PAIR DIFFERENCE CORDIAL LABELING OF PLANAR GRID AND MONGOLIAN TENT

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Abstract. Let $G = (V, E)$ be a (p, q) graph.
Define

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L = \{\pm 1, \pm 2, \pm 3, \cdots, \pm \rho\}$ called the set of labels.
Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to p-1 elements of V and repeating a label for the remaining one vertex when p is odd.The labeling as defined above is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u) - f(v)|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \leq 1$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 and number of edges not labeled with 1. A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph. In this paper we investigate pair difference cordial labeling behavior of planar grid and mangolian tent graphs.

1. Introduction

In this paper we consider only finite, undirected and simple graphs. The concept of graceful labeling was introduced by Rosa in [9] and harmonious labeling by Graham and Sloane [3] . Similar line Cachit introduced the weaker version of graceful and harmonious labeling called cordial labeling in [1]. Motivated by these labeling technique, the pair difference cordial labeling of a graph was introduced in [5]. Also pair difference cordial labeling behavior of several graphs like path, cycle, star,ladder,wheel,helm,web,fan, umberalla,(n,t)-kite, mobius ladder, slanting ladder,triangular ladder have been investigated in [5,6,7,8].In this paper we investigate the pair difference cordial labeling behavior of planar grid and mangolian tent graphs.

2. Preliminaries

Definition 2.1. [4]. *For any graph $G_1 = G_1(V_1, E_1)$ and $G_2 = G_2(V_2, E_2)$ their product $G_1 * G_2$ is defined as the graph with vertex set $V_1 \times V_2$ and two vertices (u_1, v_1) and (u_2, v_2) in $G_1 \times G_2$ are adjacent if $u_1 = u_2$ and v_1 is adjacent to v_2 (or) if $v_1 = v_2$ and u_1 is adjacent to u_2.*

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Definition 2.2. [2]. *The graph $P_n \times P_m, m, n \geq 2$ is called the planar grid graph. Let a_{ij} be the vertex in the i^{th} row and j^{th} column.*

Definition 2.3. [2]. *The Mangolian tent graph $M_{m,n}$ is obtained from the grid $P_m \times P_n$, n is odd by joinging one extra vertex above the grid and joining the vertex a with the vertices $a_{11}, a_{12}, a_{13}, \cdots, a_{1n}$.*

3. Pair difference cordial labeling

Definition 3.1. [4]. *Let $G = (V, E)$ be a (p, q) graph.*
Define

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L = \{\pm 1, \pm 2, \pm 3, \cdots, \pm \rho\}$ called the set of labels.
Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to p-1 elements of V and repeating a label for the remaining one vertex when p is odd.The labeling as defined above is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u) - f(v)|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \leq 1$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 and number of edges not labeled with 1.A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph.

4. Main Results

Theorem 4.1. *The plannar grid $P_n \times P_n$ is pair difference cordial for all values of $n \geq 2$.*

Proof. Take the vertex set and edge set from the definition 2.2
There are two cses arises.

Case 1. n is even.

Assign the labels $1, 2, 3, \cdots, n$ respectively to the vertices $a_{11}, a_{12}, a_{13}, \cdots, a_{1n}$ and assign the labels $n+1, n+2, n+3, \cdots, 2n$ to the vertices $a_{21}, a_{22}, a_{23}, \cdots, a_{2n}$ respectively. Next assign the labels $2n+1, 2n+2, 2n+3, \cdots, 3n$ respectively to the vertices $a_{31}, a_{32}, a_{33}, \cdots, a_{3n}$. Proceeding like this until we reach the vertex $a_{\frac{n}{2}n}$. Note that in this process the vertex $a_{\frac{n}{2}n}$ receive the label $\frac{n^2}{2}$.

