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Multi-Period Efficiency Analysis with Flexible Measures: Oriented and Non-Oriented Approaches

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ABSTRACT

In traditional data envelopment analysis (DEA) models, the relative efficiency of decision making units (DMUs) is usually evaluated in a particular period of time such that the status of each measure from the input or output viewpoint is certain. However, in many applications, the performance of organizations should be measured over multiple periods of time while the status of some factors called "flexible measures" from the perspective of input or output is uncertain. The purpose of this study is to propose approaches to evaluate the efficiency of the multi-period systems where flexible measures are presented. For this reason, oriented and non-oriented DEA-based approaches from the standpoints of individual DMU and aggregate efficiencies are rendered to measure the overall and period efficiency of multi-period systems with flexible measures. Also, efficiency changes between two periods are estimated using the Malmquist productivity index (MPI). A dataset is provided to validate the proposed approach.

Subject classification codes: 90C08, 90C05.

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1. Introduction

Managers plan, make decisions, and operate in order to promote other participants, sometimes at the expenditures of shareholders, to develop the substitute frames of economic achievement (Tajeddini, 2015). To reach this purpose and make appropriate decisions, efficiency analysis and proper performance measures identification are major aspects. Data envelopment analysis (DEA), first proposed by Charnes et al. (1978), is a well-known mathematical technique to evaluate the relative efficiency of decision making units (DMUs) with multiple inputs and outputs. The relative efficiency is defined as the ratio of the weighted sum of the outputs to the weighted sum of the inputs. Traditional DEA models maximize this ratio for the unit under consideration while these proportions for other DMUs do not exceed one. The fractional linear problem with positive weights can be conveniently transformed into a linear problem. Also, in some problems, the dual form of this linear model is more appropriate and informative to use. In conventional DEA models, DMUs are usually evaluated in a single period of time while performance measures play the identified input/output roles. Nevertheless, some situations exist in real-world applications in which the performance of organizations should be measured in multiple periods of time, whereas the role of factors from the input/output viewpoint is uncertain. Indeed, some measures can play the role of input or output in practical applications. These factors are called "flexible measures" in the DEA literature. For instance, trainees in hospitals, deposits in banks, and research income in universities can be deemed as flexible measures (Cook & Zhu, 2007). As stated in Cook and Zhu (2007), the deposit in banks can be considered as an output because it is a source of revenue. On the other hand, staff time extended in processing customers who are making deposits could be applied to better advantage to sell more profitable products; thus, it can be deemed as an input. As another example, in assessing the efficiency of power plant, outages can be seen as a type of output on the part of management. On the other side, it is an environmental input that has a direct influence on plant performance (Cook & Seiford, 2009). Actually, in many real issues, factors are presented whose input/output status is flexible. Therefore, determining the proper status of flexible measures is substantial for managers and policy makers in order to appropriately analyze the performance of entities. Some studies (e.g., Amirteimoori & Emrouznejad, 2011; Cook & Zhu, 2007; Tohidi & Matroud, 2017) have investigated the performance of DMUs with flexible measures in a special period of time.

However, entities such as banks, universities, and service operation units have activities continued over a span of time; thus, the examination of their performance across multiple periods of time and the evaluation of the overall and period efficiency values are significant aspects. Approaches such as window analysis (Charnes et al., 1985), Malmquist-type indexes of productivity (Caves et al., 1982; Fare & Grosskopf, 1996), dynamic DEA (Nemoto & Goto, 1999) and multi-period DEA (Park & Park, 2009) have addressed the performance of DMUs in multiple periods of time. As far as we know, among time series DEA studies, only Alizadeh Afrouzi (2020) extended the Malmquist productivity index for cases that there are flexible measures. However, according to Kao and Liu (2014), an aggregated measure of efficiency for multi-period processes is not considered in this approach. Accordingly, addressing the overall and period efficiencies of systems taking several periods of time into account and also recognizing the changing patterns of performances in a time span, containing several periods, are significant cases for long-term planning, especially where there are flexible measures with the role of either inputs or outputs in processes under investigation.

Therefore, in this research, oriented and non-oriented multi-period DEA approaches are proposed to estimate the period and overall efficiency values of multi-period processes with flexible measures and to classify flexible measures. Furthermore, efficiency changes between two periods in multi-period systems are dealt with while there are flexible measures. As far as we know, there is no DEA examination to assess the overall and period efficiency of multi-period systems in the presence of flexible measures. Consequently, this research undertakes this issue rendering oriented and non-oriented multi-period DEA models.

The rest of this paper is organized as follows. The literature review is presented in Section 2. In Section 3, the basic concepts and related models are provided. Section 4 gives several DEA-based approaches, including oriented and non-oriented ones, to evaluate the relative efficiency of multi-period systems with flexible measures. Also, the efficiency changes between two periods are analyzed in this section. A data set is applied to illustrate the proposed approach in Section 5. Conclusions are discussed in Section 6.

2. Literature Review

In this section, a review of considerations related to the efficiency analysis in the presence of flexible measures and the performance estimation of multi-period production processes is provided.

2.1 DEA Studies Along With Flexible Measures

Cook and Zhu (2007) introduced mixed integer linear programming problems to classify a flexible measure into an input or an output. Toloo (2014) claimed Cook and Zhu's approach (Cook & Zhu, 2007) may obtain incorrect efficiency scores due to a computational problem, that is, introducing a large positive number. Amirteimoori and Emrouznejad (2011) suggested an alternative method to evaluate the efficiency of DMUs in the presence of flexible measures. Also, Toloo (2012) dealt with alternative solutions to determine the status of flexible measures. Kordrostami and Jahani Sayyad Noveiri (2012) proposed a DEA-based approach to estimate the efficiency of DMUs in the presence of flexible and negative measures. Tavana et al. (2021) provided a non-radial directional distance function model to address negative data and extended it to situations in which negative and flexible measures are presented. Amirteimoori et al. (2013) proposed a flexible slacks-based measure of efficiency to classify flexible measures. Later, Boďa (2020) reconsidered the flexible slacks-based measure model and planned an alternative model with different projections for inefficient units leading probably different classifications. Furthermore, Kordrostami and Jahani Sayyad Noveiri (2014) evaluated the performance and classified inputs and outputs when there are flexible and interval factors. The classification of performance measures has been investigated in some studies (e.g., Joulaei et al., 2019; Kordrostami et al., 2014) when fuzzy data appear. Tohidi and Matroud (2017) introduced non-oriented models to handle flexible measures and also to address the type of returns to scale (constant returns to scale (CRS), increasing returns to scale (IRS), and decreasing returns to scale (DRS). CRS occurs when an increase in input causes a proportional increase in output. IRS is when output increases by a larger proportion than input. DRS is while a proportional increase in all inputs leads to a less than proportional increase in outputs. It is noticeable that in non-oriented DEA models, DMUs can manage the changes of inputs and outputs, simultaneously. But oriented DEA models are divided into input-oriented and output-oriented ones. Through input-oriented DEA models, inputs minimize while at least the given output levels are satisfied. Also, outputs increase proportionally without the change of the input proportions in output-oriented DEA models. Toloo et al. (2018) presented a directional distance DEA approach in envelopment and multiplier forms to classify input-output measures. Kordrostami et al. (2019) classified integer and non-integer flexible measures in a special period of time using their proposed integer-valued DEA approaches. Sedighi Hassan Keyadeh et al. (2019) planned the flexible Russell measure model to estimate the performance of DMUs and classify flexible measures. Abolghasem et al. (2019) evaluated the cross-efficiency of DMUs with flexible measures. Furthermore, proportional dual-role measures have been examined in some studies (e.g., Jahani Sayyad Noveiri et al., 2019; Jahani Sayyad Noveiri et al., 2020). Alizadeh Afrouzi (2020) developed the Malmquist productivity index for occasions in which flexible factors exist. Kordrostami and Jahani Sayyad Noveiri (2021) addressed the relative efficiency of entities when there are bounded, discrete, and flexible measures. Ebrahimi and Hajizadeh (2021) developed a mixed binary linear DEA model to classify the flexible measures and determine the best entities. Also, they provided a method to find a suitable epsilon value for their model and investigated the issue of ranking efficient units. As Ebrahimi and Hajizadeh (2021) have mentioned, DEA approaches with flexible measures are applied in many real world investigations such as banks, university departments, and healthcare sectors.

