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RESEARCH PAPER

Damage Detection in Truss Bridges under Moving Load Using Time History Response and Members Influence Line Curves

Kordi, A.[1] and Mahmoudi, M.[2*]

[1] Ph.D. Student, Department of Civil Engineering, Shahid Rajaee Teacher Training University, Tehran, Iran.
[2] Associate Professor, Department of Civil Engineering, Shahid Rajaee Teacher Training University, Tehran, Iran.

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ABSTRACT: In recent years, damage detection has been an important issue in the condition assessment of structures. This research presents a new method for the detection of damaged members in truss bridges under moving load using the time history response and influence line curves of the members. For this reason, two different Finite Element (FE) models of truss bridges under moving load with different damage scenarios have been investigated. The damaged members are detected by adapting the difference curve shape of displacement responses obtained from the intact and damaged models to the axial force influence line curve shape of these members. The results demonstrate that when a member of a truss bridge is damaged, the difference curve of displacement responses is similar in shape to the influence line curve of the damaged member. It should be noted that the proposed method can accurately diagnose the damaged members with the displacement response of only one desired point of the truss bridge.

Keywords: Damage Detection, Displacement, Influence Line Curve, Moving Loads, Truss Bridge.

1. Introduction

All structures such as bridges, buildings, and other kinds of onshore or offshore structures undergoing loads, continuously accumulate damages which are often caused by deterioration of members or connections during their service life. The detection of these damages and their effects on Civil Engineering structures such as bridges have been a problem that has received remarkable considerations from researchers in the last three decades (Doebling et al., 1996; Mahmoud, 2001; Sinou, 2009; Rezaifar and Doost Mohammadi, 2016; Samadi, 2021). The great use of trusses in bridge structures led some researchers to investigate damage detection in trusses and truss bridges and the effect of damage on their behavior. Brunell and Kim (2013) used an experimental program to investigate the impact of local damage on the performance of steel truss bridges. Li and Hao (2016) discussed the possibility of using the newly introduced sensors to monitor the health condition of joints in steel truss bridges. De Biagi (2016) investigated the behavior of a

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[*] Corresponding author E-mail: m.mahmoudi@sru.ac.ir

metallic truss under progressive damage and defined a strategy for designing a truss that can withstand damage acting at random on one of its elements.

Kim et al. (2016) presented a numerical algorithm for identifying the locations and extent of multiple damages in the truss structures using the free vibration analysis based on the force method. Moradipour et al. (2017) examined the application of an improved two-stage modal strain energy method to a benchmark truss bridge. Unno et al. (2019) utilized vibration signals obtained from sensors installed on the bridges and the Auto-Regressive model were then applied to the time signals to extract the structure’s soundness characteristics. Mousavi et al. (2020) presented a damage detection method for a scaled steel-truss bridge model by using Hilbert–Huang Transform based on complete ensemble empirical mode decomposition with adaptive noise and Artificial Neural Network. Perez-Ramirez et al. (2020) proposed a new methodology based on the multiple signal classification for locating different types of damages in a truss-type structure with five bays under forced dynamic excitations.

As bridge structures continuously carry moving loads, several researchers focused on the dynamic responses of bridges under moving loads. Mrechesiello et al. (1999) presented an analytical approach to the problem of vehicle bridge dynamic interaction. Law and Zhu (2004) studied the dynamic behavior of damaged reinforced concrete bridge structures subject to moving vehicular loads and investigated the effects of moving speed, road surface roughness, and oscillator parameters on the dynamic behavior. Chan and Ashebo (2006) developed a method to identify moving forces on a selected span of interest from a continuous bridge. Yu and Chan (2007) investigated the current papers on important factors in the performance of moving load identification methods. Pakrashi et al. (2010) monitored the evolution of a crack in a beam by using the beam strain response due to the moving load for investigation of increasing crack-depth ratios and estimated the damage levels using distortions of wavelet coefficients of measured strain data.

Li and Law (2012) proposed a damage identification approach based on a dynamic response reconstruction technique for a target substructure subject to moving vehicular loads. Li et al. (2013) presented a structural damage identification approach based on the dynamic response reconstruction under vehicular loads for bridge structures without knowledge of the time histories of the vehicular loads and the properties of moving vehicles and with improvements in the computation efficiency and accuracy. McGetrick et al. (2015) validated an algorithm for the estimation of global bridge stiffness by using the vehicle response during a laboratory experimental process.

Sun et al. (2016) proposed a method for detecting the damage location and extent utilizing dynamic displacement of a bridge structure and defined dynamic curvature, the second derivative of the dynamic displacement, as a damaged index. Siriwardane (2015) proposed a technique to locate the damage region of railway bridges based on measured model parameters using acceleration response under usual moving train loads. Chang and Kim (2016) conducted a field experiment on an actual simply supported steel Warren-truss bridge with four artificial damage scenarios using acceleration responses under moving load. He et al. (2017) developed a two-stage method to detect the structural damages by using the displacement response of beam bridges under quasi-static moving loads. Liu and Zhang (2018) proposed a method to detect the damage in beam bridges by using a quasi-static strain influence line based on the Brillouin optical time-domain analysis in both numerical and experimental analysis.

