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'Anbūba:

A Medieval Instrument for Measuring Relative Ground level

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Abstract

'Anbūba is a simple instrument that has been in use during the medieval Islamic age. The Arabic term *'Anbūba* literally means the tube. The instrument was a tube devised in a particular way to determine the relative height between two lands for water transportation with the purpose of irrigating the farms and gardens. In the classic Islamic age, the description and instruction of such instruments were not generally registered in a textual format, regarding that they were a component of agricultural know-how. However, the information about *'Anbūba* has reached us in four independent treatises. This article introduces the instrument *'Anbūba*, discussing its construction and application. The article also includes the edition and translation of a fifteenth-century Persian treatise written exclusively about this instrument.

Keywords: *'Anbūba*, classical Islamic Age, History of hydrology, al-Karajī, water transportation.

Introduction

The history of hydrology is a topic deprived of sufficient research; the researches are scarce, particularly about premodern hydrological knowledge. However, historical research on hydrology is essential for the modern world; it could provide us with ideas about managing water resources, which have been a part of human culture for a long time (Wescoat, 2017, 1). The traditional hydrological methods had an environment-friendly approach that was in harmony with nature. It should be said that most of the academic works in the domain of the history of hydrology are primarily based on historical monuments: ancient wells, aqueducts, dams, and other constructions. On the other hand, the historical studies on texts about hydrology in the premodern world are almost absent. In this article, we present and analyze a simple instrument that could shed some light on the characteristics of hydrology as a science in the medieval age. The main issue addressed in this article is how the minor treatises are informative about technological progress in societies, and the case study is the instrument 'Anbūba.

The subject of irrigation and water supply is a major part of the history of technology in the Islamic civilization, which could be considered the history of hydrology. The construction of dams and canals, the survey of lands and aqueducts, and Water-raising machines are the main topics of this subject (Hill, 1996, 752). The well-known sourcebook for the science of hydrology in the classic Islamic age was *'Inbāṭ al-miyāh al-khafīyya (The Extraction of Hidden Waters)* by Abū Bakr Muḥammad al-Karajī (c.953-c.1029), an engineer scientist. This book covers various subjects and instruments related to hydrology; however, this article concentrates only on a simple instrument introduced by al-Karajī and also presented by three other independent texts after him. The instrument is devised to measure the relative ground level as preparation for water transportation from upstream to downstream. The relative ground level means the difference between the height of the water source and where the water will be consumed.

In our article, Section 1 introduces the texts in which the instrument 'Anbūba is described; section 2 describes the instrument 'Anbūba, its structure, and application. The edition of the treatise *Dar*

shinākhtan-i chigūnigī-i sanjīdan-i zamīnhā wa makānhā (*On the Knowledge of How to Survey the Grounds and Locations*) of manuscript collection 772, National Library of France is in the appendix.

1. The texts

In this section, we introduce texts in which the instrument 'Anbūba is either mentioned or described. In the case of Instruments, the place and date of the manuscripts are significant, because in most cases it provides information about the region and time in which that method was probably still used, or at least it was part of agricultural circulating know-how. So, we can say the instrument 'Anbūba has been in use from Karajī in the tenth century to Bahā'ī in the seventh century in the regions of Islamic civilization, principally in the territory of nowadays Iran. A survey of details and terminology of the texts shows that they are written independently about the same instrument. These texts could be considered as registering practical knowledge in a written format. The written form could be integrated into a scholarly corpus of knowledge.

1.1. On mentioning the scales by which two grounds are scaled

Karajī's *Inbāṭ al-miyāh al-khafīyya* is a comprehensive and almost unique source in hydrology during the medieval Islamic age. The Instrument 'Anbūba is introduced in the chapter "bāb dhikr al-mawāzīn allatī yūzinu bihā al-arḍayn (On mentioning the scales by which two grounds are scaled)" of this book, 'Anbūba is the first instrument of four that could be used in scaling the grounds. The three other instruments are 1) a level with a plumb, 2) a balance-like level, and 3) the other version of 'Anbūba (Karajī, n.d., P.23-24).¹

The first instrument is an isosceles triangle frame in which a plumb is connected to its midpoint's base. The base is on the top of the instrument. It is a very simple inclinometer (Figure 1). The second one is an isosceles triangle, but the base at the bottom of the instrument and a vertical pointer. The pointer, like the plumb, is always

1. For a revision of Karajī's book in Persian see Riḍa, I., Kūrus, Gh., Imam Shūshtarī, M. & Intizāmi (1971). "Āb va Ābyarī dar Īran'i Bāstān" (The water and Irrigation in Ancient Iran). Iran: The Ministry of the Water and Electricity; For levels other than 'Anbūba in Persian see Gholam-Hosseyin Rahimi, Karajī's Levels, in *Journal for the History of Science*, Volume 7, p.55-74.

perpendicular to the horizon line because of its weight (Figure 2). The last instrument is another version of the 'Anbūba. The difference is that 'Anbūba has two open ends, and this version has two closed ends. This version is made of glass instead of reed (Figure 3). All four instruments are for the same purpose, and the surveyor can use any of them.

