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RESEARCH PAPER

Scenario-Based Hybrid Robust-Stochastic Programming for Coordinated Scheduling of Electricity & Natural Gas Supply Systems

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Abstract

Stochastic and robust optimizations have been considered as two different views of stochastic problems. While robust optimization takes optimization in the worst case, stochastic optimization regards no conservative view and merely focuses on expected value. However, a unilateral view of stochastic problems does not apply to most real problems. In this article, a hybrid robust and stochastic approach is proposed for optimization problems under uncertainty. Our major contribution is presenting different conservative levels in solving an optimization problem using a Hybrid Robust and Stochastic Optimization approach. To this end, we cluster uncertain parameters into different clusters using Latin Hypercube Sampling and k-Means clustering tools; having established various numbers of clusters of uncertain parameters, different clustering criteria and a Multi-Criteria Decision Making (MCDM) tool is employed to determine the optimal number of clusters of uncertain parameters. Then, a hybrid energy optimization model under uncertainty is applied to coordinate the scheduling of natural gas-fired electricity generation units and gas supply units (gas refinery) under natural gas and electricity demand uncertainty, with known probability distribution and uncertain parameters having different levels of conservatism. The results indicate that while no special trend is evident in the execution time as the number of clusters increases, the optimal value is decreased.

Keywords:
Coordinated Scheduling of Electricity & Natural Gas Supply Systems, Hybrid Robust and Stochastic Optimization, K-means Clustering, Latin Hypercube Sampling, Scenario Generation.

Introduction

The interdependency of electricity and natural gas systems is escalating [1-6]. This is more obvious in power systems with a considerable amount of gas-fired units where electricity generation scheduling can be directly and significantly influenced by natural gas prices and/or gas production costs [2]. Coordinated scheduling of power and natural gas systems has caught the attention of many studies [7].

On one hand, Stochastic optimization presumes that there is complete knowledge about the underlying uncertainty through a known probability distribution and minimizes a cost function [8]. There are a plethora of studies pertaining to coordinated scheduling in this approach. An integrated operational model for electricity and natural gas systems under uncertain power supply is proposed in [9] in which two-stage stochastic programming is applied to co-optimize day-ahead and real-time dispatch of both energy systems, aiming at minimizing the total expected cost. A short-term stochastic model is proposed to study the coordination of

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constrained natural gas and wind energy units in power systems in [4]. In this model, the natural gas network constraints, emission limits, and wind energy variability are integrated and solved by a Mixed Integer Programming (MIP) approach.

On the other hand, in robust optimization, it is assumed that the decision maker has no distributional knowledge about the underlying uncertainty, except for its support, and the model minimizes the worst-case cost over an uncertainty set [8, 10-12]. There is a wide array of works regarding coordinating the scheduling of power and gas systems, treating uncertain variables as robust. In [13] a tri-level optimization model is proposed to address the vulnerability of coupled gas-electric networks against malicious line interdictions by providing the optimal strategies for preventive reinforcement, increasing the resilience of the energy supply, and decreasing the operational cost of the interdependent energy systems. A robust security-constrained unit commitment (robust SCUC) model is proposed in [1] to increase the operational reliability of integrated electricity-natural gas systems against possible transmission line outages. In fact, this model optimizes electricity generation and natural gas allocation for the next day, while providing robust feasible controls over a range of possible contingency sets. In this work, a two-stage robust convex optimization model against N-k outages is derived. An integrated robust optimization model is developed in [14] for the day-ahead scheduling of electricity and natural gas systems while considering electrical load and wind generation uncertainties. In addition, an energy hub, which encompasses gas-fired units, power-to-gas facilities, and natural gas storage, is considered a large-scale electrical energy storage.

Inspired by the approach introduced in [15] for the hybridization of stochastic and robust optimization models for unit commitment in a power system, in this study, a model is proposed for the coordinated scheduling of power and gas systems.

This study intends to model and cope with an uncertain environment where the scheduling of both electricity and gas supplies is coordinated. We try to capture the uncertainty of electricity and gas demands. For this aim, generating the most likely scenarios, both stochastic and robust optimization approaches are taken into account.

The rest of this article is as follows. In section 2, the literature review is presented. In section 3 coordinated scheduling of electricity and natural gas supply systems model is represented. Section 4 is dedicated to presenting our HRSO method to solve the mentioned model. Scenario generation and clustering are represented in section 5 by introducing clustering methods and the corresponding clustering performance criteria. Simulation results are presented in section 6. In the last section, concluding remarks are provided.