2.2 DEA Studies to Address Multi-Period Processes

Park and Park (2009) estimated the aggregative efficiency of multiple-period systems with the given status of measures using a DEA-based approach. Esmaeilzadeh and Hadi-Vencheh (2013) introduced a super-efficiency model to assess aggregative efficiency of multi-period processes. Kao and Liu (2014) developed a technique based on the network DEA to measure the performance of multi-period production systems. Then, Kao and Hwang (2014) extended the Kao and Liu's approach (Kao & Liu, 2014) to analyze the multi-period efficiency in two-stage production systems. Multi-period efficiency means the efficiency of DMUs based on the time periods. Liu (2017) evaluated East Asia airport companies using multi-period network DEA. Razavi Hajiagha et al. (2015) proposed an approach founded on Chebyshev inequality bounds to evaluate the relative efficiency of multi-period systems. Liu (2016) developed a fuzzy DEA approach on the basis of the network DEA model to measure the overall and period efficiencies of DMUs with fuzzy data while taking weight restrictions into account. Kordrostami and Jahani Sayyad Noveiri (2017) also introduced an alternative fuzzy DEA approach to estimate the overall and period efficiency of multi-period processes with fuzzy and imprecise data. Later, the performance of multi-period systems in the presence of undesirable outputs with different points of disposability was addressed by Kordrostami et al. (2018). Jablonsky (2016) analyzed the efficiency in several periods of time and provided oriented and non-oriented DEA models for this aim. Kordrostami and Jahani Sayyad Noveiri (2017) suggested a multi-period DEA model to evaluate the multi-period efficiency of entities where negative measures are presented. Bansal and Mehra (2018) provided multi-period additive efficiency and super-efficiency DEA models to measure and discriminate the efficiency values of DMUs with undesirable and non-positive measures. Tavana et al. (2019) designed a fuzzy multi-objective multi-period network approach to assess the dynamic performance of Iranian oil refineries with undesirable outputs. Jahani Sayyad Noveiri and Kordostami (2019) analyzed the sustainability performance of multiple periods when discrete and bounded measures are presented. Esfidani et al. (2020) introduced a slacks-based measure model to deal with the performance of multi-period two-stage processes. Amirteimoori et al. (2020) assessed the sustainability of systems with undesirable outputs in multiple periods of time. Wang et al. (2022) introduced a multi-period two-stage DEA approach with feedback structures. As can be found, the majority of studies in the DEA context have examined multi-period systems and flexible measures, separately.

However, there are situations in which the performance of production systems with flexible measures must be evaluated in a time span covering several periods. To the authors' best knowledge, no research has been done for estimating the period and overall efficiency of multi-period systems in the presence of flexible measures. Therefore, the current paper proposes a DEA-based approach to assess the overall and period efficiency scores of multi-period systems where flexible measures are presented. Indeed, Jablonsky's (2016) approach is extended to measure the performance of multi-period systems in the presence of flexible measures. Mixed integer linear problems are provided to specify the overall and period efficiency values of multi-period systems with flexible measures. Furthermore, flexible measures are classified as inputs, outputs, or in equilibrium in the system under consideration. In summary, the contributions of this research are as follows:

- Introducing DEA-based approaches, oriented and non-oriented ones, from the perspectives of individual DMU and aggregate efficiencies to handle multi-period systems with flexible measures,
- Estimating the overall and period efficiency of multi-period frameworks with flexible factors, simultaneously,
- Classifying inputs and outputs in multi-phase organizations,
- Estimating efficiency changes between two periods in multi-period plans with flexible measures, and
- Presenting a set of data, including three periods, to explain more the approaches proposed in this study.

3. Preliminaries

In this section, some preliminaries derived from previous studies (i.e., Amirteimoori & Emrouznejad, 2011; Cook & Zhu, 2007; Jablonsky, 2016; Tohidi & Matroud, 2017) are provided that include the explanation of flexible measures, the existing models to evaluate the relative efficiency of DMUs with flexible measures, containing oriented and non-oriented ones, and the model to estimate the performance of multi-period systems. At this point, the next notations are defined that are applied hereafter in expressions:

$DMU_j \ (j = 1, ..., n)$: $j\,th$ DMU,

$x_{ij} (i = 1, ..., m)$: $i\,th$ input of DMU_j,

$y_{rj} (r = 1, ..., s)$: $r\,th$ output of DMU_j,

$w_{kj} (k = 1, ..., K)$: $k\,th$ flexible measure of DMU_j,

$\lambda_j\,(j=1,\ldots,n)$: Intensity variables,
$d_k\,(k=1,\ldots,K)$: Binary variables,
$PS_j\,(j=1,\ldots,n)$: $j\,th$ production systems,
$T\;(t=1,\ldots,T)$: Periods of time,
$x^t_{ij}(i=1,\ldots,m;\,j=1,\ldots,n;t=1,\ldots,T)$: $i\,th$ input of PS_j in period t ,
$y^t_{rj}(r=1,\ldots,s;\,j=1,\ldots,n;t=1,\ldots,T)$: $r\,th$ output of PS_j in period t ,
$w^t_{kj}(k=1,\ldots,K;\,j=1,\ldots,n;t=1,\ldots,T)$: $k\,th$ flexible measure of PS_j in period t ,
$x^t_{io}(i=1,\ldots,m;t=1,\ldots,T)$: $i\,th$ input of the system under evaluation o , PS_o , in period t ,
$y^t_{ro}(r=1,\ldots,s;t=1,\ldots,T)$: $r\,th$ output of the system under evaluation o , PS_o , in period t ,
$w^t_{ko}(k=1,\ldots,K;t=1,\ldots,T)$: $k\,th$ flexible measure of the system under evaluation o , PS_o , in period t ,
λ^t_j : Intensity variables for $j=1,\ldots,n;t=1,\ldots T$,
M : A sufficiently large positive number,
$d^t_k(k=1,\ldots,K;t=1,\ldots T)$: $k\,th$ binary variable related to period t .