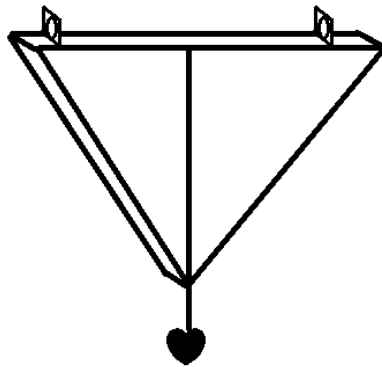


Figure 1. An illustration of the level with a plumb

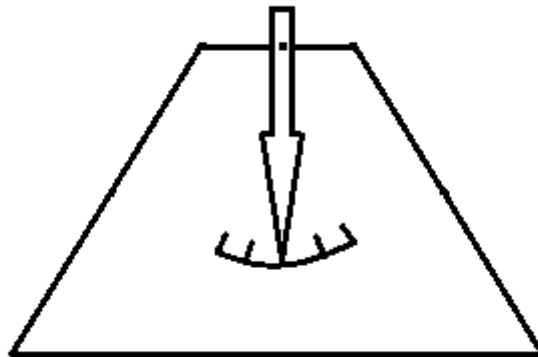


Figure 2. An illustration of the balance-like level



Figure 3. An illustration of the second version of 'Anbūba

1.2. On the knowledge of how to survey the grounds and locations

Collection no. 772 of the National Library of France is mainly known for its mathematical content. This collection includes several treatises written in Persian, among them a translation of Abū al-Wafā Būzjānī's (10 June 940-15 July 998) work is well-known because of its pivotal role in the research on geometric patterns in Islamic architecture. The only treatise in this collection that is not related to mathematics or astronomy is a treatise titled "*Dar shinākhtan-i chigūnigī-i sanjīdan-i zamīnhā wa makānhā (On the Knowledge of How to Survey the Grounds and Locations)*".

This collection is dated to the fifteenth century because it could have been written by Kūbanānī. Abū Ishāq al-Kūbunānī (d. after 886/1481), a Persian polymath, was born and grown in Kirmān, Iran, then he went to the north of Iran to be in the court of some rulers of the Ṭabaristān (Karāmātī, 2020, 1). If Kūbanānī is the scribe of this collection, which several pieces of evidence could confirm, then the manuscript should be written either in the city of Kirmān, where he studied and thought, or in the town of Sārī, where the rulers of Ṭabaristān patronized him. It is more probable that the treatise is written in the central cities of Iran, regarding the long history of water transportation in this region.

The most detailed report of 'Anbūba is in this Persian treatise. The description of the instrument is similar to Karajī's, but a survey of details shows that the treatise is not a translation of Karajī's work, and it is compiled independently. The most important difference is an introduction which is not mentioned in Karajī's book. Also, the

precise method for calculating the difference between heights is only mentioned in this treatise. There are other differences in some details, such as the material and size of the tube.

1.3. On knowing the elevations of the land during the construction of the well

The *Shinākhtan-i pastī wa bulandī-i zamīn bi hingām-i ihdāth-i qanāt* (*On Knowing the Elevations of the Land During the Construction of the Well*) is the eleventh treatise in manuscript collection no. 734 at the Goharshād Mosque Library in Mashhad. It is a collection of several treatises in various fields, most dedicated to the religious customs of Friday Pray. The scribe is Murtiḍā Ibn Moḥammad Ṣāfi Ḥosseyynī Tabrīzī, who wrote the manuscript in 1722. Oddly, the title of this Arabic text is in Persian.

This treatise is about three instruments, including 'Anbūba, briefly described and referred to as al-'Anbūba al-Mashhura (the famous tube). This adjective shows that the instrument was well-known, and it seems that two other instruments were introduced as alternatives. The two other instruments are al-Karajī's instruments: 1) the level with plumb and 2) the balance-like level.

