



# A Multi-Objective VMI Model for a Two-Echelon Single Manufacturer Multiple Buyers Supply Chain

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## Abstract

A model of a two-level single-producer multi-buyer supply chain (TSPMBSC) is focused on in this article with a single product made by the producer (or vendor) given to the buyers. The operational form of vendor managed inventory (VMI) is utilized by vendors and buyers. We assume the economic production quantity (EPQ) model used by the producer for inventory control with a limited production rate. Sales quantity and sales price are the parameters of each buyer as well as a certain production rate. Two objectives are considered for the model; the first objective is the maximization of channel profit while the second objective is the maximization of the production periods variances whereby the required storage space is minimized. Because of NP-hardness, the weighted sum multi-objective genetic algorithm (WSMOGA), the multi-objective particle swarm optimization algorithm (MOPSO) and the Non-dominated sorting genetic algorithm II (NSGA-II) are the three distinct heuristics embedded for tackling the problem. The instances of the considered problems with small, medium and large sizes are used to compare these heuristics. Considering the metrics of comparison, The MOPSO-based heuristic outperformed the other heuristics.

## Keywords:

Vendor Managed Inventory;  
Economic Production  
Quantity;  
Supply Chain;  
MOPSO, NSGA-II

## Introduction

The materials, information and cash are the three major flows in the supply chain (SC). It is the satisfaction of the customer needs that the chain is regularly aimed at where the chain operational costs are minimized. The cooperation based on novel information technology (IT) tools such as vendor managed inventory (VMI) has been of great interest amongst researchers recently. It is the supplier in the partnership of VMI by whom the decisions for replenishment of the inventory for the members as the consumers are made and the levels of inventory are monitored using electronic tools as well as the replenishment decisions are made periodically.

The model of the considered SC which is entitled TSPMBSC is formulated where the operational form of VMI is utilized by the producer and the buyers. A bounded rate of production for the producer is assumed in this paper. The EPQ system rules are followed in the period of production. The inventory control system of the economic order quantity (EOQ) is applied by the buyers. The developed model obtains the buyers' sales prices and sales quantities and the production rates of the buyers in the producer's location as decision variables. Having the decision variables' optimal values, the SC channel profit and the prices of contract among

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the producer and buyers can be found. The more complexity of the problem as a nonlinear integer program (NIP) is made by the objective function nonlinearity with some of the integer variables in the model. As claimed by Costa and Oliveira [1], the genetic algorithm (GA) is known as an evolutionary heuristic by which near optimal solutions are provided when solving NIP problems.

The external variable of the sales quantity is the variable in which the profit of the SC channel can be calculated. The sales quantity as given by Lau and Lau [3] is linearly related to the sales price. Fair pricing is believed to be a key factor by which the SC members are related reliably with VMI being the cooperation agreement.

## Review of Literature

Traditionally, single-vendor and single-buyer inventory model in SCs has been studied considerably in the literature [5-6] while multiple buyer inventory systems are of less attention. On the other hand, there exists some research in the area of VMI with only one objective and constant production rate and deterministic demand [7]; it seems the research with more than one objective will be more attractive for researchers. The operational form of VMI in the form of two-level single-vendor multi-buyer SC (TSVMBSC) is studied by Nachiappan and Jawahar [8]; the maximized channel profit results are shared among the existing organizations where the EOQ policy used by the vendor and buyers for inventory management is assumed. An SC including a single vendor and multiple buyers in which raw materials are processed by the vendor and converted to some final products which are given to the buyers is studied by Zhang et al. [9]; the VMI form is followed by the policy of inventory management. The fixed production and demand rates in a model of joint cost is proposed where different ordering cycles are considered where more than one replenishment could happen in each production cycle. A model proposed by Yao et al. [10] showed the way the cost savings of such strategies as VMI are affected by SC parameters. The best quantities of buyers' sales are given by the developed mathematical models in the operational form of the VMI case as in [8]. Abdul-Jalbar et al. [11] developed a NIP model for an SC which had one vendor and two buyers. A solution method was proposed by the combination of Karush-Kuhn-Tucker conditions and the well-known branch and bound technique.

A decentralized SC was examined by Wang [12] being composed of one manufacturer and one distributor; considering a commodity with a short life-cycle and uncertain demand, two modes of with and without VMI operational conditions being studied in the SC; Yu et al. [13] investigates how the information obtained from retailers of a manufacturer-retailers SC which utilizes Stackelberg game in the framework of VMI cooperation system could be used by the manufacturer. The VMI SC being the manufacturer multi-retailer information-asymmetric and finds their optimal profits considering the tools of marketing like pricing and advertising and inventory control policies as well was also studied by Yu et al. [14]. The consignment stock (CS) as a special case of VMI policy from which the two SC players of buyer and vendor benefit as a marketing strategy was also examined by Zavanella and Zanoni [15]. The vendor inventory suppression may be a result of the most radical form of CS as the buyer's site is used by the vendor for stocking the final commodities. Thus, the amount stored on the site of buyers is assured by the vendor to be something between the maximum and minimum amount. Somehow the commodity required is picked up by the buyer from its site which satisfies its needs. A decentralized SC was also studied by Bichescu and Fry [16] which considers VMI agreements by which the order quantity is determined and sent to the retailer who selects the reorder point. A variety of game theoretic models is used to study the power distribution effects on different players of the SC. The result of the VMI strategy is more great savings than strategies of traditional inventory management.

A multi-retailer SC with a single capacitated manufacturer multi-retailer SC was also investigated by Almehdawe and Mantin [17] in the operational mode of VMI. In order for tackling the coordination problem in this SC, a framework of the Stackelberg game was developed. In a coordination system of VMI, a single vendor and multiple retailers SC model was proposed by Darwish and Odah [18]. The theorems for tackling the problem complexities were developed and an efficient algorithm for reaching the global optimal solution was devised. The outstanding arguments representing the considerable findings were also extended by Wang et al. [19]. In the VMI cooperation program was also SC with a single vendor and a single buyer which Guan and Zhao [20] studied designing an ownership scenario for the vendor in a revenue sharing contract, and the ownership scenario for the retailer in a franchising contract. The SC with the single manufacturer single retailer in a cooperation form of VMI was also considered by Bookbinder et al. [21]. It includes three distinct scenarios: no agreement between the SC members, VMI agreement with vendor initiating orders for the retailer, and a centralized system in which the members are managed as an integrated system. The manufacturer production amount as the order quantity of the retailers obtained the optimal solutions. A model which determines the optimal sales price and quantity together with the optimal profit and price between vendor and buyer in TSVMBSC under VMI operational form was given by Goh and Ponnambalam [22]. A heuristic based on the particle swarm optimization (PSO) was developed to solve it. One supplier and one retailer SC model were developed by Pasandideh et al. [23] where the inventory management measures before VMI implementation and after that were considered. According to the results, the implementation of VMI for EOQ with back ordered unsatisfied demand is sometimes capable of decreasing the total costs of supply chains.

A model of EOQ with a single supplier and single retailer having backordered unsatisfactory demand was developed by Pasandideh et al. [24]. The supplier was with a capacitated site and the number of orders had an upper bound. It was supposed that the information of the retailer was utilized by the supplier to make decisions for the replenishments. Proposing a genetic algorithm (GA) to provide order quantities and the level of back order, it was intended that the total inventory costs of the SC be minimized.

The TSVMBSC model which Nachiappan and Jawahar (2007) provided was solved by Goh et al. [25] using GA, PSO and a new heuristic based on the immune system. A problem of multiple vendors and retailers and a single warehouse was solved by Sadeghi et al. [26] who used PSO based on the hybrid meta-heuristic where its performance was compared with the traditional GA. A model of multi-product EOQ with VMI policy was proposed by Roozbeh Nia et al. [27] for SC with single vendor single buyer. Unsatisfied demands of this model were considered backordered. In the model given, the constraints of storage capacity and delivery number were included. Fuzzy numbers were considered for demand and the existing space. In order for a near-optimal solution, a heuristic based on Ant colony was applied.

