}$ to the vertices $a_{n5}, a_{n6}, a_{n7}, \cdots, a_{nm}$ respectively.

Subcase 2. $m \equiv 2 \pmod 4$.

In this case we now assign the labels $-\frac{nm}{2}, -\frac{nm-2}{2}, -\frac{nm-4}{2}, \cdots, -\frac{mn-2m}{2}-1$ respectively to the vertices $a_{n1}, a_{n2}, a_{n3}, \cdots, a_{nm}$.

The Table 1 given below establish that this vertex labeling f is a pair difference cordial of Grid graph for the values of $n \equiv 0 \pmod 4$.
Case 2. $n \equiv 1 \pmod 4$.
There are four cases arises.

Nature of m	$\Delta_{f_1^c}$	Δ_{f_1}
$m \equiv 0 \pmod 4$	$\frac{2mn-m-n}{2}$	$\frac{2mn-m-n}{2}$
$m \equiv 1 \pmod 4$	$\frac{2mn-m-n-1}{2}$	$\frac{2mn-m-n+1}{2}$
$m \equiv 2 \pmod 4$	$\frac{2mn-m-n}{2}$	$\frac{2mn-m-n}{2}$
$m \equiv 3 \pmod 4$	$\frac{2mn-m-n-1}{2}$	$\frac{2mn-m-n+1}{2}$

Table 1:

Subcase 1. $m \equiv 0 \pmod 4$.

We know that $P_n \times P_m \cong P_m \times P_n$, then assign the labels as in Subcase 2 of case 1.

Now Consider the remaining cases.

Subcase 2. $m \equiv 1, 3 \pmod 4$.
First we assign the labels $1, 2, 3, \cdots, m$ respectively to the vertices $a_{11}, a_{12}, a_{13}, \cdots, a_{1m}$ and assign the labels $2m, 2m-1, 2m-2, \cdots, (m+1)$ to the vertices $a_{21}, a_{22}, a_{23}, \cdots, a_{2m}$ respectively. Next assign the labels $2m+1, 2m+2, 2m+3, \cdots, 3m$ respectively to the vertices $a_{31}, a_{32}, a_{33}, \cdots, a_{3m}$ and assign the labels $4m, 4m-1, 4m-2, \cdots, (3m+1)$ to the vertices $a_{41}, a_{42}, a_{43}, \cdots, a_{4m}$ respectively. Proceeding like this until we reach the $\frac{n-1}{4}^{th}$ row. Next assign the labels $4m+1, 4m+2, 4m+3, \cdots, 5m$ respectively to the vertices $a_{51}, a_{52}, a_{53}, \cdots, a_{5m}$ and assign the labels $5m+1, 5m+2, 5m+3, \cdots, 6m$ respectively to the vertices $a_{61}, a_{62}, a_{63}, \cdots, a_{6m}$. Proceed this process until we reach the $\frac{n-1}{2}^{th}$ row.

Now assign the labels $-1, -2, -3, \cdots, -m$ respectively to the vertices $a_{\frac{n+1}{2}1}, a_{\frac{n+1}{2}2}, a_{\frac{n+1}{2}3}, \cdots, a_{\frac{n+1}{2}m}$ and assign the labels $-(m+1), -(m+2), -(m+3), \cdots, -2m$ to the vertices $a_{\frac{n+3}{2}1}, a_{\frac{n+3}{2}2}, a_{\frac{n+3}{2}3}, \cdots, a_{\frac{n+3}{2}m}$ respectively. Next assign the labels $-(2m+1), -(2m+2), -(2m+3), \cdots, -3m$ respectively to the vertices $a_{\frac{n+5}{2}1}, a_{\frac{n+5}{2}2}, a_{\frac{n+5}{2}3}, \cdots, a_{\frac{n+5}{2}m}$. Proceeding this process util we reach the $n-1^{th}$ row . Now assign the labels $(\frac{mn-m}{2}+1), (\frac{mn-m}{2}+2), (\frac{mn-m}{2}+3), \cdots, \frac{mn-1}{2}$ respectively to the vertices $a_{n1}, a_{n2}, a_{n3}, \cdots, a_{n\frac{m-1}{2}}$ and assign the labels $-(\frac{mn-m}{2}+1), -(\frac{mn-m}{2}+3)$ to the vertices $a_{n\frac{m+1}{2}}, a_{n\frac{m+3}{2}}$ and assign the labels $-(\frac{mn-m}{2}+2), -(\frac{mn-m}{2}+4)$ to the vertices $a_{n\frac{m+5}{2}}, a_{n\frac{m+7}{2}}$ respectively . Next assign the labels $-(\frac{mn-m}{2}+5), -(\frac{mn-m}{2}+7)$ to the vertices $a_{n\frac{m+9}{2}}, a_{n\frac{m+11}{2}}$ respectively and assign the labels $-(\frac{mn-m}{2}+6), -(\frac{mn-m}{2}+8)$ respectively to the vertices $a_{n\frac{m+13}{2}}, a_{n\frac{m+15}{2}}$. Proceeding like this until reach the vertex $a_{n(m-1)}$. Clearly $-\frac{mn-1}{2}$ is the label of the vertex a_{nm-1}.Now assign the label $\frac{mn-3}{2}$ to the vertex a_{nm}.