Hester et al. (2020) proposed a damage identification method in beam bridges based on bridge rotation response to a moving load and showed a relation between the damage occurrence and an increase in the magnitude of rotation measurements. Yang et al. (2021) proposed a new damage detection index based on the Difference of Strain Influence Line (DSIL) gained from the long-gauge strains under moving load.

The purpose of this research is to identify the damaged members of truss bridges using influence line curves of members and the time history response of truss bridges under moving load. It should be noted that the displacement response of only one desired point of the truss bridge is needed for the process of damage detection.

2. Methodology

2.1. Preliminary Remarks

As is clear from previous studies in damage detection of bridges, the direct use of time history responses under moving load has been less observed, and often displacement or acceleration responses have been used to determine the modal characteristic changes. In other words, rarely has the damage index been defined based on the displacement or acceleration changes but in this study, FE models displacement response and influence lines are used directly and damage identification of truss bridge members is done by combining influence line and displacement response under moving load. By using accelerations from a planar Finite Element simulation model Gonzalez and Hester (2013) illustrated the relationship between the three response components contributes to establishing if damage has occurred. Zhang et al. (2017) used the distance between multi-type vibration measurements, displacement, velocity, and acceleration of undamaged and damaged structures to indicate the damage location. In this research, as the time history responses of the bridge-vehicle system are used for the process of damage detection, the system's formulation must be provided. The equation of motion for an Euler-Bernoulli beam under a load with a prescribed velocity *v(t)*, along the beam's axial direction, as depicted in Figure 1, can be written as:

$$\rho A \frac{\partial^2 u(x.t)}{\partial t^2} + C \frac{\partial u(x.t)}{\partial t} + \frac{\partial^2}{\partial \mathrm{x}^2}\left(EI(x)\frac{\partial^2 u(x.t)}{\partial x^2}\right) = P(t)\delta\big(x - \hat{x}(t)\big) \quad (1)$$

which ρA and C: are the mass per unit length and the damping of the beam. *EI(x)*, *u(x,t)* and $\hat{x}(t)$: are respectively the flexural stiffness, displacement function, and location of moving load *P(t)* at time t and *δ(t)*: is the Dirac delta function. The transverse displacement *u(x,t)* in modal coordinates is as follows:

$$u(x,t) = \sum_{i=1}^{\infty} \emptyset_i(x) q_i(t) \quad (2)$$

which $\emptyset_i(x)$ and $q_i(t)$: are the mode shape function and modal amplitude of *i*th mode. By substituting Eq. (2) into Eq. (1), multiplying by $\emptyset_i(x)$, integrating with respect to *x* between 0 and L and applying the orthogonality conditions, the following equation is obtained:

$$\frac{\mathrm{d}^2 q_i(t)}{\mathrm{d}t^2} + 2\xi_i\omega_i \frac{\mathrm{d}q(t)}{\mathrm{d}t} + \omega_i^2 q_i(t) = \frac{1}{M_i} P(t)\emptyset_i\big(\hat{x}(t)\big) \quad (3)$$

in which ω_i: is the modal frequency, $\xi_{i:}$ is the damping ratio and, M_i: is the modal mass of the *i*th mode obtained as:

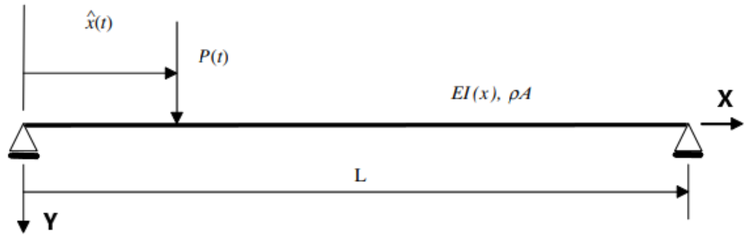


Fig. 1. A simply supported Euler–Bernoulli beam under a moving load

$$M_i = \int_0^L \rho A \phi_i^2(x)\, dx \quad (4)$$

For a continuous beam with simple supports as shown in Figure 1 with the length L, height h, and width b, the displacement response can be achieved by using the superposition principle in modal analysis and expressed as follows (Zhang et al. 2017):

$$u(x,t) = \sum_{i=1}^{\infty} \frac{\emptyset_i(x)}{M_i} \int_0^t h_i(t - \tau) P(\tau) \emptyset_i\big(\hat{x}(\tau)\big) d\tau \quad (5)$$

where $u(x,t)$: denotes the displacement of the beam at the location x and time instant t and $h_i(t) = \frac{1}{\omega_i'} e^{-\xi_i \omega_i t} sin\omega_i' t$ with $\omega_i' = \omega_i \sqrt{1 - \xi_i^2}$. As the vibration response of a bridge under a moving load is usually dominated by the fundamental frequency, the displacement response can be determined approximately with the first mode shape by using Eq. (5). If a moving constant load is considered and the damping effect is ignored, the displacement response at the mid-span of the beam can be calculated as follows (Zhang et al. 2017):

$$u\left(\frac{L}{2}, t\right) \approx \frac{\emptyset_1\left(\frac{L}{2}\right)}{M_1} \cdot \frac{P}{\omega_1} \int_0^t sin\big(\omega_1(t - \tau)\big) \emptyset_1\big(\hat{x}(\tau)\big) d\tau \quad (6)$$

Clearly, the damage location x_c is the non-smooth point of the first mode shape (Narkis 1994), which is called a singularity point in mathematics. Considering Eq. (6), the singularity point occurs in the displacement response.