The SC network of a single vendor with multiple buyers in the VMI strategy was studied by Diabat [28] where the optimal sales were found. In order to tackle the problem, the algorithm of hybrid GA /simulated annealing (SA) was provided. The decentralized three-level SC comprising a single supplier, single producer and some retailers was studied by Taleizadeh and Noori-daryan [29]. The products of the producer replenished by the supplier were required for the retailers' orders. The retailers were assumed a demand sensitive to price. The equilibrium of Stackelberg–Nash was used for analyzing the SC network total cost. The single-vendor single-buyer SC in the SC working environment was studied by Pasandideh et al. [30]. An SC with a single capacitated manufacturer and multiple retailers in an integrated VMI model was proposed by Pasandideh et al. [31]. A variety of products having a demand depending on the price were made by The Manufacturer. Proposing a profit contract, the model of bi-objective non-linear mathematical was used for formulating the problem. For obtaining fair non-dominated solutions (NDS), the lexicographic max–min technique was utilized.

The problem of TSPMBSC producing a considered product was studied by Seifbarghy et al. [32] who transported it to the buyers by the producer. The VMI operational form was used by the producer and the buyers. Considering the values of the parameters, the SC profit was obtained as well as the contract price between SC elements. A two-level single manufacturer and multiple retailers SC model was examined in other research which included the framework of a VMI system. They authorized the sale being lost in their model and utilized a GA based heuristic for solving the problem. Park et al. [33] studied an inventory-routing problem and developed a GA-based heuristic for this problem. Vehicle's routes and replenishment times and quantities are determined in this research. A tri-level model was given by Han et al. [34] for the SC system of three echelons in order to coordinate the decisions on inventory in a vendor buyer SC. Using customer relationship management (CRM) by Filho et al. [35], a comprehensive study was performed for finding the VMI system application in order to predict demand in the animal industry. Different SC contracts in five categories of a rebate, revenue sharing, quantity discount, buyback, and quantity flexibility were utilized by Sainathan and Groenevelt [36] by whom the SC coordinating approach was introduced which included a single vendor and single retailer in a working environment of VMI. In a pharmaceutical SC, a VMI model was implemented by Weraikat et al. [37]. The critical challenge of this system is the desire of vendors and buyers for keeping the inventory at the lowest possible amount leading to an increase in the number of expired drugs. The reduce in the number of excessive drugs being expired was the model's capability. A new MILP model for solving a VMI and facility location problem in an integrated manner was proposed by Golpîra [38] who used the expert systems concepts. In their study, the reduction in inventory cost and increase in benefits of SC members were shown to be the result of an increase in the replenishment frequency.

The extension of the given model by [8] is the model of the current research which assumes the vendor as a manufacturer making products for a given number of buyers at a constant rate; as a matter of fact, the policy of EPQ inventory control is followed by the vendor. The system is in VMI operational form. The rate of optimal production per buyer of the vendor together with the optimal sales quantities and sales prices of the buyers is determined by the supposed model.

## Multi-Objective Optimization Problems (MOOPs)

MOOP is considered a problem that includes two or more conflicting objectives; the objectives are tried to be satisfied simultaneously. Due to their nature, in MOOP, one should try to generate NDS [39,40]. The general formulation of MOOP includes the number of objectives with some inequalities and equalities. Mathematically, the problem can be stated as in Eq. 1.

$$\begin{aligned} & \text{Min } \{f_1(x), f_2(x), \dots, f_m(x)\} \\ & \text{s.t: } g_j(x) \leq 0; \quad j = 1, 2, \dots, J \\ & h_k(x) = 0; \quad k = 1, 2, \dots, K \end{aligned} \tag{1}$$

In the aforementioned formula,  $x$  represents the decision variables' vectors;  $f_l(x)$  represents the  $l$ th objective while  $g_j(x), h_k(x)$  are constraints' vector. It is rare to find an optimal solution for all  $f_l(x)$ . For this reason, the analyst should find an NDS.

## Notation and Problem Formulation

In this section, initially, the notations are given; then, the demand curve and its relation with the contract price are represented. Vendor and buyers' operational costs are given and finally, the mathematical model of the problem is stated.

### Notation

The used notations in the given model are:

$j$ : Index for the buyers

$n$ : The number of buyers in the SC

$a_j$ : Intercept of demand curve of buyer  $j$

$b_j$ : Slope of demand curve of buyer  $j$

$Hb_j$ : Unit inventory carrying cost of buyer  $j$  without VMI

$Hs$ : Unit inventory carrying cost of the vendor without VMI

$H_{jvmi}$ : Unit inventory carrying cost when there is VMI cooperation between buyer  $j$  and vendor

$Sb_j$ : Ordering cost of buyer  $j$  for the without VMI (i.e. the setup cost)

$Ss$ : Setup cost for the vendor without VMI

$S_{jvmi}$ : Cost of monitoring the stock of buyer  $j$  in VMI mode

$P$ : Total production rate at the vendor site

$\theta_j$ : Unit flow cost from vendor to buyer  $j$

$v_j$ : Unit transportation cost delivered from vendor to buyer  $j$

$\delta$ : Unit production cost in the vendor site

$PD_j$ : Production cost in the vendor site for buyer  $j$  and added with the corresponding distribution costs

$PR_j$ : Revenue sharing ratio in the contract of buyer  $j$  and the vendor

$Q_j$ : Order quantity for buyer  $j$  from the vendor

$W$ : Contract price of the vendor and each buyer

$W_j$ : Contract price of the vendor and buyer  $j$

$P_j$ : Production rate of buyer  $j$  at the vendor's site

$y_j$ : Sales quantity for buyer  $j$

$P(y_j)$ : Sales price of the sold product by buyer  $j$

$P(y)$ : Sales price of the product

$y_{jmin}$ : Buyer  $j$ 's Minimum sales quantity

$y_{jmax}$ : Buyer  $j$ 's Maximum sales quantity

## Demand and contract price analysis

Each vendor may have a set of direct sales channels such as retailers or buyers. The key parameters may be sales quantity 'y' and sales price by which the buyer sells in its market 'P(y)', the contract price agreed on by the vendor and the buyer 'W' and finally production rate at the vendor location for producing the products for each buyer. The sales quantity is affected by the sales price which in turn depends on the commodity's importance for the customer, the customers' buying power, and whether the commodity's nature is perishable or not. The clear assumption is that by increasing the sales price, the quantity of sales decreases and vice versa. 'P(y)' and 'y' can be deemed to be linearly related to each other as in Eq. 2 [8]:

$$P(y) = a - by \quad (2)$$

in which  $a$  and  $b$  are the intercept and slope of the mentioned curve depicted in Fig. 1.