3.1 Flexible Measures

According to Cook and Zhu (2007), "measures that can play either input or output roles are called flexible measures". For instance, research income can play the input role because it is used by the university. On the other hand, it is considered as the output because it is earned by the university. In this subsection, first the approach presented by Amirteimoori and Emrouznejad (2011), which is an input-oriented radial DEA model to deal with flexible measures, is described. Then, the non-oriented DEA method introduced by Tohidi and Matroud (2017) is briefly reviewed.

In order to evaluate the relative efficiency of DMUs in the presence of flexible measures, Amirteimoori and Emrouznejad (2011) defined production possibility set (PPS) as follows:

$$T=\{(x,y,w):x\geq\sum_{j=1}^{n}\lambda_j x_j,\,y\leq\sum_{j=1}^{n}\lambda_j y_j,(either\;w\geq\sum_{j=1}^{n}\lambda_j w_j\;or\;\;w\leq\sum_{j=1}^{n}\lambda_j w_j),\lambda_j\geq 0,\,j=1,\ldots,n\}$$

According to the defined PPS, they also proposed the following input-oriented radial DEA model incorporating flexible measures:

$$\begin{aligned}
&Min\;\;\theta\\
&s.t.\;\;\sum_{j=1}^{n}\lambda_j x_{ij}\leq\theta x_{io},\,i=1,2,\ldots,m,\\
&\qquad\sum_{j=1}^{n}\lambda_j y_{rj}\geq y_{ro},\,r=1,2,\ldots,s,\\
&\qquad\sum_{j=1}^{n}\lambda_j w_{kj}\leq\theta w_{ko}+Md_k,\,k=1,2,\ldots,K,\\
&\qquad\sum_{j=1}^{n}\lambda_j w_{kj}\geq w_{ko}-M(1-d_k),\,k=1,2,\ldots,K,\\
&\quad d_k\in\{0,1\},\,k=1,\ldots,K,\,\lambda_j\geq 0,\;j=1,\ldots,n.
\end{aligned}\tag{1}$$

If the optimal value d^*_k equals to one $(d^*_k=1)$, then the flexible measure k is an output and if $d^*_k=0$, it is an input.

Also, Tohidi and Matroud (2017) proposed the following non-oriented model under the assumption of variable returns to scale to classify flexible measures, estimate efficiency scores, and identify the returns to scale state:

$$
\begin{aligned}
& Min \quad \frac{\alpha}{\beta} \\
& s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \alpha x_{io}, i = 1,2,...,m, \\
& \qquad \sum_{j=1}^{n} \lambda_j y_{rj} \ge \beta y_{ro}, r = 1,2,...,s, \\
& \qquad \sum_{j=1}^{n} \lambda_j w_{kj} \le \alpha w_{ko} + M d_k, k = 1,2,...,K, \\
& \qquad \sum_{j=1}^{n} \lambda_j w_{kj} \ge \beta w_{ko} - M(1-d_k), k = 1,2,...,K, \\
& \qquad \sum_{j=1}^{n} \lambda_j = 1, \\
& \qquad d_k \in \{0,1\}, k = 1,...,K, \lambda_j \ge 0, j = 1,...,n, \alpha, \beta \ge 0.
\end{aligned}
\tag{2}
$$

in which α and β are scalars. In this model, $k\,th$ flexible measure is treated as an input when the optimal value $d_k^* = 0$ and it is deemed as an output where $d_k^* = 1$.

3.2 Efficiency of Multi-Period Systems

Now, Jablonsky's approach (Jablonsky, 2016) is reviewed, which was suggested to evaluate the relative efficiency of DMUs in a time span covering several periods. Jablonsky (2016) provided the following model for estimating the performance of the multi-period system o:

$$
\begin{aligned}
& e_o^* = Min \quad e_o = \sum_{t=1}^{T} \theta_o^t / T \\
& s.t. \quad \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \le \theta_o^t x_{io}^t, i = 1,2,...,m; \; t = 1,...,T, \\
& \qquad \sum_{j=1}^{n} \lambda_j^t y_{rj}^t \ge y_{ro}^t, r = 1,2,...,s; \; t = 1,...,T, \\
& \qquad \lambda_j^t \ge 0, j = 1,...,n; \; t = 1,...,T.
\end{aligned}
\tag{3}
$$

The system under evaluation, DMU_o, is called generally efficient if and only if $e_o^* = 1$. This means that it is generally efficient provided that it is efficient in each period. Notice that Jablonsky (2016) considered an output-oriented model while as can be seen in model (3), we have taken into account it as an input-oriented model. In model (3), flexible measures have not been included. Accordingly, methods are planned in the next section to evaluate the performance of multi-period structures with flexible measures.

4. Multi-Period Systems in the Presence of Flexible Measures

In this section, the status of flexible measures in multi-period production systems is tackled. To undertake this intention, oriented and non-oriented DEA models are proposed to assess the overall and period efficiency scores of multi-period systems where flexible measures are presented. These models are adopted for the following reasons:

a) Traditional input-output classification DEA models analyze the performance of entities in a special period of time while the proposed approaches appraise the efficiency of multi-period DMUs in a span of time, containing multiple periods.
b) Due to the defined structure, the overall and period efficiencies of multi-period processes with flexible measures are identified at one time when each of the models is computed.
c) Efficiency changes and input-output classification through multi-period structures are concerned in various periods of time.