Literature Review

In an array of real-world applications, the underlying probability distribution cannot be accurately determined, even when historical data are available. This distributional ambiguity might result in highly suboptimal decisions. In such cases, an alternative approach to handle such an issue is to apply distributionally robust stochastic optimization (DRSO) or distributionally robust optimization (DRO) which assumes the underlying probability distribution is unknown but lies in an ambiguity set of distributions. Many existing studies on DRO focus on how to construct the ambiguity set and how to transform the resulting DRO into equivalent models such as mixed-integer programming [16, 17]. DRO is a generalized form of a robust optimization approach [18]. This approach aims to minimize the expectation cost in the worst-case distribution rather than a particular worst-case in the traditional robust optimization. Indeed, DRO, as expressed in [8, 17, 18], has been developed as an intermediate approach to bridge the gap between the specificity of stochastic programming and the conservatism of robust optimization, and to realize a trade-off between economics and robustness. This relatively new approach has the merits of both stochastic and robust methods.

DRO applications in coordinated scheduling of gas and power systems have been seen. A data-driven distributionally robust dispatch model for the integrated electricity and natural gas system is proposed in [19]. The proposed model is a two-stage optimization one in which the day-ahead total cost for the integrated system is regarded as the optimization objective. The predicted wind power information is also taken into consideration in the first stage and the output adjustment of thermal generation units and the supply regulation of natural gas source are included in the second stage (real-time dispatch). The optimization results from stochastic, robust, and distributionally robust optimization methods are obtained and analyzed under 1,000,000 test probability distributions generated by Monte Carlo simulation. The analysis results indicate that distributionally robust optimization shows a better expected performance in both averaged and worst-case probability distribution scenarios compared with the stochastic and robust optimization methods. In [20] a two-stage distributionally robust optimization model, for which an ambiguity set is defined to capture the distribution information of wind power uncertainty, is proposed to study the coordination optimization scheduling for this multi-energy coupled system considering wind power uncertainty. The first stage of this model aims to minimize total operation cost in the base case while the second stage minimizes the expectation of wind power curtailment under the worst-case distribution of wind power forecast error defined by an ambiguity set. Then, the superiority and practicability of the proposed DRO model is demonstrated. In [21], a robust day-ahead scheduling model is proposed for electricity and natural gas system, which minimizes the total costs including the cost of fuel, spinning reserve and operational risk while the feasibility for all possible scenarios within the uncertainty set are ensured. For this aim, a risk-averse adjustable uncertainty set approach is proposed to mitigate the conservatism of robust optimization. In order to overcome distribution ambiguity, the ambiguity set is constructed by using a Wasserstein–Moment metric. Then, operational risk is the expected value under the worst-case distribution within such an ambiguity set. This approach has been at the center of attention by several authors in capturing uncertainty in power system problems while simultaneously considering gas supply factors or limitations [22-29].

Nowadays, the Hybrid Robust Stochastic Optimization (HRSO) approach has a more dominant role in dealing with uncertainty in power system challenges [30-37]. As far as we are concerned less attention has been paid to the HRSO approach for addressing the scheduling of coordinated power and gas networks. In [38] a proposed hybrid model handles the continuous uncertain variables of electricity market prices and discrete uncertainty sources of units' availability/unavailability, respectively. The uncertainty of electricity market prices is modeled by bounded intervals instead of probability distributions, aiming to derive a more tractable optimization model. Conservatism against uncertain electricity market prices is adjusted by a certain parameter, the so-called budget of robustness. Furthermore, a Markov chain approach is proposed to consider the chance of return for failed units together with the forced outage rate, in each hour of the scheduling period to produce a set of scenarios modeling the availability/unavailability of units. Simulation results demonstrate the higher effectiveness of the proposed hybrid model compared to deterministic, stochastic, robust, and CVaR-based stochastic models. A similar approach is applied in [39] by constructing scenarios from the probability distribution function.

In this study, In the first stage, the coordinated scheduling of power and gas supply systems is modeled into a MILP. Then, stochastic variables (electricity and gas demands) are quantified and uncertainty propagation is undertaken by the LHS method in order to provide the required scenarios of the problem. Next, having clustered scenarios by the k-means method to different numbers, the two most common clustering evaluation indices are applied to examine the optimal numbers of clusters. Ultimately, a simulation of the model is conducted for different numbers of clusters and the results are compared. The flowchart of the proposed approach is shown in Figure 1.