Now we assign the labels $-1, -2, -3, \cdots, -n$ respectively to the vertices $a_{\frac{n+2}{2}1}$, $a_{\frac{n+2}{2}2}, a_{\frac{n+2}{2}3}, \cdots, a_{\frac{n+2}{2}n}$ and assign the labels $-(n+1), -(n+2), -(n+3), \cdots, -2n$ to the vertices $a_{\frac{n+4}{2}1}, a_{\frac{n+4}{2}2}, a_{\frac{n+4}{2}3}, \cdots, a_{\frac{n+4}{2}n}$ respectively. Next assign the labels $-(2n+1), -(2n+2), -(2n+3), \cdots, -3n$ respectively to the vertices $a_{\frac{n+6}{2}1}, a_{\frac{n+6}{2}2}$, $a_{\frac{n+6}{2}3}, \cdots, a_{\frac{n+6}{2}n}$. Proceeding like this until we reach the vertex a_{nn}. Clearly the vertex a_{nn} receive the label $-\frac{n^2}{2}$.

Case 2. n is odd.

Assign the labels $1, 2, 3, \cdots, n$ respectively to the vertices $a_{11}, a_{12}, a_{13}, \cdots, a_{1n}$ and assign the labels $n+1, n+2, n+3, \cdots, 2n$ to the vertices $a_{21}, a_{22}, a_{23}, \cdots, a_{2n}$ respectively. Next assign the labels $2n+1, 2n+2, 2n+3, \cdots, 3n$ respectively to the vertices $a_{31}, a_{32}, a_{33}, \cdots, a_{3n}$. Proceeding like this until we reach the vertex $a_{\frac{n-1}{2}n}$. In this process $\frac{n^2-n}{2}$ is the label of the vertex $a_{\frac{n-1}{2}n}$.

Secondly assign the labels $-1, -2, -3, \cdots, -n$ respectively to the vertices $a_{\frac{n+1}{2}1}$, $a_{\frac{n+1}{2}2}, a_{\frac{n+1}{2}3}, \cdots, a_{\frac{n+1}{2}n}$ and assign the labels $-(n+1), -(n+2), -(n+3), \cdots, -2n$ to the vertices $a_{\frac{n+3}{2}1}, a_{\frac{n+3}{2}2}, a_{\frac{n+3}{2}3}, \cdots, a_{\frac{n+3}{2}n}$ respectively. Next assign the labels $-(2n+1), -(2n+2), -(2n+3), \cdots, -3n$ respectively to the vertices $a_{\frac{n+5}{2}1}, a_{\frac{n+5}{2}2}$, $a_{\frac{n+5}{2}3}, \cdots, a_{\frac{n+5}{2}n}$. Proceeding like this until we reach the vertex a_{n-1n-1}. Clearly the vertex a_{n-1n-1} receive the label $-\frac{n^2-n}{2}$.
Next assign the labels $-\frac{n^2-n+2}{2}, -\frac{n^2-n+4}{2}, -\frac{n^2-n+6}{2}, \cdots, -\frac{n^2-1}{2}$ to the vertices $a_{nn}, a_{n(n-1)}$,
$a_{n(n-2)}, \cdots, a_{n\frac{n+1}{2}}$ and assign the labels $\frac{n^2-n+2}{2}, \frac{n^2-n+4}{2}, \frac{n^2-n+6}{2}, \cdots, \frac{n^2-1}{2}$ to the vertices $a_{n\frac{n-1}{2}}, a_{n\frac{n-3}{2}}, a_{n\frac{n-5}{2}}, \cdots, a_{n2}$. Finally assign the label $\frac{n^2-3}{2}$ to the vertex a_{n1}.

□

Theorem 4.2. *The grid $P_n \times P_m$ is pair difference cordial for all values of $n \geq 2$ and $m \geq 2$.*

Proof. There are four cases arises.

Case 1. $n \equiv 0 \pmod 4$.