1.4. On leveling the land for constructing aqueducts

Bahā' al-Dīn Muḥammad ibn Ḥusayn al-Āmilī or Shiyykh-i Bahā'ī, was a Persian polymath from the sixteenth-seventeenth century, who immigrated from the Levant to the Safavid capital, Isfahan (Kohlberg, 1988, 1). His book *Al-khulāṣat al-ḥisāb* (*The Summary of Arithmetic*) was a standard textbook for arithmetics, geometry, and algebra in religious schools in the Middle East for centuries. Surprisingly, among the mathematical content of the book, the seventh section is a brief exposition of surveying the lands for hydrological purposes. The section includes the methods and instruments for finding the height of the mountains and lands. The section is divided into three chapters: 1) determining the heights of the lands to conduct the canals, 2) determining the height of the mountains, and 3) determining the widths of the rivers and the depths of the wells.

In the first chapter, “on leveling the land for constructing aqueducts” (Fī wazn al-'arḍ li 'ijrā' al-qanawāt), Bahā'ī introduces al-Karajī's level, the level with a plumb, to find the relative height of two

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lands. He remarks that it is possible to use 'Anbūba and water instead of the level with a plumb. However, he did not add any explanation on the instrument 'Anbūba, perhaps because it was famous enough at that time (Bahā'ī, 1843, p.35). *The Summary of Arithmetic* was an influential textbook, so it was the subject of several commentaries. The instrument 'Anbūba is explained in at least two recent commentaries on it: *Kanz al-ḥisāb* by Farhād Mīrzā Mu'tamid al-Dula and *Albāb* Seyyed Muḥammad Javād dhihnī Tihirānī.

2. The Instrument 'Anbūba

The instrument 'Anbūba was frequently in use during the medieval age. The information about this instrument is based on the Persian and Arabic manuscripts that reached us. In this section, its structure and applications will be discussed.

2.1. The Instrument

'Anbūba is a simple and user-friendly instrument. It is a tube made of reed (or wood) with a hole in the middle of its length (Figures 4 and 17). Evidently, any cylindrical object made of local materials could serve as the tube for the instrument. The surveyor connects this tube by two ropes to the top of the two vertically implanted sticks. The ropes should be tightly stretched without any sag on them. The surveyor drops some water into the hole: If the water comes out from two sides of the tube, the device is horizontal, so the two lands have the same height; on the other hand, if the water comes out from one side, the side where the water comes out is lower than the other side.



Figure 4. An illustration of 'Anbūba

2.2. The Measurement Units

The units are an essential part of practical knowledge. A variety of units were used in Iran depending on time and region. In the treatise *On the knowledge of how to survey the grounds and locations*, the units for measuring are anthropic. These units were prevalent in the

medieval Islamic age in central region of Iran, and some are still in use. In this section, we introduce these units.

2.2.1 Digit (ʿiṣbaʿ in Arabic and Angosht in Persian)

The digit (or finger) is an ancient unit of length approximately equal to the breadth of a human thumb or finger. It is also depended on the larger unit Cubit (dhirāʿ, see 2.2.4). The finger originally was 1/24 of the cubit, but after the 16th century, it was 1/40 or 1/41 of the cubit. It was almost 2 centimeters (Hintz, 2003, 79).

2.2.2 Palm (Qabḍa in Arabic and Vajab-e baste or Musht in Persian)

The palm is a unit of length based on the breadth of the human hand and is equivalent to four digits, almost 9 cm (Hinz, 2003, 93).

2.2.3 Span (Shibr in Arabic and Vajab in Persian)

Span is a unit of length based on the distance from the tip of the thumb to the tip of the little finger in a spread hand. It was almost a half of a cubit, so approximately equal to 25 cm.

2.2.4 Cubit (dhirāʿ in Arabic and Arsh or Gaz in Persian)

The cubit is the other ancient unit of length that had various types and could be equal to various lengths, ranging from 48.25 centimeters to 82.9 centimeters, in different civilizations. The cubit was the main length unit in the Islamic world and was generally equivalent to the length of the human arm from elbow to fingertip. It was also equal to six or seven palms (Hinz, 2003, 81).

3.3. Application of ʿAnbūba

As mentioned before, the application of ʿAnbūba is not complicated. A surveyor, who wants to use it, should tie two long ropes to the ends of the instrument. He should choose two wooden sticks almost 1.5 to 2 meters long and grade them like a ruler. Then he should implant them vertically in two points (A & B in figure 5) which he wanted to get the height difference between them. The end of the ropes is tied to the top of the sticks (Figure 5).