As a case in point, oriented and non-oriented DEA models with flexible measures – taking individual DMU and aggregate standpoints into account – are developed to classify flexible measures and analyze the efficiency of multi-period processes. For modeling approaches, the aforementioned notations are used. First, oriented models are provided to estimate the efficiency values of multi-period systems with flexible measures. Then, non-oriented approaches are extended to investigate the issue. The production possibility set (PPS) or the technology in the period t under CRS is defined as follows:

$$PPS^t = \{(x^t, y^t, w^t) : x^t \geq \sum_{j=1}^{n} \lambda_j^t x_j^t, y^t \leq \sum_{j=1}^{n} \lambda_j^t y_j^t, (either\ w^t \geq \sum_{j=1}^{n} \lambda_j^t w_j^t\ or\ w^t \leq \sum_{j=1}^{n} \lambda_j^t w_j^t), \lambda_j^t \geq 0,\ j = 1,...,n\},$$
$$t = 1,...,T.$$

Due to the definition of PPS^t for the period t, the global technology or the global production possibility set can be shown as follows:

$$PPS^G = Convex\{PPS^1 \cup PPS^2 \cup ... \cup PPS^t\}$$

Also, the following constraints in PPS^t

$$\begin{cases} either\ w_{ko}^t \leq \sum_{j=1}^{n} \lambda_j^t w_{kj}^t, k = 1,2,...,K;\ t = 1,...,T, \\ or \\ w_{ko}^t \geq \sum_{j=1}^{n} \lambda_j^t w_{kj}^t, k = 1,2,...,K;\ t = 1,...,T, \\ \lambda_j^t \geq 0,\ j = 1,...,n; t = 1,...,T, \end{cases}$$

can be substituted by the next constraints:

$$\begin{cases} \sum_{j=1}^{n} \lambda_j^t w_{kj}^t \geq w_{ko}^t - M(1 - d_k^t), k = 1,2,...,K;\ t = 1,...,T, \\ \sum_{j=1}^{n} \lambda_j^t w_{kj}^t \leq w_{ko}^t + M d_k^t, k = 1,2,...,K;\ t = 1,...,T, \\ d_k^t \in \{0,1\}, k = 1,...,K, \lambda_j^t \geq 0,\ j = 1,...,n; t = 1,...,T, \end{cases}$$

in order to incorporate either-or constraints in the model.

In Subsections 4.1 and 4.2, oriented and non-oriented multi-period models are proposed from the individual DMU viewpoint. Aggregate oriented and non-oriented multi-period approaches are also provided in Subsection 4.3.

4.1 Oriented Multi-Period Efficiency Measures

Due to the terms stated in Section 3, the following input-oriented model is introduced to deal with multi-period production systems with flexible measures:

$$\begin{aligned} e_o^* = Min \quad & e_o = \sum_{t=1}^{T} \theta_o^t / T \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \leq \theta_o^t x_{io}^t, i = 1,2,...,m;\ t = 1,...,T, \\ & \sum_{j=1}^{n} \lambda_j^t y_{rj}^t \geq y_{ro}^t, r = 1,2,...,s; t = 1,...,T, \\ & \sum_{j=1}^{n} \lambda_j^t w_{kj}^t \leq \theta_o^t w_{ko}^t + M d_k^t, k = 1,2,...,K;\ t = 1,...,T, \\ & \sum_{j=1}^{n} \lambda_j^t w_{kj}^t \geq w_{ko}^t - M(1 - d_k^t), k = 1,2,...,K;\ t = 1,...,T, \\ & d_k^t \in \{0,1\}, k = 1,...,K, \lambda_j^t \geq 0,\ j = 1,...,n; t = 1,...,T. \end{aligned} \tag{4}$$

Through computing model (4), inputs and flexible measures considered as inputs are minimized whilst the achievement of the given outputs and flexible measures treated as outputs are met in accordance with each period. In model (4), if the optimal value $d_k^{*t} = 1$, then the measure k is an output in period t and if $d_k^{*t} = 0$, it is an input in period t. The optimal value e_o^* shows the overall efficiency of the system under evaluation, PS_o. Also, the optimal value θ_o^{*t} indicates the efficiency score related to the period t. Notice that it is deemed that all periods have equal preferences for the decision maker at this stage.

If the importance of periods is different for the decision maker, model (4) is revised as follows:

$$
\begin{aligned}
& e_o^{*w} = Min \ \sum_{t=1}^{T} \rho^t \theta_o^t / \sum_{t=1}^{T} \rho^t \\
& s.t. \ \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \le \theta_o^t x_{io}^t, i = 1,2,...,m;\ t = 1,...,T, \qquad (5.1) \\
& \sum_{j=1}^{n} \lambda_j^t y_{rj}^t \ge y_{ro}^t, r = 1,2,...,s; t = 1,...,T, \qquad (5.2) \\
& \sum_{j=1}^{n} \lambda_j^t w_{kj}^t \le \theta_o^t w_{ko}^t + M d_k^t, k = 1,2,...,K;\ t = 1,...,T, \qquad (5.3) \\
& \sum_{j=1}^{n} \lambda_j^t w_{kj}^t \ge w_{ko}^t - M(1 - d_k^t), k = 1,2,...,K;\ t = 1,...,T, \ (5.4) \\
& d_k^t \in \{0,1\}, k = 1,...,K, \lambda_j^t \ge 0,\ j = 1,...,n; t = 1,...,T.
\end{aligned}
\qquad (5)
$$

ρ^t is a constant that is defined by the decision maker and indicates the importance of each period t. To illustrate in details, models (4) and (5), which are input-oriented models, calculate the period and overall efficiency scores of the system under consideration and specify the role of flexible measures.

Theorem 1. The optimal objective value e_o^{*w} in model (5) is between zero and one, i.e., $0 < e_o^{*w} \le 1$.

Proof. Let M be a sufficiently large positive number and $\rho^t (t = 1,...,T)$ the constants that are defined by users. It is clear that $\theta_o^t = 1, \lambda_o^t = 1, \ \lambda_j^t = 0, j \ne o, d_k^t = 1, t = 1,...,T, k = 1,...,K$ is a feasible solution in model (5). Accordingly, $e_o^{*w} = Min \ \sum_{t=1}^{T} \rho^t \theta_o^t / \sum_{t=1}^{T} \rho^t \le 1$. Also, it can be shown that $0 < e_o^{*w}$. Owing to semi-positive inputs, outputs, and flexible measures under investigation in each period, one j exists and λ_j^t is non-zero in (5.2). Furthermore, note that $\lambda_j^t \ge 0, j = 1,...,n, t = 1,...,T \ (\exists j, \lambda_j^t \ne 0)$ and $\mathbf{X}^t = (\mathbf{x}_j^t) \ge 0$ is non-zero. Therefore, θ_o^{t*} could not be less than or equal to zero because of (5.1). Consequently, $0 < e_o^{*w} \le 1$.

Note that similar to Theorem 1, it can be shown that $0 < e_o^* \le 1$ in model (4).

If inputs, outputs, and flexible measures can change simultaneously on condition that inputs and flexible measures considered as inputs do not increase and outputs and flexible measures deemed as outputs do not decrease, an alternative approach is needed for analysis.

Thus, in the next subsection, non-oriented DEA approaches are introduced to appraise the efficiency, treat flexible measures, and determine the returns to scale status.