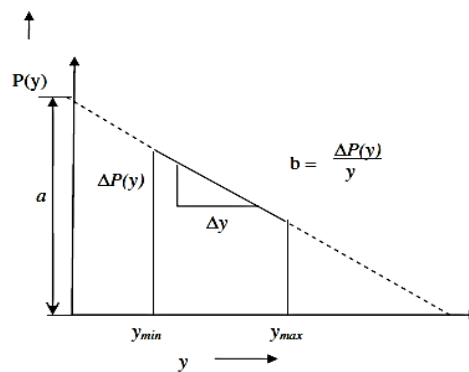


Fig. 1. The sales price and quantity relationship

Besides, sales quantity can be considered to be in a range between  $y_{j_{min}}$  and  $y_{j_{max}}$ ; in this range, the linear assumption seems to be logical. Considering a number of arbitrary buyers, the demand function for the buyer  $j$  is shown as in Eqs. 3 and 4:

$$P(y_j) = a_j - b_j y_j \quad (3)$$

$$s.t. \quad y_{j_{min}} \leq y_j \leq y_{j_{max}} \quad (4)$$

The key parameter in the SC profit is the contract price. The addressed parameter which is normally higher than the manufacturing cost and less than the sales price, can be agreed by the vendor and the buyers. The demand and nature of the product can have a serious role in finding the contract price. It is clear that commodities with a good reputation and high demand are faster bought which implies to be with lower risk; the buyer adopts a contract price that is closer to the sales price. Clearly, in cases with newer products and higher risks in the market, the contract price may be at lower levels; the levels are normally nearer to the manufacturing cost. As in [8], the contract price can be known to be dependent on the sales location, the severity of competitiveness, and the operational costs of SC. We show the contract price of vendor and buyer  $j$  by  $W_j$

## Vendor analysis

We consider the vendor as the leader in the VMI cooperation of the vendor-buyers chain as in [41]. The vendor manages the inventory system of the buyers. The existing costs are production,

distribution, ordering and inventory carrying costs. Production cost is resulted from the costs paid for producing a unit of the product ‘ $\delta$ ’, and the whole demand ‘ $y$ ’ (i.e.,  $y = \sum_{j=1}^n y_j$ ). Thus, the total production cost is  $\delta y$ . The distribution cost can be obtained from the multiplication of flow by transportation cost. The flow cost is composed of the direct mileage and the carrier contract cost per unit of product for buyer  $j$  ‘ $\theta_j$ ’, and the transportation cost is equal to the indirect cost such as mode of transport which is shown by ‘ $v_j$ ’, per unit demand for buyer  $j$  [42]. Thus, the distribution cost can be represented by ‘ $\theta_j y_j v_j y_j$ ’. Here, the value of ‘ $v_j$ ’ is taken as 0.5 as in the reference [42]. Therefore, the sum of distribution and production costs ‘ $PD_j$ ’, in order to satisfy ‘ $y_j$ ’, for buyer  $j$  is given as in Eq. 5:

$$PD_j = \delta y_j + 0.5 \theta_j y_j^2 \tag{5}$$

The vendor who is the one monitors the inventory of buyers. It replenishes the required stock for them. thus, the ordering cost per replenishment ‘ $S_{jvMI}$ ’ is the sum of the costs of monitoring the inventory status ‘ $S_s$ ’ and the ordering cost of buyer  $j$  ‘ $Sb_j$ ’, (as in Eq. 6 [8]):

$$S_{jvMI} = S_s + Sb_j \tag{6}$$

Since the replenished batch for buyer  $j$  is ‘ $Q_j$ ’, the total ordering cost for the addressed buyer is  $y_j(S_s + Sb_j)/Q_j$ . This is the same as the model in which vendor’s production rate is considered infinite [8].

The unit inventory carrying cost at vendor and buyer  $j$  is shown by ‘ $H_s$ ’, and ‘ $Hb_j$ ’. It is assumed that the vendor aggregates total inventory before sending to buyer ‘ $j$ ’. To the

policies of the EPQ model, the vendor has an average inventory equal to ‘ $\frac{Q_j(1-\frac{y_j}{P_j})}{2}$ ’, for

feeding buyer  $j$ . The average stock at the buyer will be ‘ $\frac{Q_j(1-\frac{y_j}{P_j})}{2}$ ’; for this reason, the SC members prefer to utilize the VMI strategy. In the VMI condition, the inventory carrying cost ‘ $H_{jvMI}$ ’, is equal to the sum of the corresponding costs at vendor and buyer the same as in Eq. 7 [5].

$$H_{jvMI} = H_s + Hb_j \tag{7}$$

Thus, the sum of the vendor’s ordering and inventory carrying costs on behalf of buyer ‘ $j$ ’ namely ‘ $OSM_j$ ’, is given in Eq. 8 [8].

$$OSM_j = (S_s + Sb_j) \frac{y_j}{Q_j} + \frac{Q_j(H_s + Hb_j)(1-\frac{y_j}{P_j})}{2} \tag{8}$$

Since it is the vendor which makes the products for each buyer, considering an identical cycle time ‘ $T$ ’ for the buyers, we can conclude that  $T = \frac{Q_j}{y_j}$  as in [43]. The whole vendor’s ordering and inventory carrying costs for all buyers ‘ $OSM$ ’, are given as in Eq. 9:

$$OSM = \sum_{j=1}^n \frac{(S_s + Sb_j)}{T} + \frac{(H_s + Hb_j) \times T \times y_j (1 - \frac{y_j}{P_j})}{2} \quad (9)$$

Where ‘ $T$ ’ is obtained from Eq. 10.

$$T = \sqrt{\frac{2 \sum_{j=1}^n (S_s + Sb_j)}{\sum_{j=1}^n y_j (H_s + Hb_j) (1 - \frac{y_j}{P_j})}} \quad (10)$$

We show the vendor profit obtained from its operating with buyer  $j$  by ‘ $PV_j$ ’. It is equal to the difference of vendor's revenue from buyer  $j$  ( $W_j y_j$ ) and the total costs involved, i.e., ‘ $PD_j + OSM_j$ ’. The vendor’s total profit ‘ $PV$ ’, is given as Eq. 11.

$$PV = \sum_{j=1}^n \left\{ W_j y_j - (\delta y_j + 0.5 \theta_j y_j^2) - \left[ \frac{(S_s + Sb_j)}{T} + \frac{(H_s + Hb_j) \times T \times y_j (1 - \frac{y_j}{P_j})}{2} \right] \right\} \quad (11)$$

### Buyer analysis

It is clear that the profit for buyer  $j$  shown by ‘ $Pb_j$ ’, in the VMI cooperation strategy is obtained from the difference between the obtained revenue through sales and the purchasing costs of the product from the vendor as in Eq. 12.

$$Pb_j = P(y_j) y_j - W_j y_j = (a_j - b_j y_j) y_j - W_j y_j \quad (12)$$

For a known value of revenue share ratio between the buyer  $j$  and the vendor  $PR_j = PV_j / Pb_j$ , and replacing the corresponding values from Eqs. 11 and 12, the contract price is obtained from Eq. 13.

$$W_j = \frac{a_j y_j PR_j - b_j y_j^2 PR_j + \delta y_j + 0.5 \theta_j y_j^2 + \left[ \frac{(S_s + Sb_j)}{T} + \frac{(H_s + Hb_j) \times T \times y_j (1 - \frac{y_j}{P_j})}{2} \right]}{(1 + PR_j) y_j} \quad (13)$$

It should be noted that ‘ $T$ ’ is obtained from Eq. 10.

### Objective functions of the model

We consider two objective functions for the addressed model. The former is maximization of SC profit ‘ $PC$ ’, can be obtained from Eq. 14 by considering the profits of the vendor and buyers.



$$f_1(PC) = PV + \sum_{j=1}^n Pb_j = \sum_{j=1}^n a_j y_j - b_j y_j^2 - (\delta y_j + 0.5 \theta_j y_j^2) - \left[ \frac{(S_s + Sb_j)}{T} + \frac{(H_s + Hb_j) \times T \times y_j \left(1 - \frac{y_j}{P_j}\right)}{2} \right] \tag{14}$$

The former is maximization of the variance of production periods which in turn results in less storage space requirement. It is given in Eq. 15.