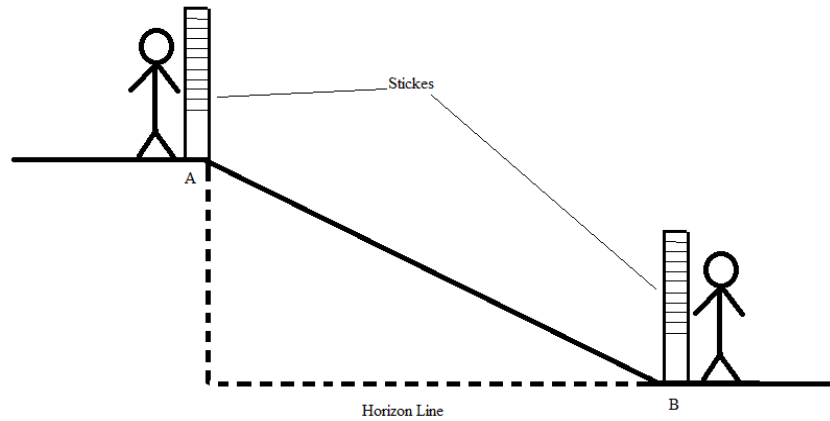


Figure 5

If the water comes from both ends of the tube, points A and B are at the same level. Otherwise, if the water comes from one side (point B in figure 5), the other side is higher (point A in figure 5). Then the difference between the two points should be measured. In this step, the surveyor on stick A pulls down the rope from C to E, where the water comes from two ends of the tube (see figure 6). The surveyor records the distance between C and E as the excess height of point A on point B.

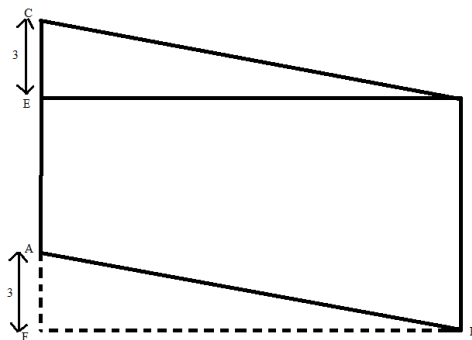


Figure 6

There is also a simple geometric image in the background of this method. ABCD is a parallelogram, and BF and DC are perpendicular on FC, so two triangles, ABF and CED, are equal and $AF=CE$. So the

grades between C (top of the stick A) and E (the new position of rope) are equal to the difference in height between points A and B.

As it is also considered in the treatise, the actual situation is more complicated because there are several lands with various heights from upstream to downstream. So the instrument should be used several times, and the result should be calculated by a procedure. The calculation seems rudimentary for a modern educated person, but for a farmer in the medieval age, the calculation process needs to be clarified. Hence, it was categorized into three cases, as follows.

3.2.1 Case A

In the case of A, points A and B are at the same level, line AB is horizontal, and the instrument would be level (figure 7). The surveyor does not need to read anything on sticks A or B. In this case, it is possible to transfer the water automatically, but it is a tough job (see section 3.3).

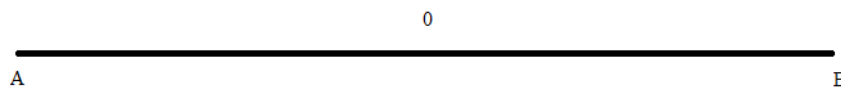


Figure 7

The surveyor needs to do a little calculation when there are ups and downs between two main points. For instance, if there are some hills between A and H (figure 8), the surveyor ignores the amounts of BC, DE, and FG because those line segments are horizontal. The suggested procedure is to add the values on stick A and the values on stick B separately and finally subtract the large number from the smaller number; the result is equal to the relative height of the stick with the large number. If we apply the concept of negative numbers, which were unknown at that age, to this calculation, then the grades on a stick with a large number are positive, and while the number of the grades on other sticks is negative, so the sum of the values could be calculated.