2.2. Moving Load Modeling

In this study, the Finite Element model of bridges in ABAQUS 6.14-3 Finite Element software (Systèmes, 2014) is used to obtain the displacement responses used for damage detection. As in the next parts of this study, in the process of damage detection, the moving load will be applied to the truss bridges. For verification of the moving load numerical modeling a valid example from a mentioned study (Zhang et al. 2017) is investigated. In this example a $\mathrm{P} = 10\mathrm{kN}$ load is moving along a beam with a speed of $2\ \mathrm{m/s}$. The geometric and material properties of the beam are listed in Table 1.

The bridge Finite Element model is built with 20 beam elements, named B22, based on the Euler-Bernoulli beam theory with 1.5 m length. In order to model the moving load in ABAQUS, a 10kN load is applied to a weightless moving cube with dimensions 0.5 × 0.5 × 0.5 m. According to Figure 3, plane strain shell element with 0.25 m width and height and depth equal to the beam is used to mesh the moving cube. There is no friction between lower area of the moving cube and the beam. The bridge-moving load system has been analyzed in ABAQUS standard solver with an implicit dynamic step with a time period of 20 seconds. The dynamic implicit step, in the process of solving matrices obtained from the Finite Element method, uses the Hilber-Hughes-Taylor Integration operator (Hilber and Hughes 1978).

The comparison of finite element analysis and theoretical calculation of the beam for validation of the moving load modeling is shown in Figure 4.

Fig. 2. A simply supported damaged beam under a moving load

Table 1. Properties of geometry and material of the beam structure Zhang et al. (2017)

Material parameters		Geometric parameters	
Density	7800 kg/m^3	Length	30 m
Young's modulus	210 Gpa	Height	1 m
Poisson's ratio	0.3	Width	0.5 m

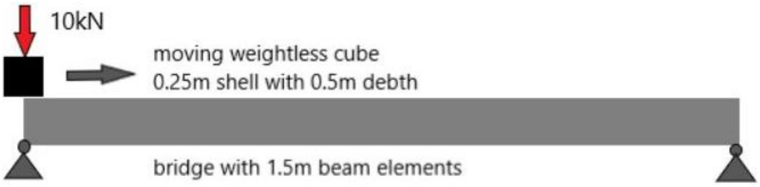


Fig. 3. Moving load modeling in the finite element software

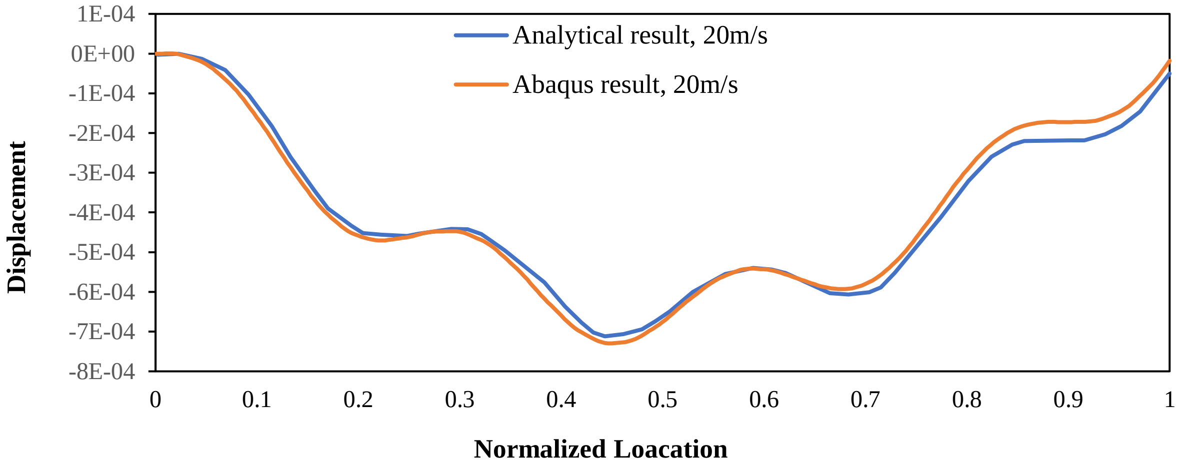


Fig. 4. Comparison of the numerical and analytical displacement response at the midpoint of the beam

The good agreement demonstrates that the finite element analysis response is finely matched with only the first mode response and this method of moving load modeling is completely usable for the process of truss bridge damage detection.

2.3. The Proposed Method for Damage Detection of Truss Bridges

It should be mentioned first that in this research, the criteria of being known as a damaged member is member's stiffness reduction compared to the initial stiffness. The mentioned damage is applied to the bridge's members by cross-section and Young's modulus reduction in truss and beam elements, respectively. This study aims to detect the damaged members of truss bridges by adapting the displacement responses difference curve obtained from the intact and damaged models and the axial force influence line curve of these members. To reach this aim, the displacement response of the truss bridge's lower chord, at the midpoint, under moving load is calculated for intact and damaged bridges. The Difference in Displacement responses (DD) is expressed as follows:

$$DD = u_i - u_d \qquad (7)$$

where u_i and u_d: stand for the displacement response at the midpoint of the intact and damaged truss bridges, respectively. After obtaining the DD curve for each truss bridge, axil Influence Lines (IL) of truss members are calculated to be compared with the truss bridge DD for identifying the damaged member. It is necessary to mention that only for members on the lower chord, members modeled with beam elements, bending moment influence line is used. The steps of the proposed method for damage detection are as follows:

- Finite Element modeling of intact and damaged truss bridge;
- Obtaining the displacement responses of one point in the truss bridge;

- Denoising the displacement responses by Wavelet toolbox in Matlab program;
- Computing the DD for the truss bridge;
- Calculating the ILs of all or damaged members;
- Comparing the DD curve with ILs curves to identify the damaged member.