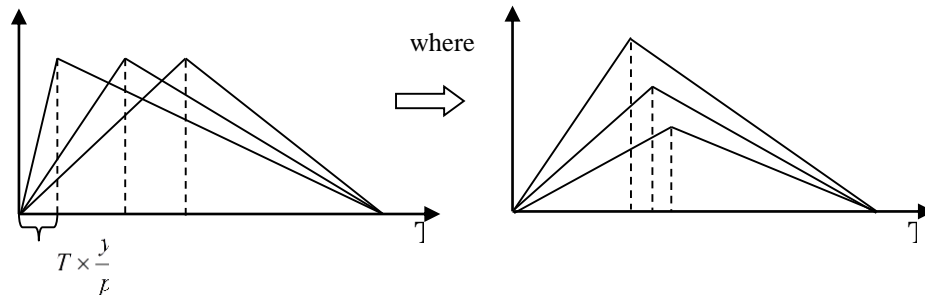


Fig. 2. Increasing the variance of production periods

$$f_2(S_T) = \frac{1}{n-1} \sum_{j=1}^n \left( T \times \frac{y_j}{P_j} - \frac{1}{n} \sum_{j=1}^n T \times \frac{y_j}{P_j} \right)^2 \tag{15}$$

**Mathematical model**

The optimal values for the sales quantity and production rate of buyer  $j$  represented by  $y_{j_{opt}}$  and  $P_{j_{opt}}$  can be obtained from the given model in Eqs. 16-20.

$$Max \{f_1(PC), f_2(S_T)\} \tag{16}$$

$$s.t. \quad y_{j_{min}} \leq y_j \leq y_{j_{max}} \quad \forall j = 1, \dots, n \tag{17}$$

$$\sum_{j=1}^n P_j = P \tag{18}$$

$$y_j \leq P_j \quad \forall j = 1, \dots, n \tag{19}$$

$$y_j \geq 0, P_j \geq 0 \quad \forall j = 1, \dots, n \tag{20}$$

Constraint (17) gives the sales quantity upper and lower bounds. Constraint (18) shows that the production rates for different buyers is equal to a whole production rate. Constraint (19) makes sure that the demand rate of each buyer is less than or equal to its corresponding production rate. Constraint (20) gives the status of the model's decision variables.

In the rest, the optimal sales price  $P(y_{j_{opt}})$ , is calculated from Eq. 21.

$$P(y_{j_{opt}}) = a_j - b_j y_{j_{opt}} \tag{21}$$

Then, the acceptable contract price  $W_{j_{opt}}$ , can be found by as in Eq. 22.

$$W_{j_{opt}} = \frac{a_j y_{j_{opt}} PR_j - b_j y_{j_{opt}}^2 PR_j + \delta y_{j_{opt}} + 0.5 \theta_j y_{j_{opt}}^2 + \left[ \frac{(S_s + Sb_j)}{T_{opt}} + \frac{(H_s + Hb_j) \times T_{opt} \times y_{j_{opt}} \left(1 - \frac{y_{j_{opt}}}{P_{j_{opt}}}\right)}{2} \right]}{(1 + PR_j) y_{j_{opt}}} \tag{22}$$

noting that  $T_{opt}$  is obtained from Eq. 23.

$$T_{opt} = \sqrt{\frac{2 \sum_{j=1}^n (S_s + S b_j)}{\sum_{j=1}^n y_{j_{opt}} (H_s + H b_j) \left(1 - \frac{y_{j_{opt}}}{P_{j_{opt}}}\right)}} \quad (23)$$

## Solution Heuristics

Here, we describe the three mentioned heuristics for the given model.

### MOPSO

Generally, PSO as a population-based algorithm was initially presented by Kennedy and Eberhart [44]. PSO simulates the social behavior of birds when searching for food. It considers a set of particles (or birds) flying through the sky in order to find the optimum point (i.e. food). The particle makes decisions based on its personal experience and social network (i.e., interaction with other birds). These two mechanisms make PSO a powerful optimizer. The main difference between PSO and other similar search heuristics is that the network's members are sharing data and this helps to the flow of optimization.

In PSO, each particle as a solution is given a random velocity dynamically adjusted based on the experiences obtained from an earlier fly. Three factors are important here: the particle's velocity, the best position reached so far '*pbest*' and the overall best position reached by the group or network of the particles '*gbest*'. We represent the number of particles by  $np$  and the position of particle  $i$  considering dimension  $j$  ( $j = 1, 2, \dots, 2n$ ) at iteration  $t$  by  $X_i^t = [x_{i,1}^t, x_{i,2}^t, \dots, x_{i,2n}^t]$ . The velocity of particle  $i$  at iteration  $t$  is shown by  $V_i^t = [v_{i,1}^t, v_{i,2}^t, \dots, v_{i,2n}^t]$ . Let  $Pb_i^t = [pb_{i,1}^t, pb_{i,2}^t, \dots, pb_{i,2n}^t]$  represent the best solution that particle  $i$  has discovered until iteration  $t$  and  $P_g^t = [p_{g,1}^t, p_{g,2}^t, \dots, p_{g,2n}^t]$  to be the best solution discovered until iteration  $t$ . A multi-objective version of PSO called MOPSO is considered for tackling the problem. The details of the given MOPSO heuristic can be shown in Algorithm 1.

#### Algorithm1: The steps of MOPSO

##### Step 1: Initialization

- Set  $t = 0$ ,  $np = 2n$ .
- Make randomly  $np$  particles and build the vector of particles as:  $X_i^0 = [y_1^0, \dots, y_n^0, p_1^0, \dots, p_n^0]$  in which  $y_{\min} \leq y_i \leq y_{\max}$ ,  $y_i \leq p_i$  and  $\sum p_i \leq P$ ; it should be noted that the continuous values for the positions are supposed to be randomly built.
- Randomly generate the initial velocities of particles,  $\{V_i^0, i=1, 2, \dots, NP\}$  where  $V_i^0 = [v_{i1}^0, v_{i2}^0, \dots, v_{i,2n}^0]$ . Initial velocities are made by utilizing the given formulae:

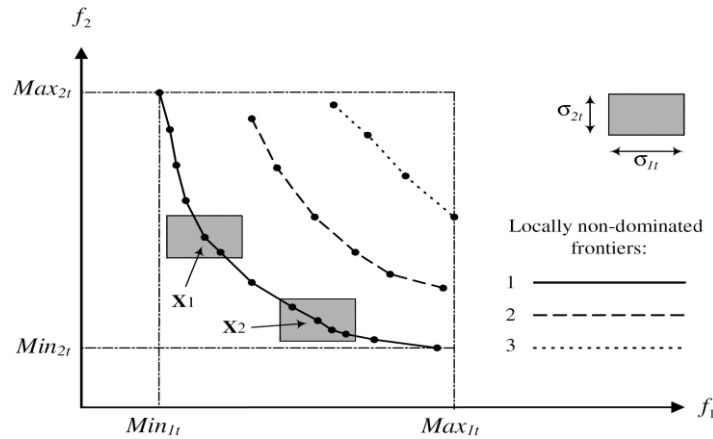
$v_{ij}^0 = v_{\min} + (v_{\max} - v_{\min}) \times r$  in which  $v_{\min} = -4, v_{\max} = 4$  and  $r$  is a random number between  $[0,$

$1]$ . Velocity values are bounded, namely  $v_{ij}^t = [v_{\min}, v_{\max}] = [-4, 4]$ , so that  $v_{\min} = -v_{\max}$ .

- Assess the particles of the swarm based on their objective functions  $f_{il}^0(\pi_{il}^0) \forall i = 1, 2, \dots, NP; l = 1, 2, \dots, m$ .
- Storing the Pareto solutions: In this section, the efficient solutions in the existing population are considered in order to be moved into an archive that holds the best NDS.
- For each particle of the swarm, set  $P_i^0 = x_i^0$ , where  $P_i^0 = [P_{i1}^0 = x_{i1}^0, P_{i2}^0 = x_{i2}^0, \dots, P_{i,2n}^0 = x_{i,2n}^0]$  with the best fitness value,  $f_i^{pb}$  for  $i = 1, 2, \dots, NP$ .
- Search for the *gbest* among the members of the swarm: If the problem has only one objective, then we only have one *gbest*. In MOOPs, the number of NDS which are placed on/near the Pareto front, is more than one; therefore, the existing NDS can be considered as the *gbest* providing its location data to the current particle. A hybrid method composed of PSO and GA is presented in the following in order to deal with the multi-objective nature of the optimization process. Several strategies to hold the diversity of NDS may be applied. Here, the concept of niche cubicle is hired in order to find the *gbest*. Niche cubicles are built per individual in the generation. A niche cubicle of an individual can be considered as a rectangular region whose center is the individual. Using Eq. 24, the size of the niche cubicle is recognized. Assuming a problem with  $m$  objectives. Let  $MAX_{lt}$  and  $MIN_{lt}$  to be the maximum and minimum of the  $l$ th objective at generation  $t$ . Now, the niche size for the  $l$ th objective  $(\sigma_{lt})$  can be obtained as in Eq. 24.