Positive values: $+2+2=+4$

Negative values: $-1-3=-4$

Final result: $+4-4=0$

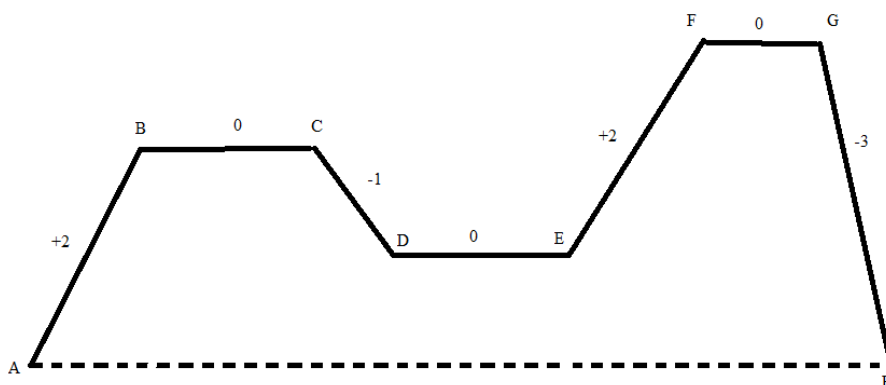


Figure 8

Another instance is when a valley (or trench) is between two main points A and D (figure 9). The calculation is in the same way.

Positive values: $+3$

Negative values: -3

Final result: $+3-3=0$

In these two instances, because points A and H are at the same level, the result shows that transferring water from A to H, or vice versa, is not possible readily.

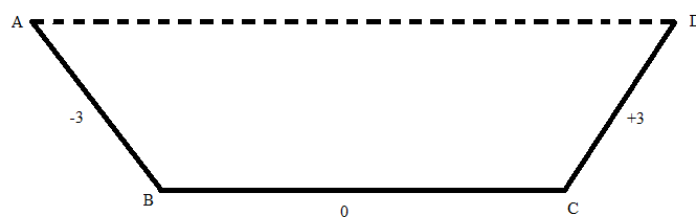


Figure 9

3.2.2 Case B

Suppose a farmer wants to transfer water from point A to point G (figure 10). In this case, the surveyor ignores horizontal line segments BC, DE, and FG, like in the previous case. According to the suggested

procedure; the values on each stick are calculated separately, and we have:

Positive values: +1

Negative values: $-3-3=-6$

Final result: $+1-6=-5$

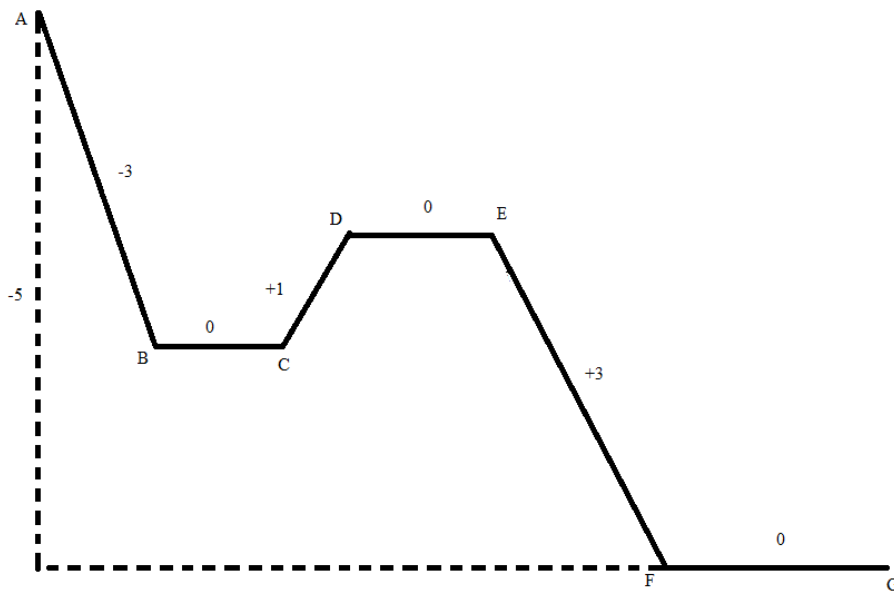


Figure 10

The final result shows that transferring water from point A to G is possible, and water can flow from A to G because point G is lower than point A.