In this study, two examples of different truss bridges are modeled numerically in the finite element software to evaluate the proposed method's capability. In addition to these two truss bridge models, Example 3 is provided to demonstrate the ability of this method to detect damage in beam bridges.

2.4. Examples

Example 1: A Pratt Type Truss Bridge with Simple Supports

In this example, a ten-span planner Pratt-type truss, as depicted in Figure 4, is considered. The elements of the lower chord are modeled as beam elements with fixed rectangular cross-sections $\mathrm{B} = 8\ \mathrm{cm}$ and $\mathrm{H} = 5\ \mathrm{cm}$ and structural mass $\mathrm{m} = 10\ \mathrm{kg}$ is attached at each node on the lower chord. The other members have modeled as truss elements sectional area $\mathrm{A} = 1\ \mathrm{cm}^2$. For all elements, Young's modulus is $\mathrm{E} = 2.1 \times 10^{11}$ Pa and material density is $\rho = 7800\ \mathrm{kg/m}^3$ (Lingyun et al., 2005). The truss configuration is depicted in Figure 5.

As in the next part of this study, the dynamic response of the truss bridge is investigated, the first five natural frequencies of the truss are extracted by applying the frequency analysis. The comparison of reference (Lingyun et al., 2005) measures and those of the present study is shown in Table 2.

In order to evaluate the proposed method for identification of damaged member, in each analysis, one member of the truss is damaged and the displacement response of lower chord midpoint, point 10, under sinusoidal moving load is extracted for intact and damaged models. The sinusoidal load moving the lower chord is as follows:

$$P = 2000(1 + 0.1\,sin(10\pi t) + 0.05\,sin(30\pi t)) \qquad (8)$$

In Civil Engineering, vehicle loads are often simulated with the time history of moving load. Since the applied moving load has two high-frequency parts, the displacement response of truss bridges is influenced by excitation frequencies. To eliminate the influences of high frequencies from displacement responses, a filter in the wavelet package of Matlab software is used to convert the displacement responses to some smoother ones (Misti et al., 2007). The displacement response of node 10 in the intact truss bridge under sinusoidal moving load and the denoised form of this curve is illustrated in Figure 6.

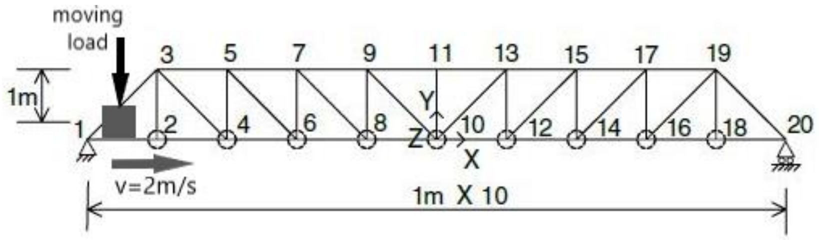


Fig. 5. Pratt simply supported bridge under moving load

Table 2. Natural frequency comparison

Frequency number	Lingyun et al.	Present
1	8.89	8.884
2	28.82	28.699
3	46.92	46.473
4	63.62	62.597
5	76.87	75.056