First we assign the labels $1, 2, 3, \cdots, m$ respectively to the vertices a_{11}, a_{12}, a_{13}, $\cdots, a_{1m}$ and assign the labels $2m, 2m-1, 2m-2, \cdots, (m+1)$ to the vertices $a_{21}, a_{22}, a_{23}, \cdots, a_{2m}$ respectively. Next assign the labels $2m+1, 2m+2, 2m+3, \cdots, 3m$ respectively to the vertices $a_{31}, a_{32}, a_{33}, \cdots, a_{3m}$ and assign the labels $4m, 4m-1, 4m-2, \cdots, (3m+1)$ to the vertices $a_{41}, a_{42}, a_{43}, \cdots, a_{4m}$ respectively. Proceeding like this until we reach the $\frac{n}{4}^{th}$ row $a_{\frac{n}{4}i}, \leq i \leq m$. Next assign the labels $4m+1, 4m+2, 4m+3, \cdots, 5m$ respectively to the vertices $a_{51}, a_{52}, a_{53}, \cdots, a_{5m}$ and assign the labels $5m+1, 5m+2, 5m+3, \cdots, 6m$ respectively to the vertices $a_{61}, a_{62}, a_{63}, \cdots, a_{6m}$. Proceed this process until we reach the row $\frac{n}{2}$. Next assign the remaining vertices.There are four subcases arises.

Subcase 1. $m \equiv 0 \pmod 4$.

Next assign the labels $-1, -2, -3, \cdots, -m$ respectively to the vertices $a_{\frac{n+2}{2}1}$, $a_{\frac{n+2}{2}2}, a_{\frac{n+2}{2}3}, \cdots, a_{\frac{n+2}{2}m}$ and assign the labels $-(m+1), -(m+2), -(m+3), \cdots, -2m$ to the vertices $a_{\frac{n+4}{2}1}, a_{\frac{n+4}{2}2}, a_{\frac{n+4}{2}3}, \cdots, a_{\frac{n+4}{2}m}$ respectively. Next assign the labels $-(2m+1), -(2m+2), -(2m+3), \cdots, -3m$ respectively to the vertices $a_{\frac{n+6}{2}1}, a_{\frac{n+6}{2}2}$, $a_{\frac{n+6}{2}3}, \cdots, a_{\frac{n+6}{2}m}$. Proceeding this process upto the row $a_{(n-1)m}$. Now we assign the labels $-(\frac{mn-2m}{2})-1, -(\frac{mn-2m}{2})-3, -(\frac{mn-2m}{2})-2, -(\frac{mn-2m}{2})-4$ respectively to the vertices $a_{n1}, a_{n2}, a_{n3}, a_{n4}$ and assign the labels $-(\frac{mn-2m}{2})-5, -(\frac{mn-2m}{2})-6, -(\frac{mn-2m}{2})-7, \cdots, -\frac{mn}{2}$ to the vertices $a_{n5}, a_{n6}, a_{n7}, \cdots, a_{nm}$ respectively.

Subcase 2. $m \equiv 1 \pmod 4$.

As in subcase 1, assign the labels to the vertices $a_{ij}, 1 \leq i \leq n, 1 \leq j \leq m$.

Subcase 3. $m \equiv 2 \pmod 4$.

Assign the labels as in subcase 1 to the vertices $a_{ij}, 1 \leq i \leq n-1, 1 \leq j \leq m$. Now we assign the labels $-\frac{nm}{2}, -\frac{nm-2}{2}, -\frac{nm-4}{2}, \cdots, -\frac{mn-2m}{2} - 1$ respectively to the vertices $a_{n1}, a_{n2}, a_{n3}, \cdots, a_{nm}$.

Subcase 4. $m \equiv 3 \pmod 4$.

Assign the labels as in subcase 1, to the vertices $a_{ij}, 1 \leq i \leq n, 1 \leq j \leq m$. The Table 1 given below establish that this vertex labeling f is a pair difference cordial of Grid graph for the values of $n \equiv 0 \pmod 4$.