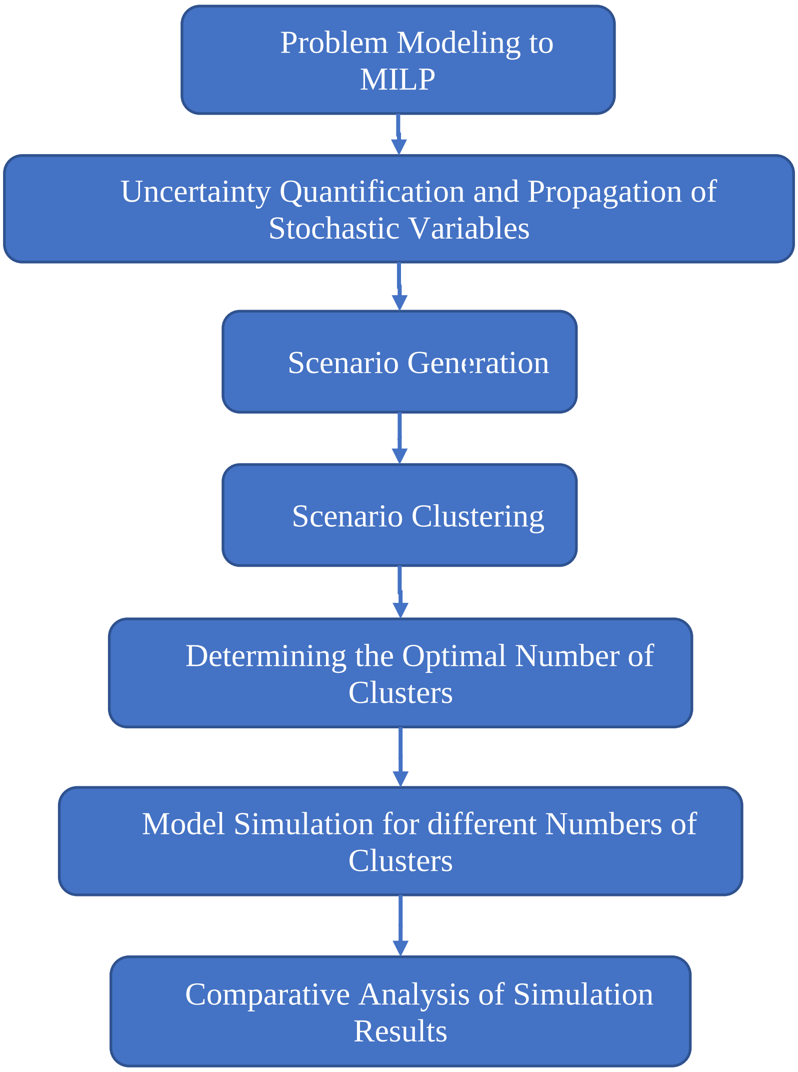

Fig. 1. Flowchart of Proposed Approach

Coordinated Scheduling of Electricity and Natural Gas Supply Systems Model

Scheduling power-generating units, widely known as unit commitment, is one of the problems in power systems. A great deal of research has been carried out in this context in the literature. It focuses on deciding the on-off status of several electricity generation units in 24 hours as well as their electricity generation amount.

Unit commitment of gas-fueled generating units is dependent upon gas flow supply. If gas flow reliability is not maintained, the reliability of power generated from gas-fueled generation units will be at risk.

However, some studies focus on the relationship and mutual impacts of these two networks as coordinated scheduling of electricity and natural gas supply systems [3, 6]. In this section, we introduce the coordinated scheduling of electricity and natural gas supply systems model developed in [40], with some changes to consider scenarios (notion ω).

$$\text{Minimize } TC = \sum_{t=1}^{24} \sum_{Ne \in Neg} \left[C_{Ne}^{SU} su_{Ne,t} + C_{Ne}^{SD} sd_{Ne,t} + C_{Ne}^{FC} F_{Ne,t}(P_{Ne,t}) \right] + \sum_{\omega \in \Omega} \pi_\omega \left[\sum_{t=1}^{24} \sum_{Ne \in Nel} C_{Ne}^{PLS} pls_{Ne,t,\omega} + \sum_{t=1}^{24} \sum_{Ng \in Ngl} C_{Ng}^{GLS} gls_{Ng,t,\omega} \right] \tag{1}$$

Subject to:

$$gp_{Ng,t,\omega} - gel_{Ng,t,\omega} - gnl_{Ng,t,\omega} + gls_{Ng,t,\omega} + gf_{Ng\to Ng,t,\omega} - gf_{Ng\to Ng,t,\omega} = 0;\ \forall\, Ng,t,\omega \tag{2}$$

$$gnl_{Ng,t,\omega} + gls_{Ng,t,\omega} = gload_{Ng,t,\omega} \quad ;\ \forall\, Ng\, \epsilon Ngl,t,\omega \tag{3}$$

$$\sum_k \left(m_{pl,k}.\, gfl_{pl,k} + b_{pl,k}.\, o_{pl,k}\right) \geq C_{pl}^2.\left(\pi_{Ng,t,\omega} - \pi_{Ng,t,\omega}\right)\ ;\ \forall\, pl\ \epsilon\ APL, (Ng,Ng)\epsilon\ pl,t,\omega \tag{4}$$

$$\sum_k \left(m_{pl,k}.\, gfl_{pl,k} + b_{pl,k}.\, o_{pl,k}\right) = C_{pl}^2.\left(\pi_{Ng,t,\omega} - \pi_{Ng,t,\omega}\right); \forall\, pl\ \epsilon\ PPL, (Ng,Ng)\epsilon\ pl,t,\omega \tag{5}$$