4.2 Non-Oriented Multi-Period Efficiency Measures

At the moment, non-oriented efficiency measures are suggested to handle multi-period systems with flexible measures in which the reduction of inputs and flexible measures determined as inputs and the expansion of outputs and flexible measures treated as outputs are considered, simultaneously. The following model is the extension of Tohidi and Matroud’s (2017) approach:

$$
\begin{aligned}
& e_o^{*no} = Min \quad \sum_{t=1}^{T} \alpha_o^t / T\beta_o^t \\
& s.t. \quad \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \le \alpha_o^t x_{io}^t, i = 1,2,\dots,m;\ t = 1,\dots,T, \\
& \sum_{j=1}^{n} \lambda_j^t y_{rj}^t \ge \beta_o^t y_{ro}^t, r = 1,2,\dots,s; t = 1,\dots,T, \\
& \sum_{j=1}^{n} \lambda_j^t w_{kj}^t \le \alpha_o^t w_{ko}^t + Md_k^t, k = 1,2,\dots,K;\ t = 1,\dots,T, \\
& \sum_{j=1}^{n} \lambda_j^t w_{kj}^t \ge \beta_o^t w_{ko}^t - M(1-d_k^t), k = 1,2,\dots,K;\ t = 1,\dots,T, \\
& \sum_{j=1}^{n} \lambda_j^t = 1, t = 1,\dots,T, \\
& d_k^t \in \{0,1\}, k = 1,\dots,K, \lambda_j^t \ge 0, j = 1,\dots,n; t = 1,\dots,T, \alpha_o^t, \beta_o^t \ge 0.
\end{aligned}
\tag{6}
$$

where α_o^t and β_o^t are scalars for each period t.

Model (6) reflects the equal preference between periods. In the presence of priority for evaluating the efficiency of systems in periods, model (6) is substituted with the following model:

$$
\begin{aligned}
& e_o^{*now} = Min \quad \sum_{t=1}^{T} \rho^t \alpha_o^t / \sum_{t=1}^{T} \rho^t \beta_o^t \\
& s.t. \quad \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \le \alpha_o^t x_{io}^t, i = 1,2,\dots,m;\ t = 1,\dots,T, \\
& \sum_{j=1}^{n} \lambda_j^t y_{rj}^t \ge \beta_o^t y_{ro}^t, r = 1,2,\dots,s; t = 1,\dots,T, \\
& \sum_{j=1}^{n} \lambda_j^t w_{kj}^t \le \alpha_o^t w_{ko}^t + Md_k^t, k = 1,2,\dots,K;\ t = 1,\dots,T, \\
& \sum_{j=1}^{n} \lambda_j^t w_{kj}^t \ge \beta_o^t w_{ko}^t - M(1-d_k^t), k = 1,2,\dots,K;\ t = 1,\dots,T, \\
& \sum_{j=1}^{n} \lambda_j^t = 1, t = 1,\dots,T, \\
& d_k^t \in \{0,1\}, k = 1,\dots,K, \lambda_j^t \ge 0, j = 1,\dots,n; t = 1,\dots,T, \alpha_o^t, \beta_o^t \ge 0.
\end{aligned}
\tag{7}
$$

To more explain, this method is based on the concept of most productive scale size (Banker, 1984). The ρ^t constant shows the importance of the period t. Models (6) and (7) are under the assumption of variable returns to scale and can determine the status of returns to scale and flexible measures. Consider $\alpha_o^{*t} = 1, \beta_o^{*t} = 1, d_k^{*t} = 1, k = 1,\dots,K, \lambda_o^{*t} = 1, \lambda_j^{*t} = 0, j \ne o$ that is a feasible solution of model (7) (model (6)), thus model (7) (model (6)) is always feasible.

Theorem 2. Assume $(\alpha^{*t}, \beta^{*t}, d^{*t}, \lambda^{*t})$ be an optimal solution of model (6) (model (7)).

If $\alpha^{*t} = \beta^{*t} = 1$, CRS exists at PS_o in period t.

If $\alpha^{*t} < \beta^{*t} < 1$, DRS prevails at PS_o in period t.

If $\alpha^{*t} > \beta^{*t} > 1$, IRS obtains at PS_o in period t.

If $\alpha^{*t} < 1 \le \beta^{*t}$ or $\alpha^{*t} \le 1 < \beta^{*t}$, PS_o is technically inefficient in period t.

Also, if $e_o^{*no} = 1$ ($e_o^{*now} = 1$), overall constant returns to scale exists at PS_o.

Proof. It can be conveniently proved similar to Banker (1984) by slight changes.

Theorem 3. The optimal value e_o^{*now} in model (7) (e_o^{*no} in model (6)) is not more than the optimal value e_o^{*w} in model (5) (e_o^{*} in model (4)).

Proof. Suppose $(\theta^{*t}, d^{*t}, \lambda^{*t})$ is an optimal solution of model (5) (model (4)). Thus, $(\alpha^{*t} = \theta^{*t}, \beta^{*t} = 1, d^{*t}, \lambda^{*t} = \lambda_j^{*t} / \sum_j \lambda_j^{*t})$ is a feasible solution of model (7) (model (6)) and the optimal value e_o^{*now} in model (7) (e_o^{*no} in model (6)) is less than or equal to the optimal value e_o^{*w} in model (5) (e_o^{*} in model (4)).

Definition 1: PS_o is called the overall efficient in models (4), (5), (6), and (7) if and only if the optimal objective values of them equal to one. It means that PS_o is the overall efficient under each of these models if and only if it is efficient in all periods.

Definition 2: In models (4) and (5), PS_o is called efficient in period t if and only if $\theta_o^{*t} = 1$. For models (6) and (7), PS_o is defined as efficient in period t if and only if $\frac{\alpha_o^{*t}}{\beta_o^{*t}} = 1$.

Moreover, the optimal objective values less than one in models (4)-(7) mean that PS_o is inefficient at least in one period. To determine the status of flexible measures, the majority rule is used. To illustrate, the role of the flexible measure in the overall system and each period is identified by taking the majority rule into account. Also, the alternative solutions are not considered for specifying the role of the flexible measure, according to Toloo (2012).

4.3 The Proposed Aggregate Approaches

According to Cook and Zhu (2007), a criterion for the general classification of each flexible measure is based on the majority choice among the DMUs as discussed before (the individual DMU viewpoint) and an alternative approach is from the viewpoint of the manager of the collection of DMUs (the aggregate perspective). In this case, the status of flexible measures is specified by the following models.