Subcase 3. $m \equiv 2 \pmod 4$.
As in subcase 2 of case 2, assign the labels to the vertices $a_{ij}, 1 \le i \le n-1, 1 \le j \le m$. Now assign the labels $-(\frac{mn-m}{2}+1), -(\frac{mn-m}{2}+2), -(\frac{mn-m}{2}+3), \cdots, -\frac{mn}{2}$ respectively to the vertices $a_{n1}, a_{n2}, a_{n3}, \cdots, a_{n\frac{m}{2}}$ and assign the labels $(\frac{mn-m}{2}+1), (\frac{mn-m}{2}+2), (\frac{mn-m}{2}+$

$3), \cdots, \frac{mn-4}{2}$ to the vertices $a_{n\frac{m+2}{2}}, a_{n\frac{m+4}{2}}, a_{n\frac{m+6}{2}}, \cdots, a_{n(m-2)}$ respectively. Next assign the labels $\frac{mn}{2}, \frac{mn-2}{2}$ to the vertices $a_{n(m-1)}, a_{nm}$ respectively.

The Table 2 given below establish that this vertex labeling f is a pair difference cordial of Grid graph for the values of $n \equiv 1 \pmod 4$.

Case 3. $n \equiv 2 \pmod 4$.

Nature of m	$\Delta_{f_1^c}$	Δ_{f_1}
$m \equiv 0 \pmod 4$	$\frac{2mn-m-n-1}{2}$	$\frac{2mn-m-n+1}{2}$
$m \equiv 1 \pmod 4$	$\frac{2mn-m-n}{2}$	$\frac{2mn-m-n}{2}$
$m \equiv 2 \pmod 4$	$\frac{2mn-m-n-1}{2}$	$\frac{2mn-m-n+1}{2}$
$m \equiv 3 \pmod 4$	$\frac{2mn-m-n}{2}$	$\frac{2mn-m-n}{2}$

Table 2:

Subcase 1. $m \equiv 1 \pmod 4$.

We know that $P_n \times P_m \cong P_m \times P_n$, then assign the labels assign in Subcase 3 of case 2. Now Consider the remaining cases.

We assign the labels $1, 2, 3, \cdots, m$ respectively to the vertices $a_{11}, a_{12}, a_{13}, \cdots, a_{1m}$ and assign the labels $2m, 2m-1, 2m-2, \cdots, (m+1)$ to the vertices $a_{21}, a_{22}, a_{23}, \cdots, a_{2m}$ respectively. Next assign the labels $2m+1, 2m+2, 2m+3, \cdots, 3m$ respectively to the vertices $a_{31}, a_{32}, a_{33}, \cdots, a_{3m}$ and assign the labels $4m, 4m-1, 4m-2, \cdots, (3m+1)$ to the vertices $a_{41}, a_{42}, a_{43}, \cdots, a_{4m}$ respectively. Proceeding like this until we reach the $\frac{n-2}{4}^{th}$ row. Next assign the labels $4m+1, 4m+2, 4m+3, \cdots, 5m$ respectively to the vertices $a_{51}, a_{52}, a_{53}, \cdots, a_{5m}$ and assign the labels $5m+1, 5m+2, 5m+3, \cdots, 6m$ respectively to the vertices $a_{61}, a_{62}, a_{63}, \cdots, a_{6m}$. Proceed this process until we reach the $\frac{n}{2}^{th}$ row.