Nature of m	$\Delta_{f_1^c}$	Δ_{f_1}
$m \equiv 0 \pmod 4$	$\frac{2mn-m-n}{2}$	$\frac{2mn-m-n}{2}$
$m \equiv 1 \pmod 4$	$\frac{2mn-m-n-1}{2}$	$\frac{2mn-m-n+1}{2}$
$m \equiv 2 \pmod 4$	$\frac{2mn-m-n}{2}$	$\frac{2mn-m-n}{2}$
$m \equiv 3 \pmod 4$	$\frac{2mn-m-n-1}{2}$	$\frac{2mn-m-n+1}{2}$

TABLE 1

Case 2. $n \equiv 1 \pmod 4$.

There are four cases arises.

Subcase 1. $m \equiv 0 \pmod 4$.

We know that $P_n \times P_m \cong P_m \times P_n$, then assign the labels as in Subcase 2 of case 1.

Subcase 2. $m \equiv 1 \pmod 4$.

First we assign the labels $1, 2, 3, \cdots, m$ respectively to the vertices $a_{11}, a_{12}, a_{13}, \cdots, a_{1m}$ and assign the labels $2m, 2m-1, 2m-2, \cdots, (m+1)$ to the vertices $a_{21}, a_{22}, a_{23}, \cdots, a_{2m}$ respectively. Next assign the labels $2m+1, 2m+2, 2m+3, \cdots, 3m$ respectively to the vertices $a_{31}, a_{32}, a_{33}, \cdots, a_{3m}$ and assign the labels $4m, 4m-1, 4m-2, \cdots, (3m+1)$ to the vertices $a_{41}, a_{42}, a_{43}, \cdots, a_{4m}$ respectively. Proceeding like this until we reach the $\frac{n-1}{4}^{th}$ row $a_{\frac{n-1}{4}i}, \leq i \leq m$. Next assign the labels $4m+1, 4m+2, 4m+3, \cdots, 5m$ respectively to the vertices $a_{51}, a_{52}, a_{53}, \cdots, a_{5m}$ and assign the labels $5m+1, 5m+2, 5m+3, \cdots, 6m$ respectively to the vertices

$a_{61}, a_{62}, a_{63}, \cdots, a_{6m}$. Proceed this process until we reach the row $\frac{n-1}{2}$.

Now assign the labels $-1, -2, -3, \cdots, -m$ respectively to the vertices $a_{\frac{n+1}{2}1}$, $a_{\frac{n+1}{2}2}, a_{\frac{n+1}{2}3}, \cdots, a_{\frac{n+1}{2}m}$ and assign the labels $-(m+1), -(m+2), -(m+3), \cdots, -2m$ to the vertices $a_{\frac{n+3}{2}1}, a_{\frac{n+3}{2}2}, a_{\frac{n+3}{2}3}, \cdots, a_{\frac{n+3}{2}m}$ respectively. Next assign the labels $-(2m+1), -(2m+2), -(2m+3), \cdots, -3m$ respectively to the vertices $a_{\frac{n+5}{2}1}, a_{\frac{n+5}{2}2}$, $a_{\frac{n+5}{2}3}, \cdots, a_{\frac{n+5}{2}m}$. Proceeding this process util we reach the row $a_{(n-1)i}, 1 \leq i \leq m$. Now assign the labels $(\frac{mn-m}{2}+1), (\frac{mn-m}{2}+2), (\frac{mn-m}{2}+3), \cdots, \frac{mn-1}{2}$ respectively to the vertices $a_{n1}, a_{n2}, a_{n3}, \cdots, a_{n\frac{m-1}{2}}$ and assign the labels $-(\frac{mn-m}{2}+1), -(\frac{mn-m}{2}+3)$ to the vertices $a_{n\frac{m+1}{2}}, a_{n\frac{m+3}{2}}$ and assign the labels $-(\frac{mn-m}{2}+2), -(\frac{mn-m}{2}+4)$ to the vertices $a_{n\frac{m+5}{2}}, a_{n\frac{m+7}{2}}$ respectively . Next assign the labels $-(\frac{mn-m}{2}+5), -(\frac{mn-m}{2}+7)$ to the vertices $a_{n\frac{m+9}{2}}, a_{n\frac{m+11}{2}}$ respectively and assign the labels $-(\frac{mn-m}{2}+6), -(\frac{mn-m}{2}+8)$ respectively to the vertices $a_{n\frac{m+13}{2}}, a_{n\frac{m+15}{2}}$. Proceeding like this until reach the vertex $a_{n(m-1)}$. Clearly $-\frac{mn-1}{2}$ is the label of the vertex a_{nm-1}.Now assign the label $\frac{mn-3}{2}$ to the vertex a_{nm}.