$$\sigma_{lt} = \frac{MAX_{lt} - MIN_{lt}}{m \sqrt{NP}} \quad ; l = 1, 2, \dots, m \quad (24)$$

Where  $NP$  is the number of particles. The niche cubicle is calculated at every generation. For more details in this regard, refer to [45]. Fig. 3 shows how the niche cubicles are made in a two-objective problem in which two niche cubicles are shown for two selected individuals  $X_1$  and  $X_2$ . The solution density of a niche cubicle can be measured by the number of individuals included in the cubicle. A solution that is in a less dense cubicle is likely to have a greater probability to stay in the next generation. For example, the niche cubicle of  $X_1$  is less dense than that of  $X_2$ ; thus,  $X_1$  will have a greater survival probability than  $X_2$ . Therefore, among all available particles in the Pareto archive, the one which enjoys the lowest density will be introduced as *gbest*.



**Fig. 3.** Niche cubicles in locally non-dominated frontier (NDF)

**Step 2:** Update the counter of iteration, i.e.,  $t = t + 1$ .

**Step 3:** Update inertia weight, i.e.,  $W = W^{t-1} \times \beta$  where  $\beta$  is decrement coefficient.

**Step 4:** Update velocity, i.e.,  $v_{ij}^t = W^{t-1} v_{ij}^{t-1} + c_1 r_1 (p_{ij}^{t-1} - x_{ij}^{t-1}) + c_2 r_2 (g_j^{t-1} - x_{ij}^{t-1})$ , where  $c_1$  and  $c_2$  are coefficients of acceleration,  $r_1$  and  $r_2$  are random numbers between  $[0, 1]$ .

**Step 5:** Update the position, i.e.,  $x_{ij}^t = x_{ij}^{t-1} + v_{ij}^t$ .

- Evaluate each particle of the swarm in iteration  $t$  considering the objective functions

$$f_{il}^t(\pi_{il}^t)$$

for  $i = 1, 2, \dots, NP$ ;  $l = 1, 2, \dots, m$ .

- Update the personal best. Each particle is assessed by using the permutation in order to

consider if the personal best improves or not. That is, if  $f_{il}^t \leq f_{il}^{pb}$ ;  $\forall l \in \{1, 2, \dots, m\}$  for  $i = 1, 2, \dots,$

$NP$ , then the personal best is updated by using  $P_i^t = X_i^t$  and  $f_{il}^{pb} = f_{il}^t$

- obtaining Pareto solutions: Efficient solutions of the existing population are moved into an archive which holds the best NDF.
- Update global best. The  $gbest$  of personal best should be found by using niche cubicle method.

**Step 6:** Stoppage criterion: If the number of iterations is higher than the maximum number of iterations or maximum CPU time, then stop; otherwise, the algorithms restarts from **step 2**.

Before giving the descriptions for the next two algorithms, we describe the solution representations for the applied algorithms. In MOPSO, particles play the role of solutions. In this article, the representation of each solution is so that a string with  $2n$  characters is designed; in the addressed string, the first  $n$  characters are the sales values of buyers while the second  $n$  characters are the vendor's production rates for the buyers. It is shown as  $\{(y_1, \dots, y_n, p_1, \dots, p_n) | y_{\min} \leq y_i \leq y_{\max}, y_i \leq p_i, \sum p_i \leq P\}$ . For example, suppose that we have

three buyers with sales values uniformly distributed as shown in  $y_1 \sim U[1600, 4800]$ ,  $y_2 \sim U[700, 1400]$ ,  $y_3 \sim U[1200, 3600]$ ; the production capacity for the vendor is considered to be  $P=18000$ . Furthermore, the constraints are  $y_i \leq p_i$ ,  $\sum_{i=1}^3 p_i \leq 18000$ ; the length of the particle is considered  $2n = 6$ . Three random numbers noting  $y_1, y_2, y_3$  is generated where  $y_{\min} \leq y_i \leq y_{\max}$ . Since the considered solutions are continuous, they should be converted to a discrete mode for being usable by the problem. Table 1 describes an example of the vector of particles  $X_i^t$  utilized by the MOPSO algorithm.

**Table 1.** Representation of solution by  $X_i^t$

Dimension, $j$						
Location	$y_1$	$y_2$	$y_3$	$P_1$	$P_2$	$P_3$
$x_{ij}^t$	4620	1333	1505	5801	2296	2640

## WSMOGA

In this algorithm, we have weighted aggregation. On the other hand, the two objectives are added considering a pre-defined weight. The steps of the WSMOGA are shown in Algorithm 2:

### Algorithm 2: The steps of WSMOGA

**1: Representation:** Encoding the solution.

**2: Initialization:**

- **Parameters tuning:** set the number of population (*popsiz*), *Max\_Gen* (the total number of generations), Probability of order crossover (*Pc*), Probability of mutation (*Pm*), Probability of reproduction (*Pr*).
- **Generating Initial population:** Generate an initial population *IPM* randomly.

**3:** Assign some weights to the objectives

**4:** counter ← 0

**5: while** counter < *Max\_Gen* or maximum CPU time **do**

**6: for**  $i=1$  to *popsiz* **do**

**7: Assessment:** Assess the fitness function of each solution; utilize and combine the min-max method with the weighting method for generating various Pareto solutions.

**8: Obtain Pareto solutions:** Efficient solutions are copied into an archive which keeps the best NDF obtained.

**9: Selection/Elitist:** Roulettewheel selection and Elitist selection are utilized to make the next generation.

**10: Crossover operation:** Select  $popsiz \times Pc$  pairs of parents of the population:

- a. Recognize the pairs of parents
- b. Employ the crossover operator to make two offspring
- c. Replace each offspring with the parents.

**11: Mutation operation:** Select  $popsiz \times Pm$  chromosome from the existing population, and mutate the individual bits

**12: Update Archive:** Efficient solutions are copied into an archive which keeps the best NDF obtained.

**13: Generate next generation:**

**14: End for**

**15:** counter ← counter + 1

**16. End while**

In this paper, similar to Behnamian et al. [46], the selection strategy is utilized as in Eq. 25.

$$F(x) = \sum_{l=1}^m w_l f_l(x)$$

$$\sum_{l=1}^m w_l = 1; w_l \geq 0 \quad (25)$$

In the aforementioned formulae,  $F(x)$  is, in fact, the aggregated function while  $w_l$  represents the  $l$ th non-negative weight corresponding to  $l$ th objective. One of the best methods of aggregation is the “dynamic weighted aggregation” (DWA) method since it is of good ability to take concave Pareto-fronts. It is defined for two objectives as in Eq. 26 in which  $t$  represents the  $t$  th population ( $t = 1, 2, \dots, N$ ) and  $R$  is considered to be 200.

$$(w_1(t), w_2(t)) = \left( \left| \sin \left( \frac{2\pi t}{R} \right) \right|, 1 - w_1(t) \right) \quad (26)$$

MOOPs may be solved by using scalarization in which the multiple objectives are transformed into a single objective one. The Min-Max method is used for this purpose. In the addressed method, minimizing the distance of each solution's objective  $f_i(x)$  from its best solution,  $f_i^*$  is considered.  $f^*$  is presented by  $f^* = (f_1^*, f_1^*, \dots, f_m^*)^T$  for different objectives. The result is shown in Eq. 27.

$$\text{Min} \left[ \sum_{i=1}^m \left( \frac{f_i(x) - f_i^*}{f_i^*} \right)^p \right]^{\frac{1}{p}}$$

$$\text{s.t. } X \in S; 1 \leq p \leq \infty \quad (27)$$

The values for  $p$  are usually from 1 to infinity using Tchebycheff norm [47]. Here, with combination of Min-Max and weighted methods, two problems are solved; the first is the mono-solution of Min-Max and the other is using weighting method as in Eq. 28:

$$\left[ w \left( \frac{f_1(x) - f_1^*}{f_1^*} \right)^p + (1 - w) \left( \frac{f_2(x) - f_2^*}{f_2^*} \right)^p \right]^{\frac{1}{p}} \quad (28)$$

In which  $f_1(x)$  and  $f_2(x)$  are the individual minima of each objective, and  $0 < w < 1$ .  $w$  represents the weight of the number of setups and usage rate. The objective functions' values are also normalized.