$$o_{pl,k}.\, \underline{gfl}_{pl,k} \leq gfl_{pl,t,\omega,k} \leq o_{pl,k}.\, \overline{gfl}_{pl,k} \quad ;\ \forall\, \omega,k,pl \tag{6}$$

$$gf_{Ng\to \acute{Ng},t,\omega} = \sum_k gfl_{pl,t,w,k} \quad ;\ \forall\, pl,\omega,\left(Ng,\acute{Ng}\right)\epsilon\ pl \tag{7}$$

$$\sum_k o_{pl,k} \leq 1 \quad ;\ \forall\, pl \tag{8}$$

$$\left(pr_{Ng}^{min}\right)^2 \leq \pi_{Ng,t,\omega} \leq \left(pr_{Ng}^{max}\right)^2 \quad ;\ \forall\, Ng,t,\omega \tag{9}$$

$$\left(\beta_{Ng}^{min}\right)^2 \leq \frac{\pi_{\acute{Ng},t,\omega}}{\pi_{Ng,t,\omega}} \leq \left(\beta_{Ng}^{max}\right)^2 \quad ;\ \forall\, pl\ \epsilon APL,\left(Ng,\acute{Ng}\right)\epsilon pl,t,\omega \tag{10}$$

$$GP_{Ng}^{min} \leq gp_{Ng,t,\omega} \leq GP_{Ng}^{max} \quad ;\ \forall\, t,\omega, Ng\epsilon Ngp \tag{11}$$

$$\sum_{Ne} P_{Ne,t,\omega} = 0 \quad ;\ \forall\, t,\omega \tag{12}$$

$$P_{Ne,t,\omega} + pls_{Ne,t,\omega} = -eload_{Ne,t,\omega} \quad ;\ \forall\, Ne\ \epsilon\ Nel,t,\omega \tag{13}$$

$$pf_{el,t,\omega} = PTDF_{el\times Ne}.\, P_{Ne,t,\omega} + PSDF_{el\times el}.\, \alpha_{el,t,\omega} + DCDF_{el\times el}.\, pf_{el,t,\omega}^{DC} \quad ;\ \forall\, el,t,\omega \tag{14}$$

$$\alpha_{el}^{min} \leq \alpha_{el,t,\omega} \leq \alpha_{el}^{max} \quad ;\ \forall\, el,t,\omega \tag{15}$$

$$pf_{el,t,\omega}^{min} \leq pf_{el,t,\omega} \leq pf_{el,t,\omega}^{min} \quad ;\ \forall\, el,t,\omega \tag{16}$$

$$I_{Ne,t}.\, Ming_{Ne} \leq P_{Ne,t,\omega} \leq I_{Ne,t}.\, Maxg_{Ne} \quad ;\ \forall\, Ne\epsilon Neg,t,\omega \tag{17}$$

$$I_{Ne,t-1} - I_{Ne,t} + su_{Ne,t} \geq 0 \quad ;\ \forall\, Ne\ \epsilon Neg,t \tag{18}$$

$$I_{Ne,t} - I_{Ne,t-1} + sd_{Ne,t} \geq 0 \quad ;\ \forall\, Ne\ \epsilon Neg,t \tag{19}$$

$$-DR_{Ne} \leq P_{Ne,t,\omega} - P_{Ne,t-1,\omega} \leq UR_{Ne} \quad ;\ \forall\, Ne\ \epsilon Neg,t,\omega \tag{20}$$

$$\sum_\tau \left(1 - I_{Ne,\tau,\omega}\right) \leq minut_{Ne}.\left(1 - su_{Ne,t,\omega}\right) \quad ;\forall\, t \leq \tau \leq t + minut_{Ne}, \forall\, Ne\ \epsilon Neg,\omega \tag{21}$$

$$\sum_\tau I_{Ne,\tau,\omega} \leq mindt_{Ne}.\left(1 - sd_{Ne,t,\omega}\right) \quad ;\forall\, t \leq \tau \leq t + mindt_{Ne}\ , \forall\, Ne\epsilon Neg,\omega \tag{22}$$

$$gel_{Ng,t,\omega} = F_{Ne,t,\omega}\left(P_{Ne,t,\omega}\right) \quad ;\ \forall\, Ng\ \epsilon Ngel, Ne\ \epsilon Neg \tag{23}$$

$$F_{Ne,t,\omega}\left(P_{Ne,t,\omega}\right) = bf_{Ne}.\, p_{Ne,t,\omega} + cf_{Ne} \quad ;\forall\, Ne\ \epsilon Neg,t,\omega \tag{24}$$

$$gp_{Ng,t,\omega}, gel_{Ng,t,\omega}, gnl_{Ng,t,\omega}, gls_{Ng,t,\omega}, \pi_{Ng,t,\omega}, pls_{Ne,t,\omega} \geq 0 \quad ;\forall\, Ng,Ne,t,\omega \tag{25}$$

$$o_{pl,k}\ , I_{Ne,t}\ , su_{Ne,t}\ , sd_{Ne,t} \in \{0,1\} \quad ;\forall\, Ne,t,pl,k \tag{26}$$