Input-oriented model (8) evaluates the overall and period efficiency scores of the set of DMUs and determines the role of flexible measure as input or output while the preference exists among periods. To clarify, the status of flexible measures in each period is determined without the need to use the majority choice. If the optimal $d_k^{*t} = 1$ then the flexible measure k is an output in period t and if $d_k^{*t} = 0$, it is an input in period t. However, the majority choice rule can be used to determine the status of each flexible measure in a whole system including several periods. Similarly, the results can be interpreted in model (9). Model (9) is an aggregate non-oriented model that considers the preference between periods.

$$
\begin{aligned}
& e^{*wa} = Min \;\; \sum_{t=1}^{T} \rho^t \theta^t / \sum_{t=1}^{T} \rho^t \\
& s.t. \;\; \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \le \theta^t \sum_{j=1}^{n} x_{ij}^t, i = 1, 2, ..., m; \; t = 1, ..., T, && (8.1) \\
& \quad \sum_{j=1}^{n} \lambda_j^t y_{rj}^t \ge \sum_{j=1}^{n} y_{rj}^t, r = 1, 2, ..., s; t = 1, ..., T, && (8.2) \\
& \quad \sum_{j=1}^{n} \lambda_j^t w_{kj}^t \le \theta^t \sum_{j=1}^{n} w_{kj}^t + M d_k^t, k = 1, 2, ..., K; \; t = 1, ..., T, && (8.3) \\
& \quad \sum_{j=1}^{n} \lambda_j^t w_{kj}^t \ge \sum_{j=1}^{n} w_{kj}^t - M(1 - d_k^t), k = 1, 2, ..., K; \; t = 1, ..., T, && (8.4) \\
& \quad d_k^t \in \{0,1\}, k = 1, ..., K, \lambda_j^t \ge 0, \; j = 1, ..., n; t = 1, ..., T.
\end{aligned}
\tag{8}
$$

And

$$
\begin{aligned}
& e^{*nowa} = Min \quad \sum_{t=1}^{T} \rho^t \alpha^t / \sum_{t=1}^{T} \rho^t \beta^t \\
& s.t. \quad \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \le \alpha^t \sum_{j=1}^{n} x_{ij}^t, i = 1,2,\dots,m;\ t = 1,\dots,T, \\
& \quad \sum_{j=1}^{n} \lambda_j^t y_{rj}^t \ge \beta^t \sum_{j=1}^{n} y_{rj}^t, r = 1,2,\dots,s; t = 1,\dots,T, \\
& \quad \sum_{j=1}^{n} \lambda_j^t w_{kj}^t \le \alpha^t \sum_{j=1}^{n} w_{kj}^t + Md_k^t, k = 1,2,\dots,K;\ t = 1,\dots,T, \\
& \quad \sum_{j=1}^{n} \lambda_j^t w_{kj}^t \ge \beta^t \sum_{j=1}^{n} w_{kj}^t - M(1-d_k^t), k = 1,2,\dots,K;\ t = 1,\dots,T, \\
& \quad \sum_{j=1}^{n} \lambda_j^t = 1, t = 1,\dots,T, \\
& \quad d_k^t \in \{0,1\}, k = 1,\dots,K, \lambda_j^t \ge 0,\ j = 1,\dots,n; t = 1,\dots,T, \alpha^t, \beta^t \ge 0.
\end{aligned} \tag{9}
$$

Theorem 4. In model (8), $0 < e_o^{*wa} \le 1$.

Proof. It is clear to see that $\theta_o^t = 1, \lambda_j^t = 1,\ j = 1,\dots,n, d_k^t = 1, t = 1,\dots,T, k = 1,\dots,K,$ is a feasible solution in model (8) while M is a large positive number and $\rho^t (t = 1,\dots,T)$ are parameters. As a result, $e_o^{*wa} = Min \sum_{t=1}^{T} \rho^t \theta_o^t / \sum_{t=1}^{T} \rho^t \le 1$. Also, it can be shown that $0 < e_o^{*w}$. The constraint (8.2) makes $\lambda_j^t, j = 1,\dots,n, t = 1,\dots,T$ to be non-zero because of $Y^t = (y_j^t) \ge 0, Y^t \ne 0$. Note that inputs, outputs, and flexible measures under investigation are semi-positive. Therefore, from the constraint (8.1), $\theta^t, t = 1,\dots,T$ cannot be less than or equal to zero (i.e., $\theta^t > 0$). In this regard, $0 < e_o^{*wa} \le 1$.

4.4 Efficiency Changes

Estimating the efficiency change of production systems over time is a significant aspect for managers and decision makers. One of the popular measures for this purpose is the Malmquist productivity index (MPI), originated by Caves et al. (1982). In the literature, different forms of the MPI can be found. Fare et al. (1994) used the non-parametric DEA methodology to compute MPI. Pastor and Lovell (2005) introduced a global MPI to construct a global frontier, which consists of all observations of all periods. This index is applied in the current paper due to several favorable properties.

By considering $\bar{E}_o^t$ and $\bar{E}_o^{t+1}$ as the relative efficiency scores of periods t and $t+1$, the global MPI of PS_o is defined as follows:

$$
MPI_o^{t,t+1} = \frac{\bar{E}_o^{t+1}}{\bar{E}_o^t} \tag{10}
$$

Thus, we have:

- $MPI_o^{t,t+1} > 1$, the efficiency increases and the progress is observed,
- $MPI_o^{t,t+1} < 1$, the efficiency decreases and the regress is observed, and
- $MPI_o^{t,t+1} = 1$, no change is observed in the relative efficiency between two periods.

5. Explanatory Example

As known, evaluating the efficiency of universities as knowledge organizations is a major perspective. Thus, the proposed approach in this research is used to analyze the efficiency of ten universities within three periods from 2013 to 2015. As Kuah and Wong (2011) mentioned, there is no explicit standard to select the inputs/outputs in order to assess the efficiency of universities. Therefore, after reviewing the DEA literature and consulting directors and considering information availability, two inputs (i.e., general expenditure and the number of academic staff), two outputs (i.e., the number of undergraduate students and postgraduate students), and one flexible measure (i.e., research income) have been considered. For more illustration, a research income was considered as both the input and the output in some studies (i.e., Beasley, 1990; Chen, 2014; Cook & Zhu, 2007) and has been stated in Amirteimoori and Emrouznejad (2011) that many considerations deem it as the input due to the fact that it is gained and utilized by universities and also it is a revenue obtained by universities. Thus, it is dealt with as the output. Variables are more described in Table 1 and the data are provided in Table 2. The data were gathered by referring to available documents and consulting with responsible individuals.