3.2.3 Case C

Suppose a farmer wants to transfer water from point A to point H (figure 11). Again, the surveyor can ignore horizontal line segments BC, DE, and FG. According to the calculation procedure:

Positive values: $+3+3+4=10$

Negative values: -2

Final result: $+10-2=+8$

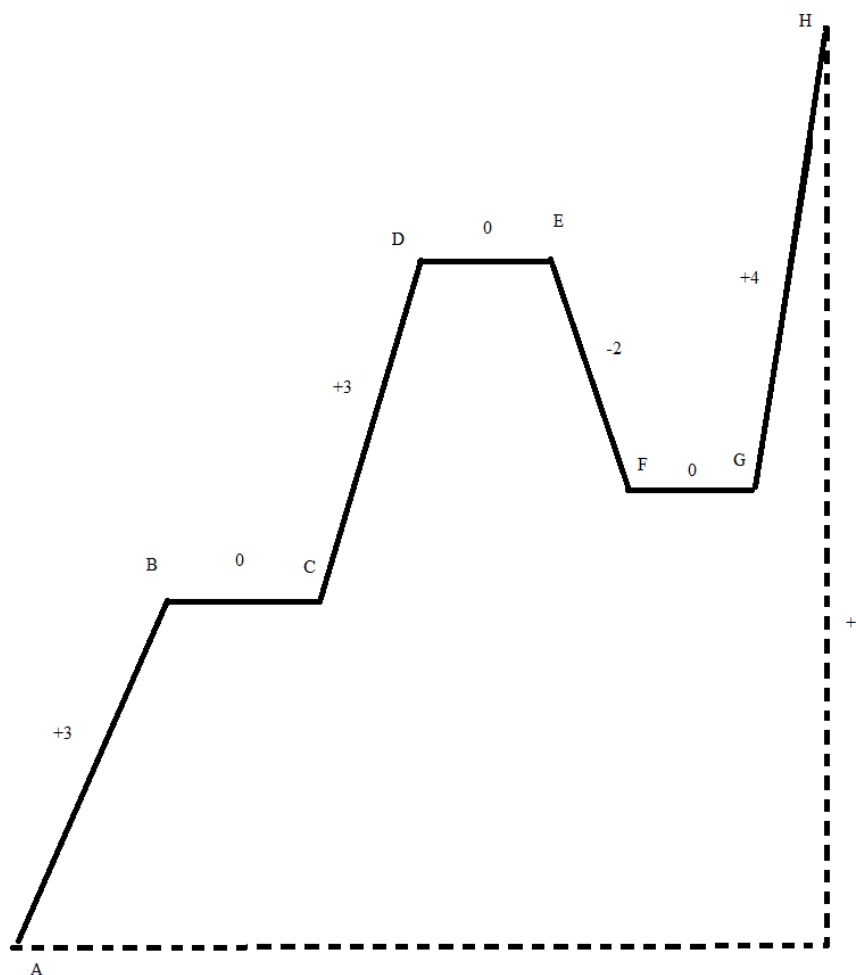


Figure 11

As the final result shows, transferring water from point A to G is impossible, and the water must be pumped to flow between A and H because point H is higher than point A. In the Persian treaties, the expression '*Amal-e Mīrābī*' is used for the process of water transportation in this case. In section 3.3, this expression is explained.

3.3. '*Amal-e Mīrābī*'

The Persian Treatise on 'Anbūba is distinguished from three others by including a theoretical passage on the possibility of water transportation by the relative height of source and destination. When

the source is higher, then it is possible to transfer water automatically; when the source and destination are at the same level, it is possible but not easily, and finally, when the destination is higher, it is only possible by what he called '*Amal-e Mīrābī*'. In this expression, '*Amal*' means work, and *Mīrābī* is the adjective for *Mīrāb*. *Mīrāb* is a combination of two other words *Mīr* is a contraction of the word *Amīr*, which means emir, the ruler; the other word *āb* simply means water. *Mīrāb* was a person who was responsible for the distribution of water between farmers in a village. The expression here refers to the application of the old water pumps, for instance, *Sāqiya* (*Dūlāb* in Persian). The *Sāqiya* uses buckets, jars, or scoops fastened to a vertical wheel. A shaft connects the vertical wheel to a horizontal wheel, which usually is rotated by animals (oxen, donkeys, and even humans). *Noria* (*Nā'ūra* in Arabic) is the other water pump that uses the kinetic energy of the water stream instead of the animals to rotate the pump. The *sāqiyah* and *Noria* are still used in the Middle East and North Africa (Nātiq, 2014, 1).

3. Conclusion

The instrument 'Anbūba is a simple instrument to measure the relative height of the lands for water transportation. The water only flows naturally from highlands toward lowlands, and the water should be pumped in other situations. This fact was known to laypeople, and the instrument was also as basic as this fact, so the question is why a written version of them was needed. The answer is that hydrology, as a science, was formed by the priority of application to knowledge. The farmers discovered and applied the primary methods, which were then generalized and developed lately in the scientific structure. The texts on the instrument 'Anbūba, introduced in this article, are pieces of evidence for such transmission from everyday know-how to the registered corpus of knowledge.