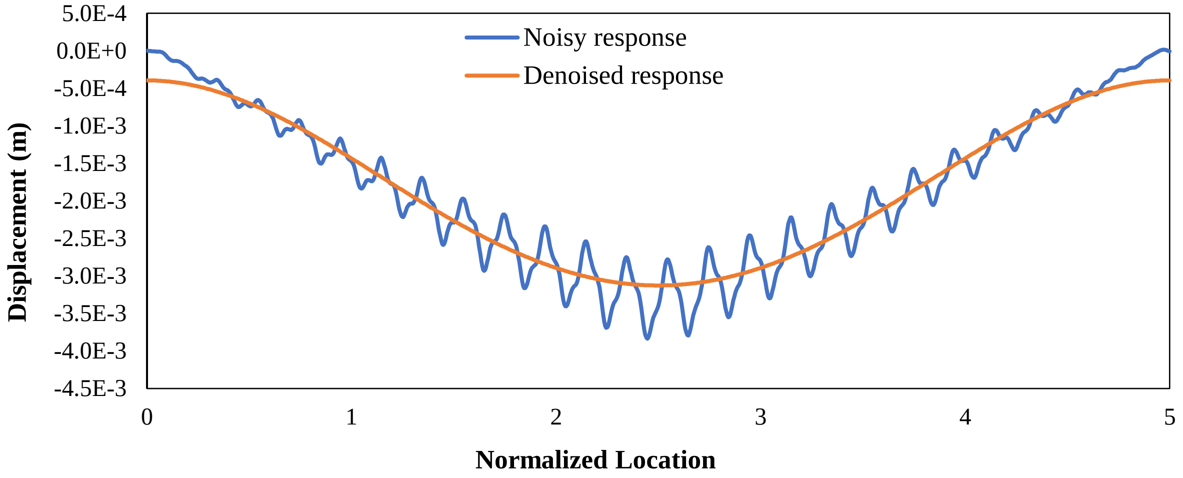


Fig. 6. Pratt truss node 10 displacement response under moving load

In this bridge, for members with truss elements, the damage is modeled by decreasing the sectional area in the whole length of the member and for members with beam elements, by decreasing the elasticity modulus at the center of a member with 0.1 m length. Four different scenarios including all kinds of members such as vertical, diagonal, up and down horizontal, in different parts of the truss are investigated. The damage scenarios and damaged member's features are illustrated in Table 3. Except for M4-6 which is the member between nodes 4 and 6 with beam elements, the other three members are modeled with truss elements.

The IL curves of damaged and some other members are obtained by applying a unit force to the points on the lower chords, 1 up to 20, and calculating each member's axial load or bending moment amounts for truss or beam elements respectively.

Example 2: A Warren Type Truss Bridge with Simple Supports

In the second example, a four-span planner Warren truss with material properties similar to example number one is investigated. The elements of the lower chord are modeled as beam elements with fixed rectangular cross-sections $B = 2$ cm and $H = 4$ cm. The other members have modeled as bar elements sectional area $A = 4\ \text{cm}^2$. The bridge truss is depicted in Figure. 7.

The above truss bridge undergoes a sinusoidal moving load (Eq. (5)), and the displacement response of the lower chord midpoint, point 4, is extracted for intact and damaged models. The sinusoidal load moving the lower chord is as follows:

$$P = 100(1 + 0.1\,sin(20\pi t) + 0.05\,sin(40\pi t)) \tag{9}$$

As is shown in Table 3, three different scenarios are investigated and for each scenario, only one member is damaged. In this table, the selected members' features before and after damage are shown. For example, the M2-5 stands for the member with truss element between nodes 2 and 5, and the M4-6 for the member with beam element between nodes 4 and 6.

Table 3. Damage features

Damage scenarios	**Intact members features**		**Damaged members features**	
	cm^2	**Mpa**	**cm^2**	**Mpa**
M6-7	A = 1	E = 2.1E5	A = 0.5	E = 2.1E5
M11-13	A = 1	E = 2.1E5	A = 0.5	E = 2.1E5
M14-17	A = 1	E = 2.1E5	A = 0.5	E = 2.1E5
M4-6	A = 40	E = 2.1E5	A = 40	E = 1.89E5

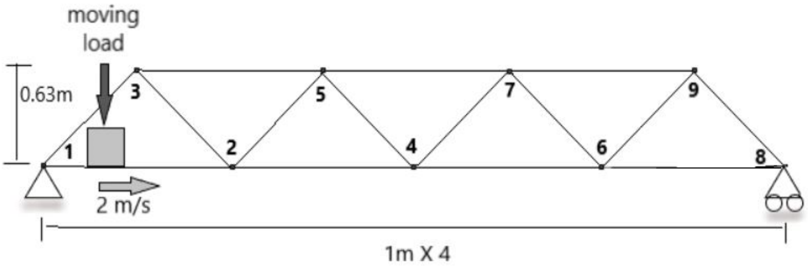


Fig. 7. Warren simply supported bridge under a moving load

Table 4. Damage features

Damage scenarios	Intact members features		Damaged members features	
	cm^2	Mpa	cm^2	Mpa
M2-5	A = 4	E = 2.1E5	A = 2	E = 2.1E5
M5-7	A = 4	E = 2.1E5	A = 2	E = 2.1E5
M4-6	A = 8	E = 2.1E5	A = 8	E = 1.89E5

The good agreement of frequency measures, especially the first mode frequency, illustrates that the truss modeling is correct and its dynamic behavior is valid for the continuation of study and evaluation of the newly proposed method.

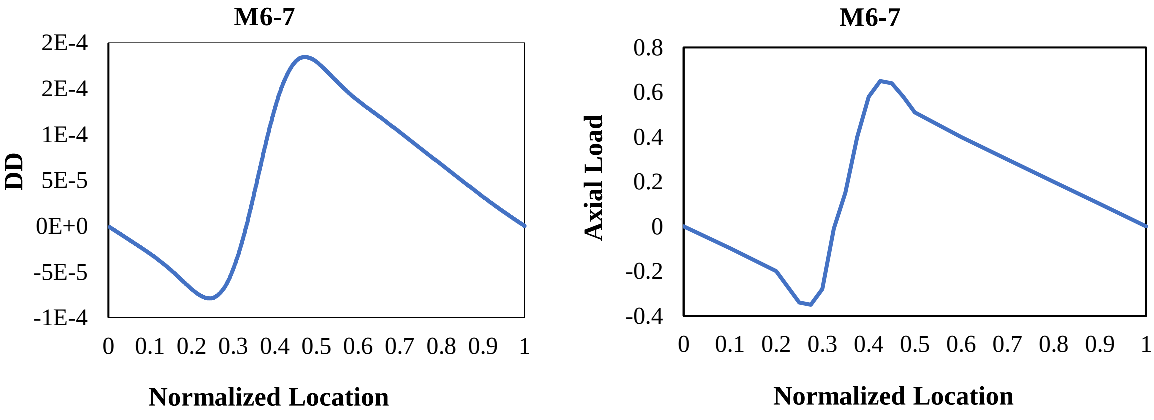
Example 3: A Simply Supported Beam Bridge

In this example, the beam bridge with the geometrical and material properties of Table 1 is modeled in the Finite Element software and the DD and IL curve pattern is presented. The damage is simulated at the location 2L/3 from the left support of the beam bridge. Damage with 0.1 m length by changing Young's modulus, E, to E/8 is created. The filtered displacement responses of intact and damaged beams and DD and IL curves are illustrated in Figures 10 and 11, respectively.