Subcase 3. $m \equiv 2 \pmod 4$.

As in subcase 2 of case 2, assign the labels to the vertices $a_{ij}, 1 \leq i \leq n-1, 1 \leq j \leq m$. Now assign the labels $-(\frac{mn-m}{2}+1), -(\frac{mn-m}{2}+2), -(\frac{mn-m}{2}+3), \cdots, -\frac{mn}{2}$ respectively to the vertices $a_{n1}, a_{n2}, a_{n3}, \cdots, a_{n\frac{m}{2}}$ and assign the labels $(\frac{mn-m}{2}+1), (\frac{mn-m}{2}+2), (\frac{mn-m}{2}+3), \cdots, \frac{mn-4}{2}$ to the vertices $a_{n\frac{m+2}{2}}, a_{n\frac{m+4}{2}}, a_{n\frac{m+6}{2}}, \cdots$, $a_{n(m-2)}$ respectively. Next assign the labels $\frac{mn}{2}, \frac{mn-2}{2}$ to the vertices $a_{n(m-1)}, a_{nm}$ respectively.

Subcase 4. $m \equiv 3 \pmod 4$.

As in subcase 2 of case 2, assign the labels to the vertices $a_{ij}, 1 \leq i \leq n, 1 \leq j \leq m$.
The Table 2 given below establish that this vertex labeling f is a pair difference cordial of Grid graph for the values of $n \equiv 1 \pmod 4$.

Nature of m	$\Delta_{f_1^c}$	Δ_{f_1}
$m \equiv 0 \pmod 4$	$\frac{2mn-m-n-1}{2}$	$\frac{2mn-m-n+1}{2}$
$m \equiv 1 \pmod 4$	$\frac{2mn-m-n}{2}$	$\frac{2mn-m-n}{2}$
$m \equiv 2 \pmod 4$	$\frac{2mn-m-n-1}{2}$	$\frac{2mn-m-n+1}{2}$
$m \equiv 3 \pmod 4$	$\frac{2mn-m-n}{2}$	$\frac{2mn-m-n}{2}$

TABLE 2

Case 3. $n \equiv 2 \pmod 4$.

Subcase 1. $m \equiv 0 \pmod 4$.

First we assign the labels $1, 2, 3, \cdots, m$ respectively to the vertices $a_{11}, a_{12}, a_{13}, \cdots, a_{1m}$ and assign the labels $2m, 2m-1, 2m-2, \cdots, (m+1)$ to the vertices $a_{21}, a_{22}, a_{23}, \cdots, a_{2m}$ respectively. Next assign the labels $2m+1, 2m+2, 2m+3, \cdots, 3m$ respectively to the vertices $a_{31}, a_{32}, a_{33}, \cdots, a_{3m}$ and assign the labels $4m, 4m-1, 4m-2, \cdots, (3m+1)$ to the vertices $a_{41}, a_{42}, a_{43}, \cdots, a_{4m}$ respectively. Proceeding like this until we reach the $\frac{n-2}{4}^{th}$ row $a_{\frac{n-2}{4}i}, \leq i \leq m$. Next assign the labels $4m+1, 4m+2, 4m+3, \cdots, 5m$ respectively to the vertices $a_{51}, a_{52}, a_{53}, \cdots, a_{5m}$ and assign the labels $5m+1, 5m+2, 5m+3, \cdots, 6m$ respectively to the vertices $a_{61}, a_{62}, a_{63}, \cdots, a_{6m}$. Proceed this process until we reach the row $\frac{n}{2}$.