Table 1. Variable Descriptions

Variables	Terms	Description
Inputs		
General expenditure	GE	Expenditure on staff and student facilities, general educational expenditure, and service expenditure like computer network and library
Academic staff	AS	The number of full-time academic staff
Outputs		
Undergraduate students	UGS	The total number of undergraduate students
Postgraduate students	PGS	The total number of postgraduate students
Flexible measures		
Research income	RI	Incomes obtained from funding council grants and research grants

Table 2. Data

#University	Period 1					Period 2					Period 3				
	GE	AS	UGS	PGS	RI	GE	AS	UGS	PGS	RI	GE	AS	UGS	PGS	RI
1	270	55	330	469	689	300	60	320	470	700	340	66	300	490	720
2	922	43	550	517	681	700	36	530	523	710	760	30	510	570	720
3	878	47	570	573	704	845	44	575	576	720	830	40	560	580	730
4	549	78	435	389	606	576	85	420	395	650	585	87	415	400	660
5	306	94	188	551	680	265	97	170	560	695	250	86	150	570	690
6	431	97	323	406	914	543	99	320	410	927	540	99	310	415	929
7	209	97	280	476	872	250	94	260	465	880	230	97	265	475	870
8	611	44	265	423	823	650	36	240	410	840	640	33	230	420	846
9	274	96	268	350	748	290	98	270	370	760	260	98	250	372	762
10	452	38	203	286	860	489	45	190	295	880	440	44	190	300	886

We assume there is no preference between three periods. At the first stage, model (4), which is the input-oriented model, is calculated for evaluating the efficiency of universities. The results can be found in Table 3. Columns 2-4 show the relative efficiency scores of periods 1, 2, and 3. The overall efficiency scores of universities are indicated in column 5. As can be seen, 3 universities, (1, 2, and 7) are efficient overall. It means they are efficient in all periods. To illustrate, 40% of universities are efficient in each period while 30% of them are efficient overall. Columns 6-8 show the input/output role of the income research for periods 1, 2 and 3, respectively. Without considering alternative solutions, it can be seen that the most universities consider the research income as an input factor. Thus, according to the majority rule, the role of research income is specified as the input in the multi-period system of the university. In the spans 1 and 3, the number of universities that determined the status of research income as the input and the output is equal. Therefore, in these cases, if the manager desires to classify the flexible measure for each period, the aggregate approaches can be used. In the period 2, 4 universities have specified the role of research income as the input. Accordingly, the role of the flexible measure is determined as the input considering the majority rule.

In the next stage, we calculate the non-oriented model (6). By comparing Tables 3 and 4, it can be found that the results of the period and overall efficiency scores of models (4) and (6) are similar. Also, the role of research income is determined as the input in model (6) according to the majority rule that is analogous to model (4). Due to Theorem 1, the status of returns to scale for periods 1, 2, and 3 is also obtained and shown in columns 9-11 of Table 4. Some universities have the same returns to scale condition in different periods under consideration such as universities 1, 2, and 7 that show CRS, universities 8 and 10 that indicate IRS, and the university 4 with DRS. However, some universities such as universities 3, 5 and 9 prevail different returns to scale situations in these years. Universities with DRS experience improper use of their resources. These universities can decrease their inputs (and the flexible measure with the input role) while they produce the same proportion or more outputs (and the flexible measure with the output role). For the increase of inputs (and the flexible measure with the input role), universities with IRS produce more outputs (and the flexible measure with the output role) by a higher proportion than the increase of inputs. These universities are appropriate to increase resources to raise outcomes. Efficient universities with CRS operate at the optimal scale size and the change of inputs does not have the influence on the efficiency.

Furthermore, considering the efficiency values, the managers of inefficient universities should pay attention to their performance in these periods and make more attempt to improve their efficiency. As the results show, the university 6 has the least efficiency in the period 1 and the university 10 gains the least efficiency scores in periods 2 and 3 and generally. Therefore, it is essential that the managers of these universities explore their efficiency and also their strengths and weaknesses deeply to progress the performance. Due to these efficiency levels and the accurate consideration of merits and demerits, managers can make appropriate plans and decisions.

Table 3. Results of Model (4)

#University	Efficiency				d		
	Period 1	**Period 2**	**Period 3**	**Overall**	**Period 1**	**Period 2**	**Period 3**
1	1.000	1.000	1.000	1.000	0 or 1	0 or 1	0 or 1
2	1.000	1.000	1.000	1.000	0 or 1	0 or 1	0 or 1
3	1.000	0.908	0.988	0.965	0 or 1	1	1
4	0.803	0.762	0.865	0.810	1	1	1
5	0.923	1.000	1.000	0.974	1	0 or 1	0 or 1
6	0.685	0.641	0.704	0.677	0	0	0
7	1.000	1.000	1.000	1.000	0 or 1	0 or 1	0 or 1
8	0.881	0.820	0.809	0.837	0	0	0
9	0.787	0.873	0.875	0.845	1	0	1
10	0.725	0.621	0.649	0.665	0	0	0

Table 4. Results of Model (6)

#University	Efficiency				d			1	2	3
	1	**2**	**3**	**Overall**	**1**	**2**	**3**			
1	1.000	1.000	1.000	1.000	0 or 1	0 or 1	0 or 1	CRS	CRS	CRS
2	1.000	1.000	1.000	1.000	0 or 1	0 or 1	0 or 1	CRS	CRS	CRS
3	1.000	0.908	0.988	0.965	0 or 1	1	1	CRS	DRS	DRS
4	0.803	0.762	0.865	0.810	1	1	1	DRS	DRS	DRS
5	0.923	1.000	1.000	0.974	1	0 or 1	0 or 1	DRS	CRS	CRS
6	0.685	0.641	0.704	0.677	0	0	0	Tec. Inef	Tec. Inef	Tec. Inef
7	1.000	1.000	1.000	1.000	0 or 1	0 or 1	0 or 1	CRS	CRS	CRS
8	0.881	0.820	0.809	0.837	0	0	0	IRS	IRS	IRS
9	0.787	0.873	0.875	0.845	1	0	1	Tec. Inef	IRS	Tec. Inef
10	0.725	0.621	0.649	0.665	0	0	0	IRS	IRS	IRS

Furthermore, the efficiency changes between two periods are calculated using the formula (10). Results are indicated in Table 5. As can be seen, universities, 1, 2, and 7 are without efficiency changes in these years. In the periods 1 and 2, 5 universities have regressed and 2 universities 5 and 9 have progressed as shown in columns 2 and 3. In the periods 2 and 3, 5 universities have progressed and only the university 8 has regressed. And for the three-year period, $MPI^{1,3}$, the progress has been observed in 4 universities and the regress has been seen in 3 universities. The average of MPIs for periods 1-2, 2-3, and 1-3 are displayed in the last row of Table 5. During the period 1-2, universities have regressed on average, while the efficiency of these universities has improved in periods 2-3 and 1-3 because of the average MPIs greater than one.

Table 5. Efficiency Changes

#University	$MPI^{1,2}$	Changes	$MPI^{2,3}$	Changes	$MPI^{1,3}$	Changes
1	1	No change	1	No change	1	No change
2	1	No change	1	No change	1	No change
3	0.908	Regress	1.088106	Progress	0.988	Regress
4	0.948941	Regress	1.135171	Progress	1.07721	Progress
5	1.083424	Progress	1	No change	1.083424	Progress
6	0.935766	Regress	1.098284	Progress	1.027737	Progress
7	1	No change	1	No change	1	No change
8	0.93076	Regress	0.986585	Regress	0.918275	Regress
9	1.109276	Progress	1.002291	Progress	1.111817	Progress
10	0.856552	Regress	1.045089	Progress	0.895172	Regress
Average	0.977272	Regress	1.035553	Progress	1.0101635	progress

In this part, the aggregate models, i.e., models (8) and (9), are also applied to evaluate the efficiency scores and to classify the research income from the viewpoint of the manager of the set of universities. Results are provided in Table 6. Period and overall efficiency scores are shown in columns 2-5. According to the majority rule, both models consider the research income as the input. Moreover, for each period, decreasing returns to scale is determined using model (9).