Now assign the labels $-1, -2, -3, \cdots, -m$ respectively to the vertices $a_{\frac{n+1}{2}1}, a_{\frac{n+1}{2}2}, a_{\frac{n+1}{2}3}, \cdots, a_{\frac{n+1}{2}m}$ and assign the labels $-(m+1), -(m+2), -(m+3), \cdots, -2m$ to the vertices $a_{\frac{n+3}{2}1}, a_{\frac{n+3}{2}2}, a_{\frac{n+3}{2}3}, \cdots, a_{\frac{n+3}{2}m}$ respectively. Next assign the labels $-(2m+1), -(2m+2), -(2m+3), \cdots, -3m$ respectively to the vertices $a_{\frac{n+5}{2}1}, a_{\frac{n+5}{2}2}, a_{\frac{n+5}{2}3}, \cdots, a_{\frac{n+5}{2}m}$. Proceeding this process util we reach the $n-1^{th}$ row .

Subcase 2. $m \equiv 0 \pmod 4$.

Next we assign the labels $-\frac{mn}{2}, -\frac{mn-2}{2}, -\frac{mn-4}{2}, -\frac{mn-6}{2}, \cdots \frac{-mn+2m-10}{2}$ to the vertices $a_{n1}, a_{n2}, a_{n3}, \cdots, a_{n(m-4)}$ respectively and assign the labels $\frac{-mn+2m-8}{2}, \frac{-mn+2m-4}{2}, \frac{-mn+2m-6}{2}, \frac{-mn+2m-2}{2}$ to the vertices $a_{n(m-3)}, a_{n(m-2)}, a_{n(m-1)}, a_{nm}$.

Subcase 3. $m \equiv 2 \pmod 4$.

Next assign the labels to the last row vertices. We assign the labels $-(\frac{mn-2m}{2}+1), -(\frac{mn-2m}{2}+2), -(\frac{mn-2m}{2}+3), \cdots - \frac{mn-m}{2}$ to the vertices $a_{n1}, a_{n2}, a_{n3}, \cdots, a_{n\frac{m}{2}}$ respectively. Now assign the labels $-(\frac{mn-m}{2}+1), -(\frac{mn-m}{2}+3)$ respectively to the vertices $a_{n\frac{m+2}{2}}, a_{n\frac{m+4}{2}}$ and assign the labels $-(\frac{mn-m}{2}+2), -(\frac{mn-m}{2}+4)$ to the vertices $a_{n\frac{m+6}{2}}, a_{n\frac{m+8}{2}}$ respectively. Next assign the labels $-(\frac{mn-m}{2}+5), -(\frac{mn-m}{2}+7)$ respectively to the vertices $a_{n\frac{m+10}{2}}, a_{n\frac{m+12}{2}}$ and assign the labels $-(\frac{mn-m}{2}+6), -(\frac{mn-m}{2}+8)$ to the vertices $a_{n\frac{m+14}{2}}, a_{n\frac{m+16}{2}}$ respectively . Proceeding like this until reach the vertex $a_{n(m-3)}$. Clearly the vertex $a_{n(m-3)}$ receive the label $-\frac{mn-m-4}{2}$. Now we assign the labels $-\frac{mn-m-6}{2}, -\frac{mn-m}{2}, -\frac{mn-m-2}{2}$ respectively to the vertices $a_{n(m-2)}, a_{n(m-1)}, a_{nm}$.

Subcase 4. $m \equiv 3 \pmod 4$.