Now assign the labels $-1, -2, -3, \cdots, -m$ respectively to the vertices $a_{\frac{n+1}{2}1}, a_{\frac{n+1}{2}2}, a_{\frac{n+1}{2}3}, \cdots, a_{\frac{n+1}{2}m}$ and assign the labels $-(m+1), -(m+2), -(m+3), \cdots, -2m$ to the vertices $a_{\frac{n+3}{2}1}, a_{\frac{n+3}{2}2}, a_{\frac{n+3}{2}3}, \cdots, a_{\frac{n+3}{2}m}$ respectively. Next assign the labels $-(2m+1), -(2m+2), -(2m+3), \cdots, -3m$ respectively to the vertices $a_{\frac{n+5}{2}1}, a_{\frac{n+5}{2}2}, a_{\frac{n+5}{2}3}, \cdots, a_{\frac{n+5}{2}m}$. Proceeding this process util we reach the row $a_{(n-1)i}, 1 \leq i \leq m$. Next we assign the labels $-\frac{mn}{2}, -\frac{mn-2}{2}, -\frac{mn-4}{2}, -\frac{mn-6}{2}, \cdots \frac{-mn+2m-10}{2}$ to the vertices $a_{n1}, a_{n2}, a_{n3}, \cdots, a_{n(m-4)}$ respectively and assign the labels $\frac{-mn+2m-8}{2}$, $\frac{-mn+2m-4}{2}, \frac{-mn+2m-6}{2}, \frac{-mn+2m-2}{2}$ to the vertices $a_{n(m-3)}, a_{n(m-2)}, a_{n(m-1)}, a_{nm}$.

Subcase 2. $m \equiv 1 \pmod 4$.

We know that $P_n \times P_m \cong P_m \times P_n$, then assign the labels assign in Subcase 3 of case 2.

Subcase 3. $m \equiv 2 \pmod 4$.

Assign the labels as in subcase 1 of case 3, to the vertices $a_{ij}, 1 \leq i \leq n-1$ and $1 \leq i \leq m$. Next assign the labels to the last row vertices. We assign the labels $-(\frac{mn-2m}{2}+1), -(\frac{mn-2m}{2}+2), -(\frac{mn-2m}{2}+3), \cdots - \frac{mn-m}{2}$ to the vertices $a_{n1}, a_{n2}, a_{n3}, \cdots, a_{n\frac{m}{2}}$ respectively. Now assign the labels $-(\frac{mn-m}{2}+1), -(\frac{mn-m}{2}+3)$ respectively to the vertices $a_{n\frac{m+2}{2}}, a_{n\frac{m+4}{2}}$ and assign the labels $-(\frac{mn-m}{2}+2), -(\frac{mn-m}{2}+4)$ to the vertices $a_{n\frac{m+6}{2}}, a_{n\frac{m+8}{2}}$ respectively. Next assign the labels $-(\frac{mn-m}{2}+5), -(\frac{mn-m}{2}+7)$ respectively to the vertices $a_{n\frac{m+10}{2}}, a_{n\frac{m+12}{2}}$ and assign the labels $-(\frac{mn-m}{2}+6), -(\frac{mn-m}{2}+8)$ to the vertices $a_{n\frac{m+14}{2}}, a_{n\frac{m+16}{2}}$ respectively . Proceeding like this until reach the vertex $a_{n(m-3)}$. Clearly the vertex $a_{n(m-3)}$ receive the label $-\frac{mn-m-4}{2}$. Now we assign the labels $-\frac{mn-m-6}{2}, -\frac{mn-m}{2}, -\frac{mn-m-2}{2}$ respectively to the vertices $a_{n(m-2)}, a_{n(m-1)}, a_{nm}$.