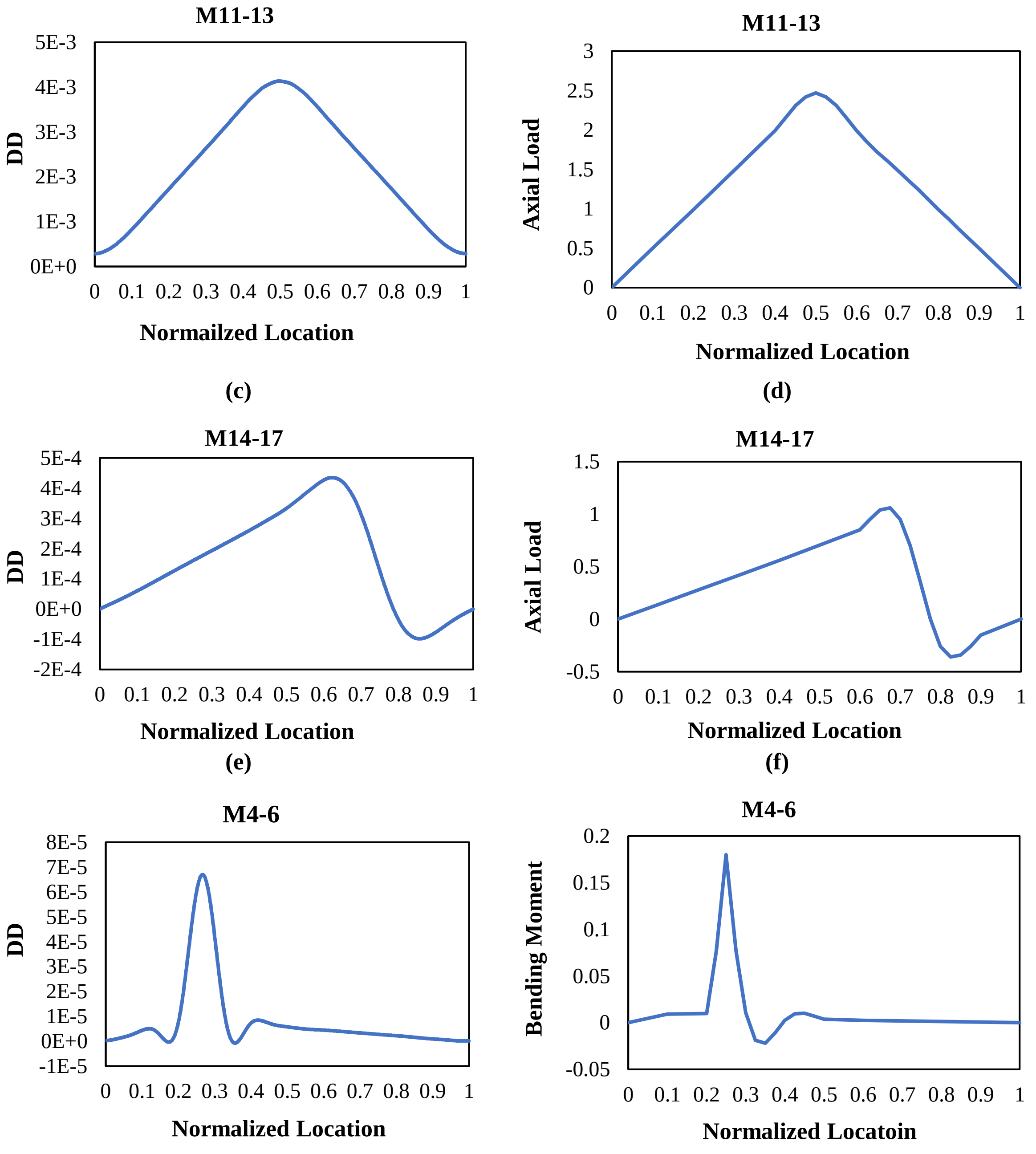
3. Verification of The Proposed Method

As expressed in the previous parts of this study, in the Pratt truss bridge, Example 1, four different scenarios are investigated by damaging vertical, diagonal, upper chord horizontal, and lower chord horizontal member respectively, one by one. In order to evaluate the ability of the new method for the identification of damaged members, the DD curve pattern of the truss bridge for each scenario should be compared with IL curve patterns of damaged members. The comparison of displacement response difference curves and influence line curves of Pratt truss bridge under moving load for each scenario is illustrated in Figure 8.

The left column curves illustrate the displacement difference of the truss bridge lower chord midpoint for each scenario, and the right columns curves show the influence line of damaged members. The results of Example 2 are illustrated in Figure 9.