Equation (1) is the objective function, which is the total cost of gas and electricity generation to respond to the demand, consists of start-up and shut-down costs (or commitment costs), load shedding costs, and fuel (gas) costs of electricity generation units, and natural gas load shedding penalty costs. Constraint (2) indicates natural gas supply and demand equilibrium in the corresponding node in the natural gas network. Constraint (3) ensures natural gas demands other than electricity generation plants are in priority. On the other hand, compressor fuel consumption to compensate for pressure loss in gas pipelines is indicated in constraint (4). Constraints (5) to (9) imply natural gas flow constraints in the underlying natural gas pipeline network. Constraint (11) assigns natural gas supply limitations. Constraint (12) addresses electricity demand load and supply equilibrium to their corresponding node in the underlying electricity network. Electricity load shedding is considered in constraint (13). Electricity flow modeling in the underlying network is conducted in constraints (14), (15), and (16).

Electricity generation capacity limitations are reflected in constraint (17), start-up and shut-down of electricity generation units are indicated in constraints (18) and (19), and ramp-up and ramp-down of electricity generation units are derived in constraint (20). Minimum down-time and minimum up-time of electricity generation units are formulated in constraints (21) and (22). While the gas fuel demand constraint is indicated in constraint (23), constraint (24) reflects the corresponding gas fuel consumption function of the electricity generation units. Constraint (25) ensures its corresponding variables are non-negative, and constraint (26) indicates its corresponding variables are binary.

Proposed Hybrid Robust Stochastic Optimization Method

Our proposed approach is inspired by [15] which is based on scenario generation and clustering of the generated scenarios. In this article, we study the effects of the number of scenario clusters in the optimum solution. We further study the clustering evaluation in terms of their corresponding number.

In this approach, one may control the extent of conservation of the solution by obtaining the expected value of several robust solutions to the stochastic problem. Intuitively, the HRSO approach, which is based on discrete feasible space by discretizing probability space, separates or splits the feasible space of uncertain parameters into two or more feasible spaces. The solution is next determined in the worst possible case in each cluster. Mathematically speaking, the robust solution is found in each cluster. Stochastic values of the resulting robust solutions are eventually obtained by finding the expectation of the robust solutions. Indeed, the way by which the feasible space is separated substantially impacts the optimal value of the HRSO solution. Therefore, how discrete scenarios are clustered has a key role in the hybrid robust-stochastic optimization approach.

In stochastic programming, different ways to discretize random parameters or random space are employed to construct a scenario fan. In this study, the Latin Hypercube Sampling (LHS) method is employed for discretization [41], hence the generation of samples of random parameters in an efficient manner. Moreover, the LHS method with randomized multi-variate normal distribution is employed to form multi-variate samples, considering the linear correlation between the random variables.

A scenario tree is then constructed from the resulting scenario fan. The most widely used approach for constructing a scenario tree has been proposed by Heitsch and Römisch [42] who used a hierarchal method. In this study, we apply the k-means clustering method in order to split the random space.

In addition, the evaluation of the clustering pattern is taken into account in order to gain the optimal number of clustering, investigating whether any relationship between clustering size and the solution values is evident.

In this case study, all values of random parameters are feasible as a penalty function is considered in the objective function in case demand is not met. Hence, there would be no concern about the feasibility of the discretized values. More precisely speaking, our problem is robust in terms of the feasibility of the solutions.

We also compare the number of iterations to achieve the optimal solution in each case (cluster number) since it shows the computational burden of the solution in a better way as it does not depend on the computer system features (computer processor and RAM size) in which the operations are conducted.

As both gas and electricity demand are uncertain, simultaneous scheduling of gas and power supply is the center of attention in this study. This problem involves solving a stochastic programming model under gas and electricity demand uncertainty. Hence, we deal with investigating the HRSO approach to solve this problem by considering stochastic and robust approaches. For this aim, we first solve the coordinated scheduling of electricity and natural gas supply systems from an absolutely robust viewpoint. Next, the mentioned model is solved from an absolutely stochastic point of view. In the following stage, the clustering method is proposed to implement the HRSO approach.

Carrying out the above-mentioned approaches requires discretizing the random parameters in each time step to obtain abundant samples to achieve well-reliable solutions.