Next we assign the labels $-(\frac{mn-2m}{2}+1), -(\frac{mn-2m}{2}+2), -(\frac{mn-2m}{2}+3), \cdots,$ - $\frac{mn-m-1}{2}$ to the vertices $a_{n1}, a_{n2}, a_{n3}, \cdots, a_{n\frac{m-1}{2}}$ respectively. Now we assign the labels $-(\frac{mn-2m}{2}+1), -(\frac{mn-2m}{2}+3)$ to the vertices $a_{n\frac{m+1}{2}}, a_{n\frac{m+3}{2}}$ and assign the labels $-(\frac{mn-2m}{2}+2), -(\frac{mn-2m}{2}+4)$ to the vertices $a_{n\frac{m+5}{2}}, a_{n\frac{m+7}{2}}$ respectively. Proceeding like this until we reach the vertex $a_{n(m-1)}$. Clearly the vertex $a_{n(m-1)}$ receive the label $-\frac{mn-m-1}{2}$. Now we assign the label $\frac{mn-m-1}{2}$ to the vertex a_{nm}.
The Table 3 given below establish that this vertex labeling f is a pair difference cordial of Grid graph for the values of $n \equiv 2 \pmod 4$.
Case 4. $n \equiv 3 \pmod 4$.

Nature of m	Δ_{f_1}	$\Delta_{f_1^c}$
$m \equiv 0 \pmod 4$	$\frac{2mn-m-n}{2}$	$\frac{2mn-m-n}{2}$
$m \equiv 1 \pmod 4$	$\frac{2mn-m-n+1}{2}$	$\frac{2mn-m-n-1}{2}$
$m \equiv 2 \pmod 4$	$\frac{2mn-m-n}{2}$	$\frac{2mn-m-n}{2}$
$m \equiv 3 \pmod 4$	$\frac{2mn-m-n-1}{2}$	$\frac{2mn-m-n+1}{2}$

Table 3:

Subcase 1. $m \equiv 0 \pmod 4$.

Since $P_n \times P_m \cong P_m \times P_n$, assign the labels assign in Subcase 4 of case 1.

Subcase 2. $m \equiv 1 \pmod 4$.

Clearly $P_n \times P_m \cong P_m \times P_n$. We now assign the labels assign in Subcase 4 of case 2.

Subcase 3. $m \equiv 2 \pmod 4$.

We know that $P_n \times P_m \cong P_m \times P_n$. The vertex labeling in Subcase 4 of case 3 is also pair difference cordial labeling for this case also.

Subcase 4. $m \equiv 3 \pmod 4$.

First we assign the labels $1, 2, 3, \cdots, m$ respectively to the vertices $a_{11}, a_{12}, a_{13}, \cdots, a_{1m}$ and assign the labels $2m, 2m-1, 2m-2, \cdots, (m+1)$ to the vertices $a_{21}, a_{22}, a_{23}, \cdots, a_{2m}$ respectively. Next assign the labels $2m+1, 2m+2, 2m+3, \cdots, 3m$ respectively to the vertices $a_{31}, a_{32}, a_{33}, \cdots, a_{3m}$ and assign the labels $4m, 4m-1, 4m-2, \cdots, (3m+1)$ to the vertices $a_{41}, a_{42}, a_{43}, \cdots, a_{4m}$ respectively. Proceeding like this until we reach the $\frac{n-3}{4}^{th}$ row $a_{\frac{n-3}{4}i}, \leq i \leq m$. Next assign the labels $4m+1, 4m+2, 4m+3, \cdots, 5m$ respectively to the vertices $a_{51}, a_{52}, a_{53}, \cdots, a_{5m}$ and assign the labels $5m+1, 5m+2, 5m+3, \cdots, 6m$ respectively to the vertices $a_{61}, a_{62}, a_{63}, \cdots, a_{6m}$. Proceed this process until we reach the $\frac{n-1}{2}^{th}$ row.