(a) (b)

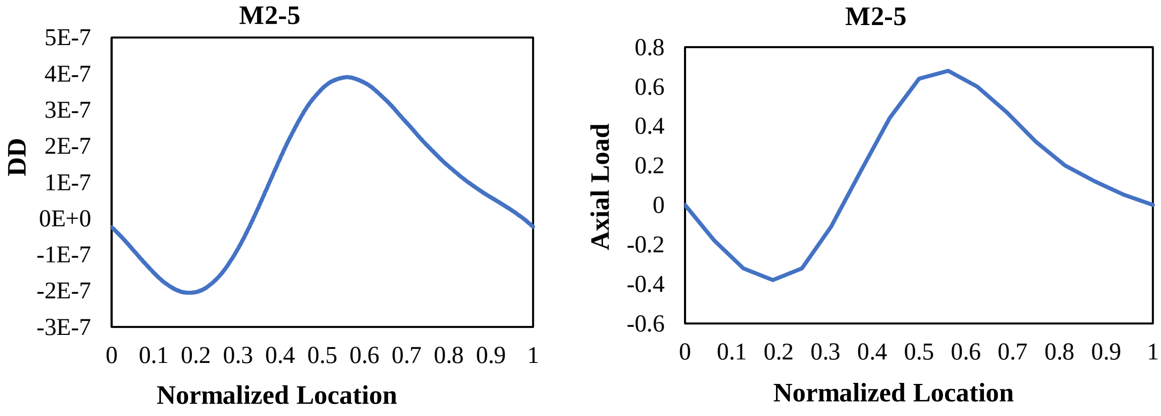


(c) **(d)**

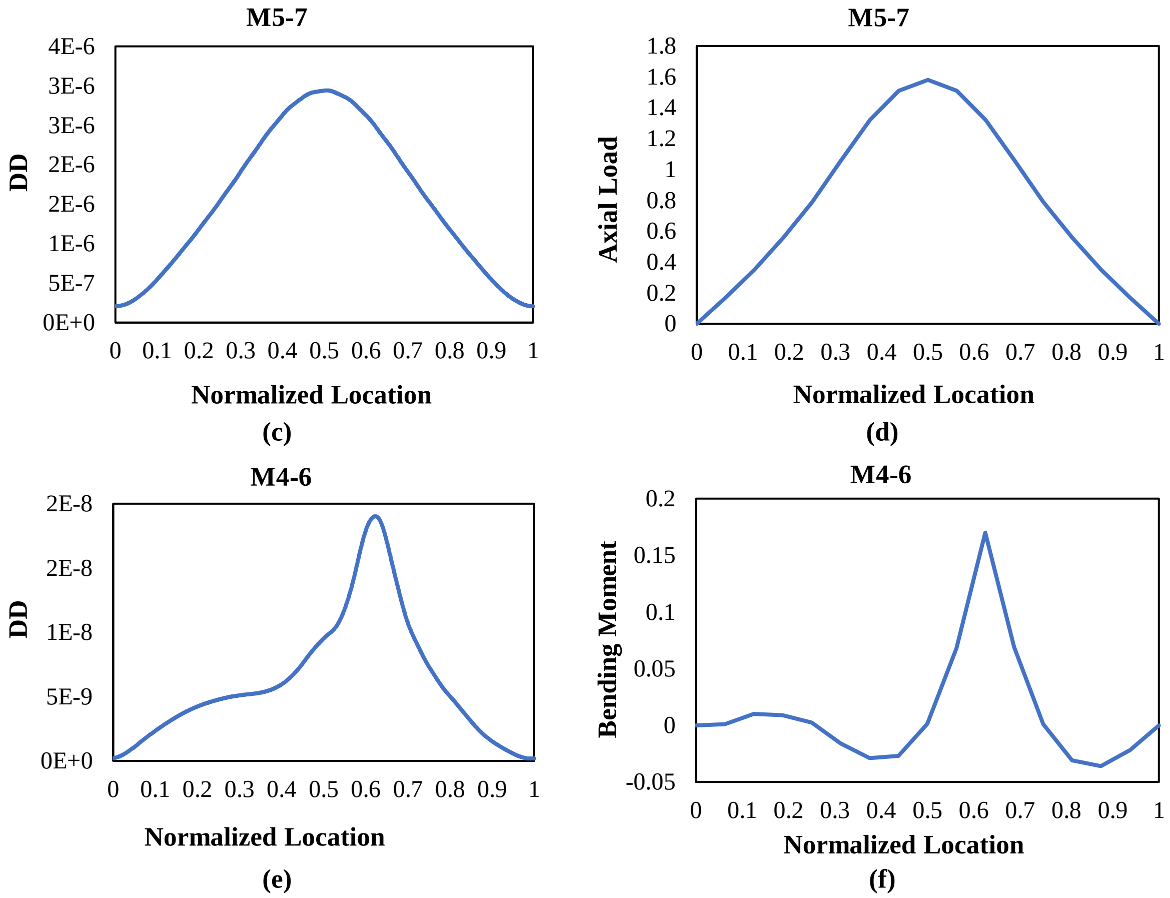
(e) **(f)**

(g) **(h)**

Fig. 8. Adaptation of DDs and ILs: a,b) Vertical; c,d) Up horizontal; e,f) Diagonal; and g,h) Down horizontal



(a) **(b)**



(c) **(d)**

(e) **(f)**

Fig. 9. Adaptation of DDs and ILs: a,b) Diagonal; c,d) Up horizontal; and e,f) Down horizontal

As shown in above figures, in the diagonal and vertical members of truss bridges, the curves experience a jump along an almost constant slope. Although in all horizontal members of truss bridges a peak is seen, in the lower chord members simulated with beam elements, a sharper peak is seen because in these flexural members unlike the upper chord members, simulated with truss elements, bending moment influence line is investigated, not axil load influence line.