Appendix. The treatise and its Translation¹

The text

در شناخت چگونگی سنجیدن زمین‌ها و مکان‌ها

[۱] مراد به این، شناختن بلندی بعضی از زمین‌هاست از بعضی، یا پستی بعضی از بعضی، یا برابری بعضی به بعضی. و فایده این وقتی معلوم شود که خواهند که آبی را از موضعی به موضع دیگر نقل کنند. هر گاه که موضعی که آب از او نقل می‌کنند بلندتر از موضعی باشد که آب به او نقل می‌کنند احتیاج به عمل میرایی نباشد. و اگر برعکس این صورت باشد، ممکن نباشد که آب را نقل توان کرد. و اگر هر دو موضع برابر باشند در بلندی و پستی، نقل کردن آب ممکن باشد اما به دشواری و زحمت. و در این صورت از آلتی که بواسطه آن مطلوب توان رسید چاره نباشد.

[۲] و آسان‌ترین آلت‌هایی که از برای این کار وضع کرده اند آلتی است که آن را انبویه می‌گویند و آن تیری است راست که بندهاش گشاده باشد و در میان آن سوراخ کرده، چنانچه اگر آب از آن سوراخ بریزند به هر دو سرش بیرون تواند رفت. و درازی آن نی قریب پنج وجب. و اگر این آلت را از چوب بسازند هم به این وضع که گفته شد خوب‌تر باشد.

[۳] و طریقه سنجیدن زمین‌ها آن است که دو چوب برابر در درازی، که درازی هر یک نزدیک به درازی انبویه باشد، بگیرند و هر یک را ازین دو چوب به یک مقدار قبضه یا انگشت قسمت‌ها کنند و به موضع قسمت‌ها نشان‌ها سازند. بعد از آن ریسمان همواری که درازای آن قریب بیست و چهار گز باشد در آن انبویه درآورند و دو شخص را بفرمایند تا هر یکی یک سر ریسمان را با یکی از این دو چوب بگیرد و یکی در موضعی که آب را از آن موضع نقل خواهند کرد بایستد و دیگری در موضعی که آب را بدان موضع نقل خواهند کرد هم به این موضع بایستد.

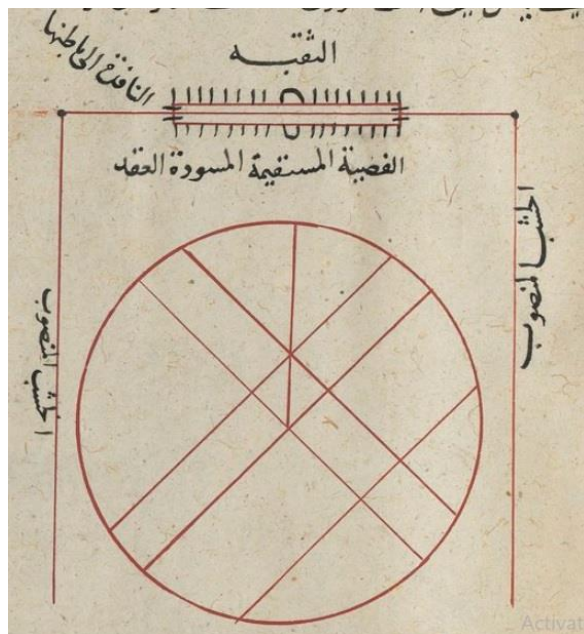
[۴] و هر یکی چوب خود را بر زمین راست کند و شاقولی از سر چوب بیاویزند تا معلوم شود که چوب راست ایستاده است یا نه. و انبویه را در میان ریسمان بدارند. بعد

1. "Dar shinākhtan-i chigūnig-i sanjidan-i zamīnā wa makānhā" (On the Knowledge of How to Survey the Grounds and Locations), collection 772, National Library of Franc, pp.63-64.

از آن، دو شخص دیگر را بفرمایند تا ظرف آب و پاره پنبه یا پشم بگیرد و قطره قطره آب به این پنبه پاره در سوراخ که میان انبویه است می چکاند تا آب از انبویه بیرون آید. و اگر چناچه آب از هر دو طرف انبویه بیرون آید، این هر دو موضع در بلندی و پستی برابر باشند. و اگر از یک طرف بیرون آید، آن موضع که در آن طرف است بیشتر از آن موضع دیگر باشد. پس آن شخص را که در طرف بلندتر باشد بفرمایند تا سر ریسمان را اندک اندک از سر چوب به زیر می آورد تا از هر دو جانب انبویه آب بیرون آید. بعد از آن، ما بین سر چوب و موضع ریسمان را ببینند که چه مقدارست از قبضه‌ها یا از انگشت‌ها؟ آنچه باشد، آن قدر بلندی موضع بلندتر باشد بر موضع پست تر. آنرا نگاه دارند.