## NSGA-II

NSGA-II is widely used for MOOPs with acceptable performance [48]. Its pseudo-code is given in Algorithm 3. NSGA-II takes the fast non-dominated sorting mechanism to make sure for convergence.

<p><b>Algorithm3</b> The Pseudo-Code of NSGA-II</p> <p><b>step.1:</b> Set the parent vector <math>P = \emptyset</math>; the offspring vector <math>Q = \emptyset</math>; the collect vector <math>R = \emptyset</math> and <math>t = 0</math>.</p> <p><b>step.2:</b> Initialize the parent vector <math>P_0</math>.</p> <p><b>step.3: While</b> <math>t &lt;</math> the terminate generation number <b>do</b></p> <p>(1) Combine the parent and offspring population via <math>R_t = P_t \cup Q_t</math>.</p> <p>(2) Sort all solutions of <math>R_t</math> to get all NDFs <math>F =</math> fast-non-dominated-sort (<math>R_t</math>) where <math>F = (F_1, F_2, \dots)</math>.</p> <p>(3) Set <math>c</math> and <math>i = 1</math>.</p> <p>(4) <b>While</b> the parent population size <math> P_{t+1}  +  F_i  &lt; N</math> <b>do</b></p> <p>(a) Calculate crowding-distance of <math>F_i</math>.</p> <p>(b) Add the <math>i</math>th NDF <math>F_i</math> to the parent pop <math>P_{t+1}</math>.</p> <p>(c) <math>i = i + 1</math>.</p> <p><b>end while</b></p> <p>(5) Sort the <math>F_i</math> according to the crowding distance.</p> <p>(6) Fill the parent pop <math>P_{t+1}</math> with the first <math>N -  P_{t+1} </math> elements of <math>F_i</math>.</p> <p>(7) Generate the offspring population to <math>Q_{t+1}</math>.</p> <p>(8) Set <math>t = t + 1</math>.</p> <p><b>end while</b></p> <p><b>step.4:</b> the population in vector <math>P</math> is the NDS.</p>
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### Computational Experiments

Since the SC structure in this paper is close to that of [8], we have made a number of numerical examples inspired by those in [8]. The numerical examples are in three classes considering three, five, and eight buyers in the model. The buyer parameters are assumed to be constant, while the vendor parameters are changeable. The buyer parameters' values are in Tables 2 and 3. The parameter values of the vendor are in Table 4.

**Table 2.** Values of the buyers' parameters for three and five buyers

	three buyers (n=3)			five buyers (n=5)				
$j$	1	2	3	1	2	3	4	5
$Hb_j$	8	10	10	8	10	10	6	7
$Sb_j$	24	11	29	24	11	29	14	25
$a_j$	31	35	37	31	35	37	32	39
$b_j$	0.008	0.004	0.006	0.008	0.004	0.006	0.003	0.004
$y_{j_{min}}$	1600	700	1200	1600	700	1200	1500	900
$y_{j_{max}}$	4800	1400	3600	4800	1400	3600	3000	2700
$\theta_j$	0.004	0.008	0.005	0.004	0.008	0.005	0.005	0.007

**Table 3.** Values of the buyers' parameters for eight buyers

	eight buyers (n=8)							
$j$	1	2	3	4	5	6	7	8
$Hb_j$	8	10	10	6	7	12	13	14
$Sb_j$	24	11	29	14	25	12	30	22
$a_j$	31	35	37	32	39	33	36	38
$b_j$	0.008	0.004	0.006	0.003	0.004	0.005	0.007	0.006
$y_{j_{\min}}$	1600	700	1200	1500	900	700	800	1200
$y_{j_{\max}}$	4800	1400	3600	3000	2700	3500	4900	3000
$\theta_j$	0.004	0.008	0.005	0.005	0.007	0.005	0.007	0.006

**Table 4.** Vendor's parameters and values

Level	$H_s$	$S_s$	$\delta$	$P$
Low (-1)	3	5	5	18000
Up (+1)	15	40	10	27000

### Performance measures

The performance measurement of obtained solutions in MOOPS is done in different ways [49]. There are some metrics to compare algorithms in this matter such as the number of non-dominated solutions (NOS) and spacing metric [50]. The *diversification metric* may be also utilized to show the solution set spread. Here, we have utilized:

- NOS: It Counts the number of Pareto solutions in the front. Higher NOS means the DM can select among higher existing solutions.
- Spacing Metric (SM): This metric measures the uniformity of the solution spread as in Eq. 29.

$$S = \sqrt{\frac{\sum_{i=1}^n (d_i - \bar{d})^2}{n-1}} \quad (29)$$

In which  $d_i$  represents the Euclidean distance between consecutive solutions and can be defined as in Eq. 30.

$$d_i = \min_j \{ |f_1^i(x) - f_1^j(x)| + |f_2^i(x) - f_2^j(x)| \} \quad \forall i, j=1,2,3,\dots,n \quad (30)$$

And  $\bar{d}$  is the expected value of all  $d_i$ s.  $S = 0$  means that all solutions spread uniformity.

- Mean Ideal Distance (MID): MID is the closeness between Pareto solution and ideal point as in Eq. 31.

$$MID = \frac{\sum_{i=1}^n c_i}{n} \quad (31)$$

In which  $n$  is the number of non-dominated sets and  $c_i = \sqrt{f_1^2 + f_2^2}$ . The lower the value of MID is better.



• Area under Linear Regression Curve (ALC): It means finding the best smooth line in the NDS set. If the triangle area under the estimation line is less, the set of NDS pertinent to that line is of higher desirability.

### Parameters tuning

The parameters' values have an important role in the final solutions found by the meta-heuristic algorithms. We have used the try and error technique for this purpose. The results are given in Table 5.

**Table 5.** Parameters tuning result

NSGA-II		WSMOGA	
Parameter	Best Value	Parameter	Best Value
popsize	500	popsize	600
Max_Gen	750	Max_Gen	800
Crossover rate	0.7	Crossover rate	0.7
Mutation rate	0.2	Mutation rate	0.2
Reproduction rate	0.1	Reproduction rate	0.1
Elitist	top 20% of population	Elitist	top 20% of population

Regarding MOPSO, we have implemented tuning for only the inertia weight set with the value of  $w^{\circ} = 0.9$ , and the acceleration coefficients equal as  $c_1 = c_2 = 2$ . The decrement factor  $\beta$  was considered equal to 0.98 and the population size was considered to be twice the dimension number.

### Comparative results

In this subsection, all the proposed heuristics are compared with each other. Table 6 gives the values of the four given performance measures; the first 10 problems are considered as small-sized problems (SP), the second 10 problems are considered as medium-sized problems (MP) and the last 10 problems are considered as large-sized problems (LP).