Scenario Generation

In this section, the scenario generation method for the stochastic optimization model is presented. It is assumed that electricity demand and gas demand are stochastic parameters of our model. In order to generate the scenarios related to these parameters, we assume a normal probability distribution with two known parameters, mean and standard deviation, for each period, which is 24 hours for both electricity and gas demand. These values are shown in Tables 1 and 2.

Latin hypercube sampling is often the preferred sampling procedure in Monte Carlo analyses due to the efficient manner in which it stratifies across the range of each sampled variable [41]. The use of Latin hypercube sampling, which is part of a Monte Carlo procedure for the propagation of uncertainty, is extensive and growing [41].

Then applying the Latin Hypercube Sampling (LHS) method, assuming independence between gas and electricity demands, 50 samples are taken from electricity and gas demands based on this method. Figure 2 illustrates electricity and gas demand scenarios.

Table 1. Electricity Demand Distribution Parameters (MW)

Hours	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Mean	175	165	158	155	155	160	173	190	206
Std.	50	50	50	50	60	40	80	30	40
Hour	10	11	12	13	14	15	16	17	18
Mean	217	229	236	242	244	249	256	256	247
Std.	70	20	80	20	30	40	30	50	60
Hour	19	20	21	22	23	24			
Mean	246	237	237	227	201	197			
Std.	90	90	90	70	60	40			

Table 2. Gas Demand Distribution Parameters (KCF/H)

Hours	1	2	3	4	5	6	7	8	9
Mean	5220	4920	4680	4740	5100	5640	5580	6060	6180
Std.	150	150	150	150	160	240	280	230	240
Hour	10	11	12	13	14	15	16	17	18
Mean	6240	6120	6120	6000	5700	5760	5880	6060	6240
Std.	170	220	180	220	230	240	230	150	160
Hour	19	20	21	22	23	24			
Mean	6540	6780	6660	6540	6060	5520			
Std.	190	290	290	270	160	140			

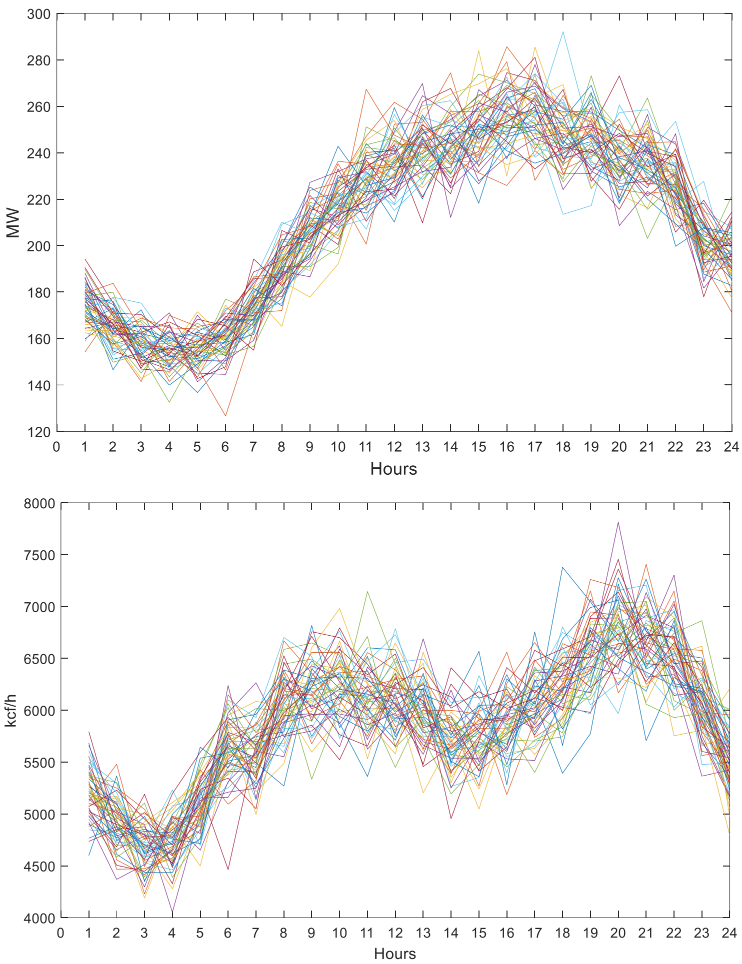

Fig. 2. Electricity demand (top) and Gas demand (bottom) scenarios

Clustering Stochastic Scenarios

The generated scenarios in the last section are grouped into different numbers of clusters to form the corresponding scenario tree. In essence, stochastic space is partitioned into several subspaces.