The Figure 11a's curve illustrates the displacement difference of the beam bridge at the midpoint of the beam, and Figure 11b's curve shows the bending moment influence line of the beam.

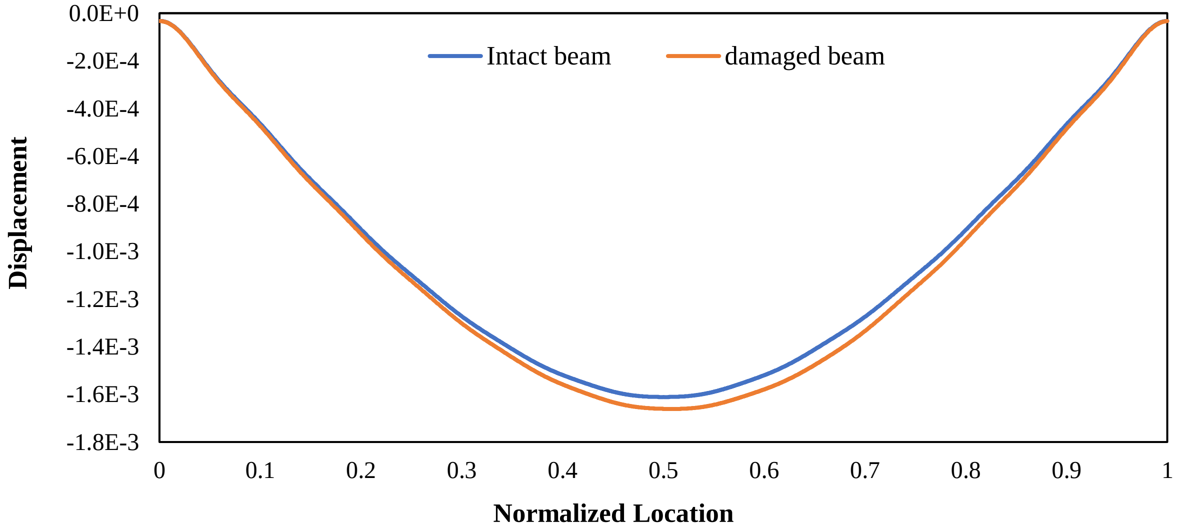
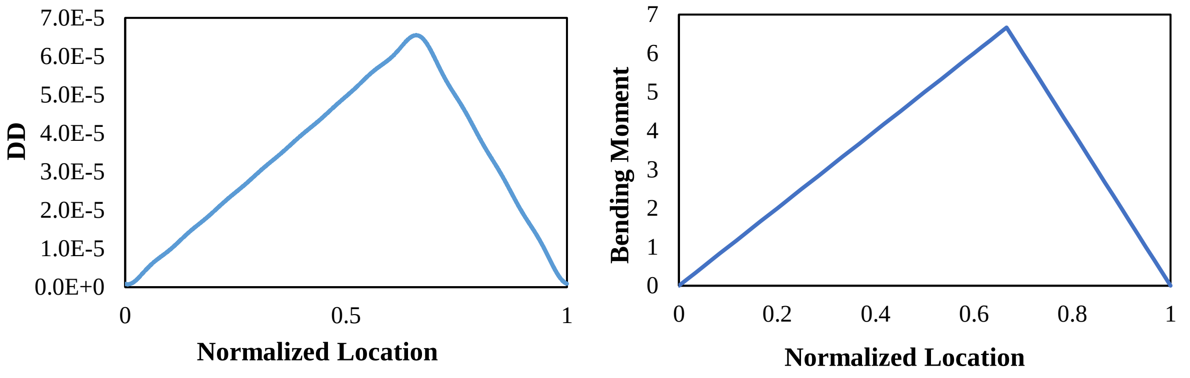


Fig. 10. Filtered displacement response of intact and damaged beam bridges



(a) (b)

Fig. 11. Adaptation of DD and IL of the 30-meter beam bridge

4. Conclusions

The results presented in this paper indicate that the new proposed method of combining truss bridges members influence line and displacement responses difference is able to simply identify the damaged members. The results demonstrate that when a member in the truss bridge is damaged, the difference curve of intact and damaged truss bridge displacement responses is apparently similar to the influence line curve of the damaged member. In other words, the shape similarity of displacement responses difference curve and the damaged member IL curve is very prominent. It is worth mentioning that the proposed method can accurately diagnose the damaged members with the displacement response of only one point of the truss bridge. Based on the many models investigated, the proposed method is able to identify the members with very minor damages. It is clear that, for intact members, when there is no damage, the DD measure is zero. So there would be no curve for similarity comparison with the IL curves. Obviously, the proposed method is also effective for beam bridges. The third example confirms this. So the peak points of curves in Figure 11, show the location of damage in the beam. Further studies could be required to detect the multiple damages and amount of each member’s damage.

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