From the results given in Table 10, it can be concluded that MOPSO outperforms other heuristics considering the given metrics for almost all numerical problems, especially for LPs. NSGAI generally outperforms WSMOGA for all the metrics. We use analysis of variances (ANOVA) for exactly comparing the heuristics from a statistical viewpoint. For this purpose, the results in Table 10 are normalized using the relative percentage deviation (RPD) as given in Eq. 32.

$$RPD_{ij} = \frac{|Alg_{sol}(ij) - Best_{sol}(i)|}{Best_{sol}(i)} \times 100 \quad \forall i = 1, \dots, n \quad (32)$$

$RPD_{ij}$  represents the RPD of algorithm  $j$  for problem  $i$ ,  $Alg_{sol}(ij)$  represents the metric's value for algorithm  $j$  in problem  $i$ ,  $Best_{sol}(i)$  represents the best value of the metric among all algorithms;  $n$  represents the number of problems. The ANOVA results are illustrated in Tables 7-10. Not that a P-value less than 0.05 results in not accepting the null hypothesis implying that the algorithms work differently.

**Table 6.** Results of the algorithms for all the test problems

No	NOS			Spacing			MID			ALC ( $10^4$ )		
	WSMOG	NSGA	MOPS	WSMOG	NSGA	MOPS	WSMOG	NSGA	MOPS	WSMOG	NSGA	MOPS
	A	II	O	A	II	O	A	II	O	A	II	O
1	6	6	6	9.12	9.02	9.02	62.08	62.08	62.08	0.227	0.219	0.210
2	6	6	6	1.03	1.03	1.1	50.05	44.05	39.05	0.183	0.177	0.194
3	10	10	10	1.22	1.12	1.02	59.21	58.54	59.19	0.194	0.191	0.182
4	8	8	8	3.96	3.56	3.16	70.66	71.48	70.66	0.254	0.249	0.249
5	9	9	10	5.45	5.45	5.07	83.98	83.98	83.98	0.271	0.264	0.279
6	9	8	8	5.21	4.33	3.48	69.10	63.68	59.49	0.522	0.519	0.506
7	11	10	8	4.17	2.54	1.83	68.41	64.71	55.31	0.752	0.745	0.739
8	10	7	11	3.91	3.44	2.71	69.16	67.34	48.16	0.520	0.511	0.507
9	8	9	11	5.9	4.32	1.05	64.71	66.51	57.60	0.646	0.634	0.620
10	11	9	9	5.37	3.51	2.03	61.20	60.46	68.67	0.620	0.513	0.368
11	18	18	21	4.03	4.01	3.9	95.60	95.85	95.09	0.420	0.410	0.368
12	10	10	12	2.34	1.66	0.76	100.17	96.84	87.45	0.535	0.456	0.454
13	18	18	21	3.28	3.35	3.12	90.85	89.81	89.38	0.595	0.516	0.372
14	9	9	9	4.26	4.26	4.26	95.31	95.31	95.31	0.311	0.268	0.205
15	11	14	14	2	2.43	1.67	93.45	92.63	88.55	0.812	0.783	0.679
16	10	12	12	4.68	3.08	2.62	103.43	100.23	84.24	0.279	0.257	0.171
17	12	11	15	4.40	4.18	4.32	97.82	94.61	88.07	0.326	0.319	0.142
18	13	13	14	4.51	2.72	1.53	91.31	99.40	91.01	0.465	0.426	0.194
19	11	13	13	4.24	4.65	1.42	99.65	100.61	96.41	0.661	0.582	0.483
20	11	14	15	5.88	4.23	3.31	98.29	96.85	85.30	0.488	0.407	0.339
21	35	42	49	9.63	9.39	7.16	523.57	527.84	502.84	8.248	8.154	8.140
22	58	56	51	17.38	10.33	6.34	752.52	629.58	565.30	7.166	7.044	6.966
23	48	50	50	17.7	6.06	5.43	815.01	552.33	574.06	6.457	6.358	5.258
24	45	44	55	16.76	10.15	7.34	704.47	672.93	668.70	7.564	7.469	6.322
25	46	51	58	17.93	11.23	6.36	817.00	713.93	677.45	11.357	9.355	7.222
26	43	47	58	17.10	15.34	14.00	814.40	738.28	767.78	9.228	8.909	9.109
27	49	52	54	17.93	15.27	12.47	794.92	738.52	717.56	10.875	11.778	7.704
28	39	50	55	18.25	14.44	10.73	797.28	704.16	746.86	12.882	12.881	9.811
29	46	51	58	17.29	15.05	13.80	778.65	750.08	743.43	10.506	9.421	8.319
30	50	55	56	19.31	15.27	13.39	808.97	787.23	718.31	9.999	9.017	7.874
Average	22.33	23.73	25.90	8.47	6.51	5.15	307.71	280.66	272.91	3.45	3.29	2.80

**Table 7.** ANOVA results considering NOS

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Factor	2	1572	786.17	10.26	0.000
Error	87	6664	76.60		
Total	89	8237			

**Table 8.** ANOVA results considering SM

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Factor	2	115566	57783	11.43	0.000
Error	87	439760	5055		
Total	89	555326			

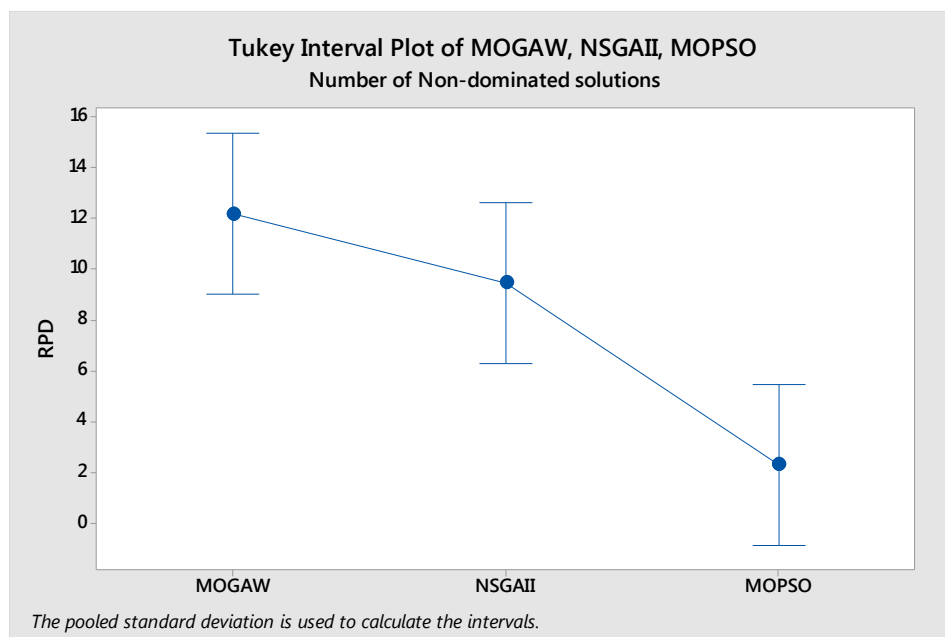
**Table 9.** ANOVA results considering MID

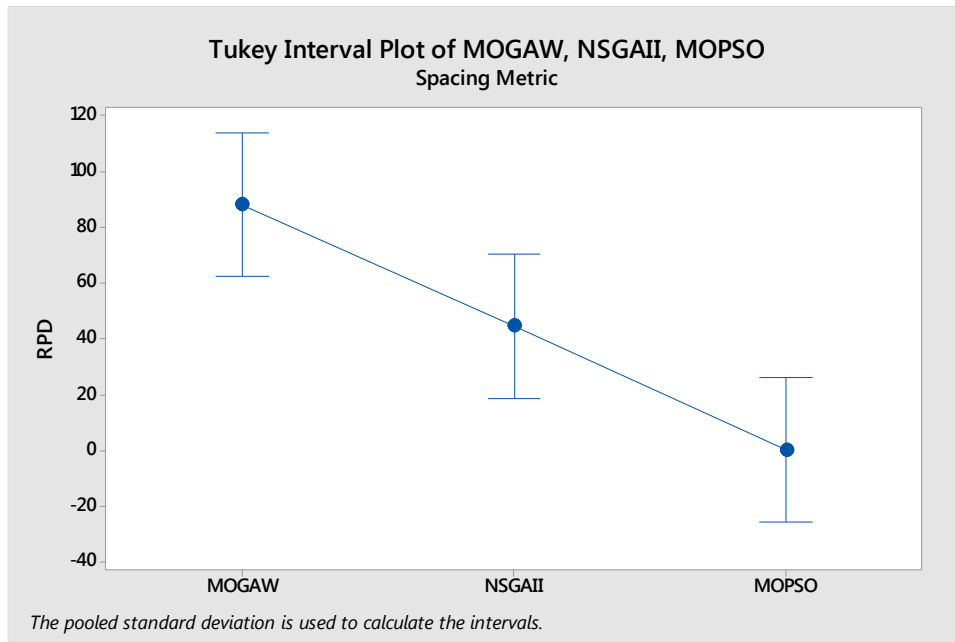
Source	DF	Adj SS	Adj MS	F-Value	P-Value
Factor	2	1871	935.70	11.38	0.000
Error	87	7156	82.25		
Total	89	9028			