Subcase 4. $m \equiv 3 \pmod 4$.

As in subcase 1 of case 3, assign the labels to the vertices $a_{ij}, 1 \leq i \leq n-1$ and $1 \leq i \leq m$. Next we assign the labels $-(\frac{mn-2m}{2}+1), -(\frac{mn-2m}{2}+2), - -(\frac{mn-2m}{2}+3), \cdots, -\frac{mn-m-1}{2}$ to the vertices $a_{n1}, a_{n2}, a_{n3}, \cdots, a_{n\frac{m-1}{2}}$ respectively. Now we assign the labels $-(\frac{mn-2m}{2}+1), -(\frac{mn-2m}{2}+3)$ to the vertices $a_{n\frac{m+1}{2}}, a_{n\frac{m+3}{2}}$ and assign the labels $-(\frac{mn-2m}{2}+2), -(\frac{mn-2m}{2}+4)$ to the vertices $a_{n\frac{m+5}{2}}, a_{n\frac{m+7}{2}}$

respectively. Proceeding like this until we reach the vertex $a_{n(m-1)}$. Clearly the vertex $a_{n(m-1)}$ receive the label $-\frac{mn-m-1}{2}$. Now we assign the label $\frac{mn-m-1}{2}$ to the vertex a_{nm}.
The Table 3 given below establish that this vertex labeling f is a pair difference cordial of Grid graph for the values of $n \equiv 2 \pmod 4$.

Nature of m	Δ_{f_1}	$\Delta_{f_1^c}$
$m \equiv 0 \pmod 4$	$\frac{2mn-m-n}{2}$	$\frac{2mn-m-n}{2}$
$m \equiv 1 \pmod 4$	$\frac{2mn-m-n+1}{2}$	$\frac{2mn-m-n-1}{2}$
$m \equiv 2 \pmod 4$	$\frac{2mn-m-n}{2}$	$\frac{2mn-m-n}{2}$
$m \equiv 3 \pmod 4$	$\frac{2mn-m-n-1}{2}$	$\frac{2mn-m-n+1}{2}$

TABLE 3

Case 4. $n \equiv 3 \pmod 4$.

Subcase 1. $m \equiv 0 \pmod 4$.

Since $P_n \times P_m \cong P_m \times P_n$, assign the labels assign in Subcase 4 of case 1.

Subcase 2. $m \equiv 1 \pmod 4$.

Clearly $P_n \times P_m \cong P_m \times P_n$. We now assign the labels assign in Subcase 4 of case 2.

Subcase 3. $m \equiv 2 \pmod 4$.

We know that $P_n \times P_m \cong P_m \times P_n$. The vertex labeling in Subcase 4 of case 3 is also pair difference cordial labeling for this case also.

Subcase 4. $m \equiv 3 \pmod 4$.

First we assign the labels $1, 2, 3, \cdots, m$ respectively to the vertices $a_{11}, a_{12}, a_{13}, \cdots, a_{1m}$ and assign the labels $2m, 2m-1, 2m-2, \cdots, (m+1)$ to the vertices $a_{21}, a_{22}, a_{23}, \cdots, a_{2m}$ respectively. Next assign the labels $2m+1, 2m+2, 2m+3, \cdots, 3m$ respectively to the vertices $a_{31}, a_{32}, a_{33}, \cdots, a_{3m}$ and assign the labels $4m, 4m-1, 4m-2, \cdots, (3m+1)$ to the vertices $a_{41}, a_{42}, a_{43}, \cdots, a_{4m}$ respectively. Proceeding like this until we reach the $\frac{n-3}{4}^{th}$ row $a_{\frac{n-3}{4}i}, \leq i \leq m$. Next assign the labels $4m+1, 4m+2, 4m+3, \cdots, 5m$ respectively to the vertices $a_{51}, a_{52}, a_{53}, \cdots, a_{5m}$ and assign the labels $5m+1, 5m+2, 5m+3, \cdots, 6m$ respectively to the vertices $a_{61}, a_{62}, a_{63}, \cdots, a_{6m}$. Proceed this process until we reach the row $\frac{n-1}{2}$.