Now assign the labels $-1, -2, -3, \cdots, -m$ respectively to the vertices $a_{\frac{n+1}{2}1}, a_{\frac{n+1}{2}2}, a_{\frac{n+1}{2}3}, \cdots, a_{\frac{n+1}{2}m}$ and assign the labels $-(m+1), -(m+2), -(m+3), \cdots, -2m$ to the vertices $a_{\frac{n+3}{2}1}, a_{\frac{n+3}{2}2}, a_{\frac{n+3}{2}3}, \cdots, a_{\frac{n+3}{2}m}$ respectively. Next assign the labels $-(2m+1), -(2m+2), -(2m+3), \cdots, -3m$ respectively to the vertices $a_{\frac{n+5}{2}1}, a_{\frac{n+5}{2}2}, a_{\frac{n+5}{2}3}, \cdots, a_{\frac{n+5}{2}m}$. Proceeding this process util we reach the $n-1^{th}$row . Now assign the labels $(\frac{mn-m}{2}+1), (\frac{mn-m}{2}+2), (\frac{mn-m}{2}+3), \cdots, \frac{mn-1}{2}$ respectively to the vertices $a_{n1}, a_{n2}, a_{n3}, \cdots, a_{n\frac{m-1}{2}}$ and assign the labels $-(\frac{mn-m}{2}+1), -(\frac{mn-m}{2}+3)$ to the vertices $a_{n\frac{m+1}{2}}, a_{n\frac{m+3}{2}}$ and assign the labels $-(\frac{mn-m}{2}+2), -(\frac{mn-m}{2}+4)$ to the vertices $a_{n\frac{m+5}{2}}, a_{n\frac{m+7}{2}}$ respectively . Next assign the labels $-(\frac{mn-m}{2}+5), -(\frac{mn-m}{2}+7)$ to the vertices $a_{n\frac{m+9}{2}}, a_{n\frac{m+11}{2}}$ respectively and assign the labels $-(\frac{mn-m}{2}+6), -(\frac{mn-m}{2}+8)$ respectively to the vertices $a_{n\frac{m+13}{2}}, a_{n\frac{m+15}{2}}$. Proceeding like this until reach the vertex $a_{n(m-1)}$.In this process $\frac{mn-1}{2}$ is the label of the vertex $a_{n(m-1)}$. Now assign the label $\frac{mn-1}{2}$ to the vertex a_{nm}. Note that the vertex $a_{n(m-1)}$ receive the label $\frac{mn-1}{2}$.
The Table 4 given below establish that this vertex labeling f is a pair difference cordial of Grid graph for the values of $n \equiv 3 \pmod 4$.

□

Nature of m	$\Delta_{f_1^c}$	Δ_{f_1}
$m \equiv 0 \pmod 4$	$\frac{2mn-m-n-1}{2}$	$\frac{2mn-m-n+1}{2}$
$m \equiv 1 \pmod 4$	$\frac{2mn-m-n}{2}$	$\frac{2mn-m-n}{2}$
$m \equiv 2 \pmod 4$	$\frac{2mn-m-n+1}{2}$	$\frac{2mn-m-n-1}{2}$
$m \equiv 3 \pmod 4$	$\frac{2mn-m-n}{2}$	$\frac{2mn-m-n}{2}$

Table 4:

Theorem 7. *The mangolian tent $M_{m,n}$ is pair difference cordial for all m and odd values of $n \geq 3$.*

Proof. Let the vertex set and edge set taken from the definition 2.3 .
Case 1. m is even.

Assign the labels $1, 2, 3, \cdots, n$ respectively to the vertices $a_{11}, a_{12}, a_{13}, \cdots, a_{1n}$ and assign the labels $2n, 2n-1, 2n-2, \cdots, n+1$ to the vertices $a_{21}, a_{22}, a_{23}, \cdots, a_{2n}$ respectively. Next assign the labels $2n+1, 2n+2, 2n+3, \cdots, 3n$ respectively to the vertices $a_{31}, a_{32}, a_{33}, \cdots, a_{3n}$ and assign the labels $4n, 4n-1, 4n-2, \cdots, 3n+1$ respectively to the vertices $a_{41}, a_{42}, a_{43}, \cdots, a_{4n}$. Proceeding like this until we reach the $\frac{m}{2}^{th}$ row.

Now we assign the labels $-1, -2, -3, \cdots, -n$ respectively to the vertices $a_{\frac{m+2}{2}1}, a_{\frac{m+2}{2}2}, a_{\frac{m+2}{2}3}, \cdots, a_{\frac{m+2}{2}n}$ and assign the labels $-(n+1), -(n+2), -(n+3), \cdots, -2n$ to the vertices $a_{\frac{m+4}{2}1}, a_{\frac{m+4}{2}2}, a_{\frac{m+4}{2}3}, \cdots, a_{\frac{m+4}{2}n}$ respectively. Next assign the labels $-(2n+1), -(2n+2), -(2n+3), \cdots, -3n$ respectively to the vertices $a_{\frac{m+6}{2}1}, a_{\frac{m+6}{2}2}, a_{\frac{m+6}{2}3}, \cdots, a_{\frac{m+6}{2}n}$ and assign the labels $-(3n+1), -(3n+2), -(3n+3), \cdots, -4n$ respectively to the vertices $a_{\frac{m+8}{2}1}, a_{\frac{m+8}{2}2}, a_{\frac{m+8}{2}3}, \cdots, a_{\frac{m+8}{2}n}$. Proceeding like this until we reach the $m-1^{th}$ row.