**Table 10.** ANOVA results considering ALC ( $\times 10^4$ )

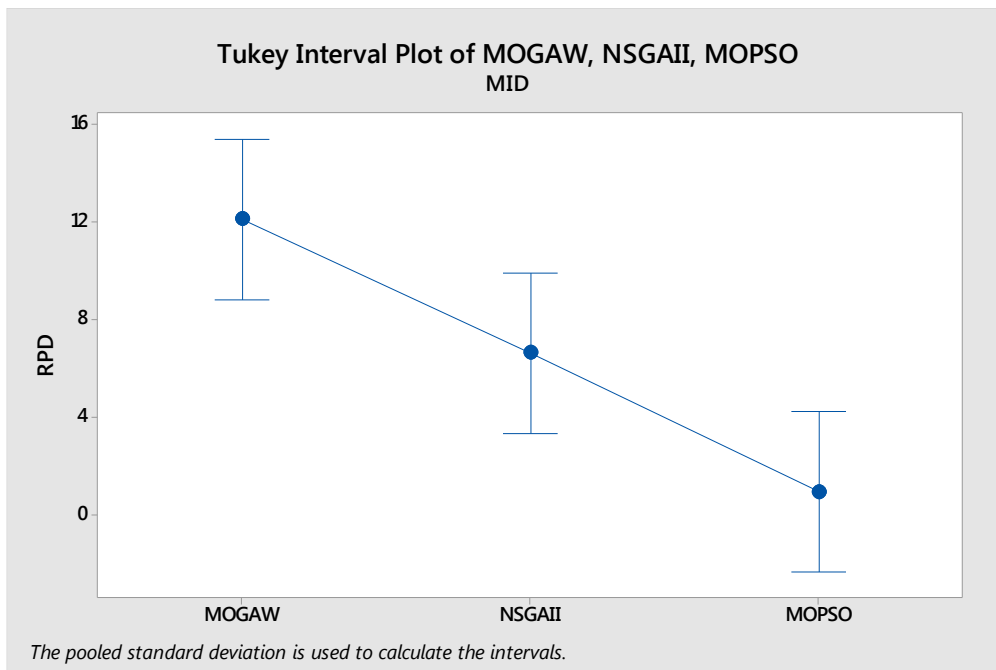
Source	DF	Adj SS	Adj MS	F-Value	P-Value
Factor	2	14260	7130.0	9.55	0.000
Error	87	64975	746.8		
Total	89	79235			

According to Tables 7-10, the heuristics have significant differences with respect to all metrics. The 95% Tukey simultaneous confidence intervals are computed and the results are given in Fig. 4-7. In Fig. 4, it is clear that based on the NOS metric, MOPSO has the best performance among all. NSGAII is relatively better than WSMOGA. Regarding the Spacing metric, Fig. 5 shows that MOPSO and NSGAII have the same quality. However, MOPSO statistically performs better than WSMOGA. According to Fig. 6, it can be known that the efficiency of the MOPSO and NSGAII is at the same level for the MID metric. Finally, regarding ALC, MOPSO outperforms the two other heuristics; NSGAII and WSMOGA are at the same level.

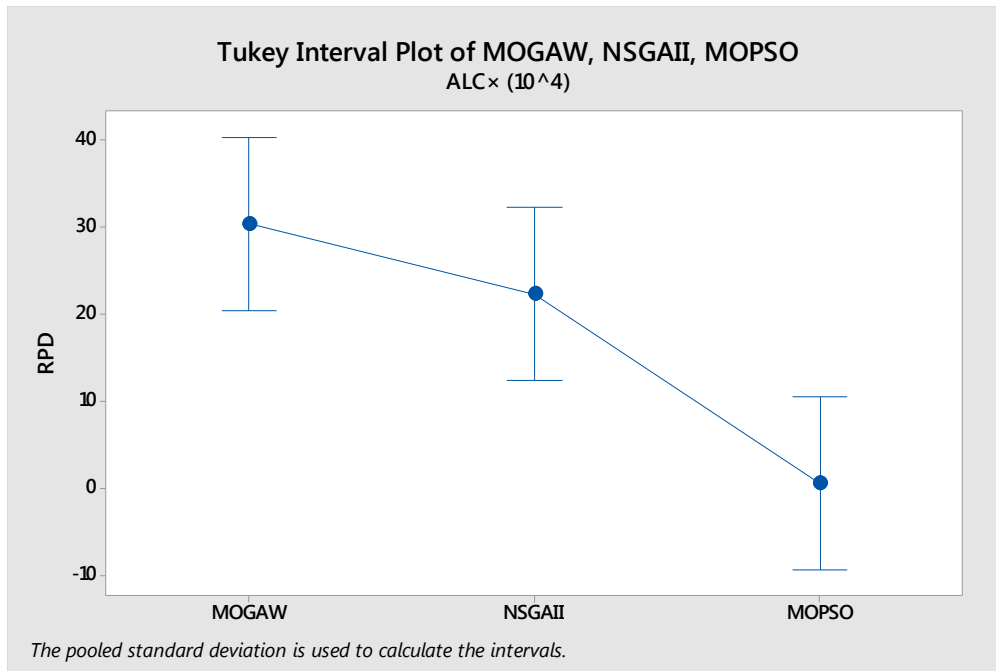
**Fig. 4.** Tukey simultaneous confidence intervals of the algorithms for NOS metric



**Fig. 5.** Tukey simultaneous confidence intervals of the algorithms for Spacing metric



**Fig. 6.** Tukey simultaneous confidence intervals of the algorithms for MID metric



**Fig. 7.** Tukey simultaneous confidence intervals of the algorithms for ALC metric

Furthermore, the given heuristics are run at equal iterations. On the other hand, each algorithm stops when there is no improvement in the best solution after a pre-specified iteration. [Table 11](#) presents the average CPU time using different heuristics.

**Table 11** the average CPU time when using different heuristics (per second)

Algorithms	PS (problems 1:10)	PM (problems 10:20)	PL (problems 20:30)
WSMOGA	9.749403	92.32132	593.2265
NSGAI	11.47660	119.2277	597.2863
MOPSO	10.93250	103.6383	601.4178

Although CPU times show some differences, such a difference is not considerable and the Pareto solutions generated from MOPSO outperforms the others.

### Conclusion and Suggestion

This article proposes a TSPMBSC model under the VMI strategy between the SC members. It was a development for the model given by [8] when the vendor replenished orders as EPQ. The model was formulated as a MOOP; the first objective was SC profit maximization while the second was the production periods variance maximization. These two objective functions Acted inversely. Considering the NP-hard nature of the problem, we gave three different heuristics MOPSO, WSMOGA and NSGA-II for the problem. Numerical examples showed that the proposed MOPSO-based heuristic outperformed the other two given heuristics. To show the efficiency of the given MOPSO heuristic and its superiority to the other two heuristics in terms of the MOOP comparison metrics.

Further research could be conducted to study more complex SCs with higher echelons. Demand can be assumed to be uncertain.

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