There are different clustering methods. Data clustering algorithms can be either hierarchical or partitional. Hierarchical algorithms determine successive clusters using previously established clusters, whereas partitional algorithms determine all clusters at a time. Hierarchical algorithms are agglomerative (bottom-up) or divisive (top-down). Agglomerative algorithms begin with each element as a separate cluster and merge them into successively larger clusters. Divisive algorithms begin with the whole set and proceed to divide it into successively smaller clusters. Besides, clustering methods are composed of Partitional Clustering (including K-Mean and K-Medoids algorithms), Density-Based Clustering (including DBSCAN and SSN algorithms), Grid-Based Clustering (including STING and CLIQUE algorithms), and Grid-Based Clustering (including Statistical and Neural network approaches) [43].

Furthermore, there are different criteria to evaluate the suitability of the number of clusters of clustered data. The criteria are categorized into internal and external indices. In this article, two common clustering indexes are applied: the Dunn index and the Davies-Bouldin (DBI) index [44].

Here, scenarios are clustered from 2 to 15 clusters through the k-means clustering method, as one of the most common clustering methods is k-means clustering [45, 46]. Thereafter, clustering evaluation indices are attained for each clustered scenario. The results are demonstrated in Table 3 and Figure 3. Accordingly, while the Dunn index for different numbers of clusters ranges from approximately 0.5 to 0.65, this is approximately 2.6 to 4 for the DBI index. As it is evident from Table 3, the dominant trend in the Dunn index is decreasing while there are some exceptions. From the DBI index's perspective, there is no special trend.

In order to determine the optimal number of clusters, we apply a multi-criteria decision analysis method so-called TOPSIS [47], as one of the most common methods used, using Dunn and DBI index values. Figure 4 depicts the TOPSIS value for each number of clusters. Again, no obvious trend is observed in TOPSIS values, with the 8-cluster having the least value, and number 13 the largest one. Following this analysis, the simulation is carried out, and the results are described in the next section. Then, TOPSIS indicates the best number of clusters is the maximum value that here is 13. This implies that the best partitioning state of feasible space relates to the 13-cluster case in HRSO.

Table 3. Clustering Evaluation Indices For Clustered Scenarios

Index \ Clusters	2	3	4	5	6	7
Dunn	0.5700	0.5761	0.6026	0.6208	0.6070	0.5696
DBI	3.9690	3.4066	3.4451	3.2860	3.1224	3.0613
	8	**9**	**10**	**11**	**12**	**13**
Dunn	0.5999	0.5726	0.5416	0.5580	0.5191	0.6424
DBI	2.9021	2.7446	2.7035	2.8844	2.7589	2.7810
	14	**15**				
Dunn	0.6307	0.5999				
DBI	2.9378	2.8568				

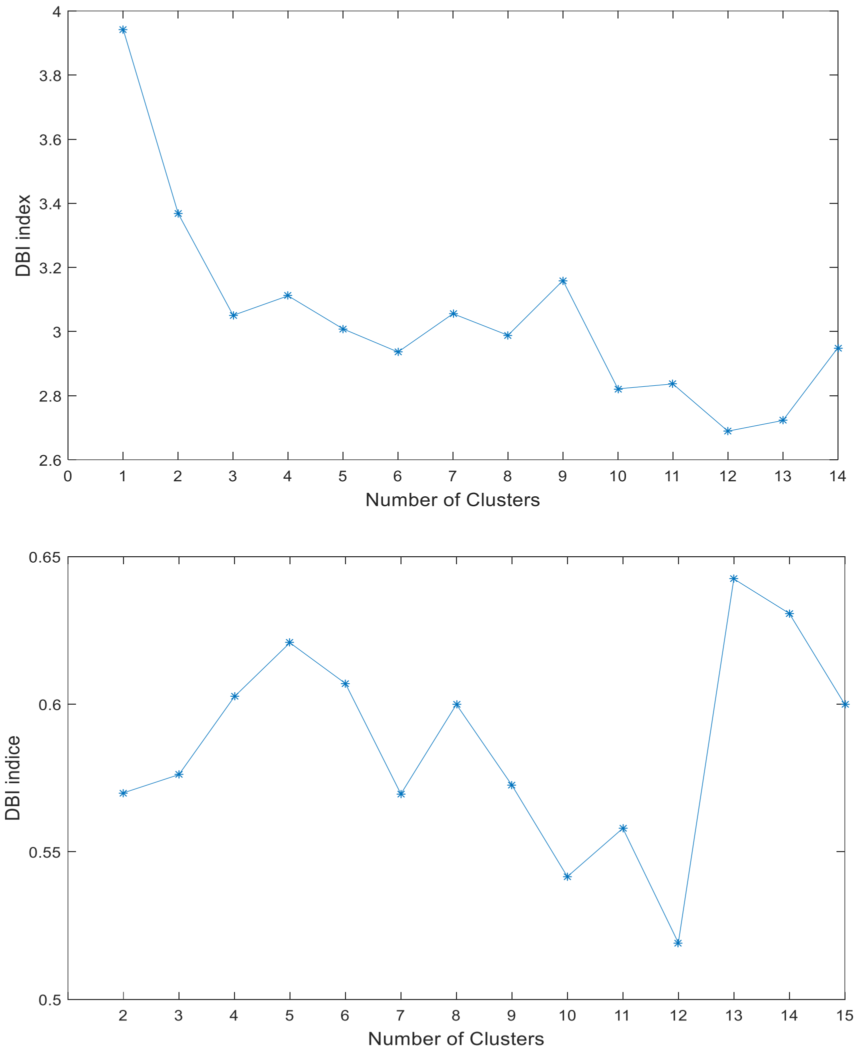

Fig. 3. Clustering evaluation indices for different clustered scenarios: DBI index (top) and Dunn index (bottom)