Table 6. Results of Models (8) and (9)

Model	Efficiency				d		
	Period 1	Period 2	Period 3	Overall	Period 1	Period 2	Period 3
Model (8)	0.803	0.799	0.826	0.81	0	0	0
Model (9)	0.772 DRS	0.799 DRS	0.826 DRS	0.799	1	0	0

Finally, model (3) is computed to compare the existing models with the proposed approach. For this purpose, the research income is deemed as the output to calculate model (3). The results obtained are denoted in Table 7.

Table 7. Results of Model (3)

#University	Efficiency			
	Period 1	Period 2	Period 3	Overall
1	1	1	1	1
2	1	1	1	1
3	1	0.908	0.988	0.965
4	0.803	0.762	0.865	0.81
5	0.923	1	1	0.974
6	0.779	0.745	0.766	0.763
7	1	1	1	1
8	1	1	1	1
9	0.787	0.885	0.875	0.849
10	1	1	1	1

The examination of the results computed shows that

- By using model (3), 60% of universities are assessed as efficient in each period while 40% of universities are determined as efficient by model (4).
- 50% of universities are overall efficient in model (3) while this amount reaches 30% in model (4).
- The efficiency scores obtained from model (3) are more than or equal to model (4).
- The proposed model does not overestimate the efficiency scores and it discriminates the efficient universities more rationally.

We can find the overall and period performances of universities using one programming problem while the efficiency of each period can be measured individually by computing either model (1) or model (2). Also, model (3) ignores the effect of flexible measures on the estimation of multi-period efficiency, thus, as illustrated; there are differences between the efficiency values achieved from models (3) and (4).

It is clear that to assess the performance of multi-period processes in the presence of flexible measures, the proposed models are more beneficial and practical in comparison with models (1)-(3) that either classify flexible measures and analyze the efficiency in one period or only estimate the multi-period efficiency without addressing flexible measures.

From the management aspect, the efficiency evaluation and the analysis of efficiency changes are important topics. Moreover, the classification of measures as inputs and outputs is essential to specify accurate and rational results of the efficiency. In DEA models, decision variables are also significant components in order to obtain rational efficiency findings. In this study, approaches based on DEA have been introduced to measure the overall and period efficiency scores of systems with flexible measures, simultaneously. Actually, alternative DEA models are provided for benchmarking. The approaches proposed in this study aid managers in measuring the performance of firms in multiple periods of time when measures with uncertain input/output status exist. The time domain of the examination permits supervisors to consider the impacts of diverse structural alterations on the consequential efficiency of the entities over time periods. Also, the role of flexible measure in multi-period structures is identified using the introduced techniques. The results show incorporating flexible measures in multi-period systems effects on efficiency scores. As shown in Figure 1, the overall efficiency scores estimated by models (3) and (4) are different in some universities, i.e., universities 6, 8, 9, and 10. Considering the ability of decision makers to prioritize the importance of periods, the parameters have been presented in the objective functions to show the preferences of the managers.

To analyze the multi-period efficiency of systems with the unknown input-output status of some measures, each of the models (4)-(9) can be utilized by practitioners. The selection among these models to apply depends on some matters such as:

- The ability of decision makers to control performance measures,
- The preference of periods,
- Classifying flexible measures based upon the individual or aggregate perspective.

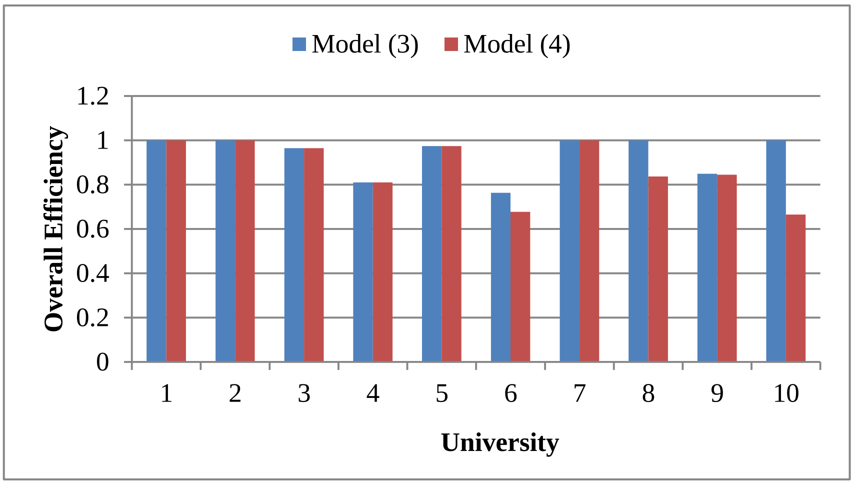

Figure 1. The Comparison of the Overall Efficiency Scores

6. Conclusion

Efficiency analysis in multiple periods of time is a significant aspect for managers in order to make a decision and plan for the future. Also, determining the status of input/output of factors is vital to calculate the accurate efficiency. The present paper has been designed to identify the status of flexible measures and evaluate the relative efficiency scores of multi-period production systems where there are flexible measures. Actually, the DEA-based approaches have been introduced to estimate the overall and period efficiencies of firms with flexible measures, simultaneously. To illustrate in more details, oriented and non-oriented models have been proposed to deal with the subject. Also, the majority rule has been used to determine the role of flexible measures. The aggregate models were further suggested establishing the appropriate status of flexible measures from the aggregate efficiency perspective. To analyze efficiency changes between two periods, MPI has been utilized. A dataset has also been presented to clarify and demonstrate the proposed approaches. The results obtained on the proposed techniques have shown incorporating flexible measures influences on the efficiency scores of multi-period systems. One of the advantages of the suggested approaches is to have reasonable computational efforts. Actually, period and overall efficiency scores of systems with flexible measures are evaluated by only one programming problem. Ignoring interconnections among periods is a disadvantage of the rendered models that can be addressed in future. Another limitation is the consideration of only non-negative performance measures. Accordingly, classifying flexible measures and evaluating the performance of multi-period systems with negative and non-negative data can be deemed as another matter for additional discussion.

Further research may investigate imprecise information in multi-period systems when there are flexible measures. Also, discriminating and ranking of multi-period systems with flexible measures is an area of focus for future study.

Conflicts of Interest

There is no conflict of interest.

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