Now assign the labels $-1, -2, -3, \cdots, -m$ respectively to the vertices $a_{\frac{n+1}{2}1}, a_{\frac{n+1}{2}2}, a_{\frac{n+1}{2}3}, \cdots, a_{\frac{n+1}{2}m}$ and assign the labels $-(m+1), -(m+2), -(m+3), \cdots, -2m$ to the vertices $a_{\frac{n+3}{2}1}, a_{\frac{n+3}{2}2}, a_{\frac{n+3}{2}3}, \cdots, a_{\frac{n+3}{2}m}$ respectively. Next assign the labels $-(2m+1), -(2m+2), -(2m+3), \cdots, -3m$ respectively to the vertices $a_{\frac{n+5}{2}1}, a_{\frac{n+5}{2}2}$,

$a_{\frac{n+5}{2}3}, \cdots, a_{\frac{n+5}{2}m}$. Proceeding this process util we reach the row $a_{(n-1)i}, 1 \le i \le m$. Now assign the labels $(\frac{mn-m}{2}+1), (\frac{mn-m}{2}+2), (\frac{mn-m}{2}+3), \cdots, \frac{mn-1}{2}$ respectively to the vertices $a_{n1}, a_{n2}, a_{n3}, \cdots, a_{n\frac{m-1}{2}}$ and assign the labels $-(\frac{mn-m}{2}+1), -(\frac{mn-m}{2}+3)$ to the vertices $a_{n\frac{m+1}{2}}, a_{n\frac{m+3}{2}}$ and assign the labels $-(\frac{mn-m}{2}+2), -(\frac{mn-m}{2}+4)$ to the vertices $a_{n\frac{m+5}{2}}, a_{n\frac{m+7}{2}}$ respectively . Next assign the labels $-(\frac{mn-m}{2}+5), -(\frac{mn-m}{2}+7)$ to the vertices $a_{n\frac{m+9}{2}}, a_{n\frac{m+11}{2}}$ respectively and assign the labels $-(\frac{mn-m}{2}+6), -(\frac{mn-m}{2}+8)$ respectively to the vertices $a_{n\frac{m+13}{2}}, a_{n\frac{m+15}{2}}$. Proceeding like this until reach the vertex $a_{n(m-1)}$.In this process $\frac{mn-1}{2}$ is the label of the vertex $a_{n(m-1)}$. Now assign the label $\frac{mn-1}{2}$ to the vertex a_{nm}. Note that the vertex $a_{n(m-1)}$ receive the label $\frac{mn-1}{2}$.
The Table 4 given below establish that this vertex labeling f is a pair difference cordial of Grid graph for the values of $n \equiv 3 \pmod 4$.

□

Nature of m	$\Delta_{f_1^c}$	Δ_{f_1}
$m \equiv 0 \pmod 4$	$\frac{2mn-m-n-1}{2}$	$\frac{2mn-m-n+1}{2}$
$m \equiv 1 \pmod 4$	$\frac{2mn-m-n}{2}$	$\frac{2mn-m-n}{2}$
$m \equiv 2 \pmod 4$	$\frac{2mn-m-n+1}{2}$	$\frac{2mn-m-n-1}{2}$
$m \equiv 3 \pmod 4$	$\frac{2mn-m-n}{2}$	$\frac{2mn-m-n}{2}$

TABLE 4

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