Next assign the labels $-\frac{mn}{2}, -\frac{mn-2}{2}, -\frac{mn-4}{2}, \cdots, -\frac{mn-2m}{2}$ to the vertices $a_{m1}, a_{m2}, a_{m3}, \cdots, a_{mn}$.Next assign the label $\frac{mn}{2}$ to the vertex a.

Case 2. m is odd.
Assign the labels $1, 2, 3, \cdots, n$ respectively to the vertices $a_{11}, a_{12}, a_{13}, \cdots, a_{1n}$ and assign the labels $2n, 2n-1, 2n-2, \cdots, n+1$ to the vertices $a_{21}, a_{22}, a_{23}, \cdots, a_{2n}$ respectively. Next assign the labels $2n+1, 2n+2, 2n+3, \cdots, 3n$ respectively to the vertices $a_{31}, a_{32}, a_{33}, \cdots, a_{3n}$ and assign the labels $4n, 4n-1, 4n-2, \cdots, 3n+1$ respectively to the vertices $a_{41}, a_{42}, a_{43}, \cdots, a_{4n}$. Proceeding like this until we reach the $\frac{m-1}{2}^{th}$ row.

We now assign the labels $-1, -2, -3, \cdots, -n$ respectively to the vertices $a_{\frac{m+1}{2}1}, a_{\frac{m+1}{2}2}, a_{\frac{m+1}{2}3}, \cdots, a_{\frac{m+1}{2}n}$ and assign the labels $-(n+1), -(n+2), -(n+3), \cdots, -2n$ to the vertices $a_{\frac{m+3}{2}1}, a_{\frac{m+3}{2}2}, a_{\frac{m+3}{2}3}, \cdots, a_{\frac{m+3}{2}n}$ respectively. Next assign the labels $-(2n+1), -(2n+2), -(2n+3), \cdots, -3n$ respectively to the vertices $a_{\frac{m+5}{2}1}, a_{\frac{m+5}{2}2}, a_{\frac{m+5}{2}3}, \cdots, a_{\frac{m+5}{2}n}$ and assign the labels $-4n, -(4n-1), -(4n-2), \cdots, -(3n+1)$ respectively to the vertices $a_{\frac{m+7}{2}1}, a_{\frac{m+7}{2}2}, a_{\frac{m+7}{2}3}, \cdots, a_{\frac{m+7}{2}n}$. Proceeding like this until we reach the $(m-2)^{th}$ row.

Next assign the labels $-\frac{mn-n}{2}, -\frac{mn-n-2}{2}, -\frac{mn-n-4}{2}, \cdots, -\frac{mn-3n}{2}$ to the vertices $a_{(m-1)1}, a_{(m-1)2}, a_{(m-1)3}, \cdots, a_{(m-1)n}$ and assign the labels $-(\frac{mn-n}{2}+1), -(\frac{mn-n}{2}+2), -(\frac{mn-n}{2}+3), \cdots, -\frac{mn+1}{2}$ to the vertices $a_{m1}, a_{m2}, a_{m3}, \cdots, a_{m\frac{n+1}{2}}$. Finally assign the labels $-(\frac{mn-n}{2}+1), -(\frac{mn-n}{2}+2), -(\frac{mn-n}{2}+3), \cdots, \frac{mn-1}{2}$ to the vertices $a_{m\frac{n+3}{2}}, a_{m\frac{n+5}{2}}, a_{m\frac{n+7}{2}}, \cdots, a_{mn}$ and assign the label $\frac{mn+1}{2}$ to the vertex a.

The Table 5 given below establish that this vertex labeling f is a pair difference cordial of the mangolian tent for all m and odd values of n.

□

Nature of m	$\Delta_{f_1^c}$	Δ_{f_1}
m is odd	$\frac{2mn-m+1}{2}$	$\frac{2mn-m-1}{2}$
m is even	$\frac{2mn-m}{2}$	$\frac{2mn-m}{2}$

Table 5:

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