[۵] آنگاه آن شخص را که در جانبی است که آب را به آن جانب نقل خواهند کرد، بفرمایند تا از مکانی که ایستاده است حرکت نکند. و آن شخص دیگر را بفرمایند تا از مکانی که ایستاده است به جانبی رود که آب را به آن جانب نقل خواهند کرد. و باز همین عمل بکنند. و اگر طرفی از طرف دیگر بلندتر باشد، از قبضه‌ها یا انگشت‌ها، آن را نگاه دارند. و هم چنین عمل می کنند و مقدار بلندی جانبی را بر جانب دیگر نگاه می دارند تا به آن موضع برسند که آب را به آنجا نقل خواهند کرد. آنگاه ببینند که مقدار بلندی‌ها که نگاه داشته اند از یک جانب است [یا] از هر دو جانب. و اگر از یک جانب باشد، آن جانب بآن قدر از آن جانب دیگر بلندتر باشد. و اگر بعضی بلندی‌ها از یک جانب باشد و بعضی از جانب دیگر: اگر هر دو بعض برابر باشند، هر دو جانب در بلندی و پستی برابر باشند. و اگر یکی از این دو بعض بیشتر باشند صاحب بیشتر از دو جانب، بلندتر از جانب دیگر باشد به قدر زیادتی مقرر شد که میان هر دو بعض است. و چون برابری یا قدر زیادتی مقرر شد، معلوم شد نقل آب ممکن است به آسانی یا ممکن نیست. اینست صورت آلاتی که ذکر رفت. و الله اعلم.

101/ 'Anbūba: A Medieval Instrument for ...



The translation:

[1] The main goal of that job is knowing the height of some lands from some [others], or the lowliness of some, or the equality of some related to some. This [job] benefits when they want to transfer water from one place to another. If the position that the water transferred from it is higher than the position that the water would be transferred, there is no need for irrigation. Vice versa, it is not possible to transfer water. If both positions are equal in height, it is possible to transfer water, but it is not easy. Hence should use a tool to achieve the goal in this case.

[2] The most straightforward tools they have set up for this job, [measuring the heights], are called 'Anbūba (the tube), and it is a piece of reed with a hole in the middle, so if they drop water into that hole, water can go out through both sides. The length of the reed should be about five spans. If this instrument is made of wood, it would be better.

[3] The way to measure the [height of] the lands [relative to each other] is to take two sticks of equal length, close to the length of the

tube, grade both sticks by palm or digit, and mark the divisions on both. Then, they get a rope about twenty-four cubits long and connect it to the tube, and they send two men, each one should connect one end of the rope [to] one the stick. One man should be at the position where water will transfer from and another at the position to which the water will be transferred into.

[4] They should implant their sticks into the ground vertically and connect a plumb from the stick's top to determine whether it is vertical. Then, they ask the other two men to take a water container and a piece of cotton or wool and drop water into the tube's hole until the water comes out. If the water comes through both sides of the tube, those [two] men are at the same height. If it comes through one side, the other position is higher. So, the person in the higher place should lower the rope's end on the stick until the water comes out from both sides of the tube. Next, they measure how many palms or digits are between the head of the stick and the position of the rope. What is measured is the difference in the height of the higher position to the lower. They record it.

[5] Then, they tell the person on the side where the water would be transferred not to move. Then, they tell the other person to go from his standpoint to where the water would be transferred. They measure the height again. They record it according to the palms or digits if one side is higher than the other. They keep going through the procedure. They record the height of one side related to the other side until they reach the position where they will transfer the water. Then, they notice that the records are on one side [or] on both sides*. If they are on one side, that side is higher than the other in proportion to [sum of] the records. [What should you do] if some records are on one side and others on the other*: if [the sum of] both are equal, both sides are equal in height. If [the sum of] one of them is more, that one is higher than the other in proportion to [the absolute value of] the difference between records. If equality or difference is known, it becomes clear that water transfer may or may not be easy. These are the pictures of the tools mentioned. Moreover, God knows [better].

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