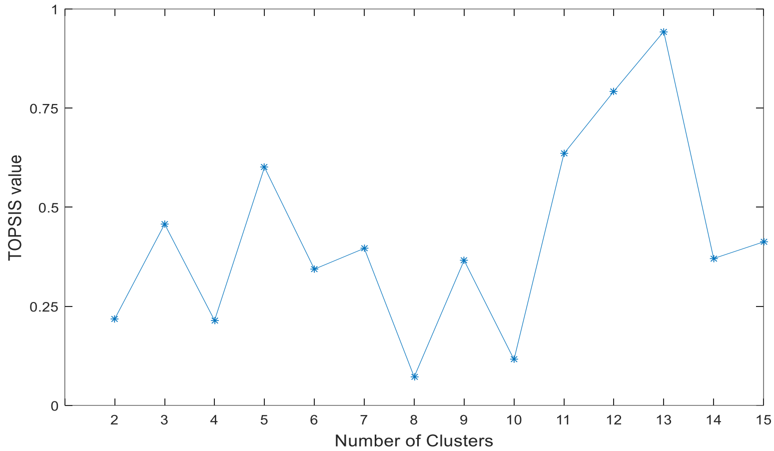

Fig. 4. Clustering evaluation through the TOPSIS method

Simulation Results

Our solution performance is tested by applying a sample system including a 7-node natural gas network and a 6-node power grid, which originally appeared in [48]. The power grid has three demand nodes and three generating units, all of which are gas-fired and receive their fuels from three nodes of the natural gas network. The natural gas network has two refineries as production units and three demand nodes. Figure 5 depicts the sorted total cost (objective function value) for different clusters. As expected, notwithstanding some exceptions, the more the number of clusters, the less the cost of energy scheduling is. The general trend of the combined energy scheduling cost is decreasing. The robust case has the most expensive scheduling cost, while the stochastic case indicates the least cost. As observed in Figure 5, the 13-cluster case of HRSO, as the best number of clustering, stands in 14th position among all cases.

Total penalties consisting of power and gas penalties for each HRSO case are depicted in Figure 6. Yellow parts of bars indicate gas penalties. According to this plot, excluding some HRSO cases, power penalties generally decrease as the number of clusters increases. With solely 5-, 9- and 10-clusters having gas penalty, no special trend is observed in this penalty. The same decreasing trend as the power penalty can be considered in total penalty amounts, but more exceptions. Again, we can see the 13-cluster case in the same position, with no gas penalty.

Table 4 indicates the ranking of the cost, power penalty, gas penalty, and sum of penalties for each cluster. It is evident that the ranking in cost and power penalty is almost decreasing, while the rankings of different clusters are not decreasingly in order, more specifically in the case of a gas penalty.

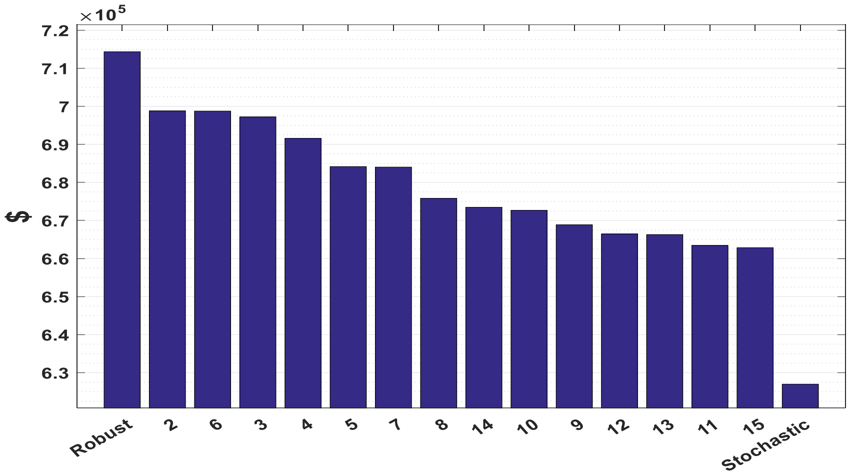

Fig. 5. Combined gas and power scheduling cost of all HRSO cases ($)

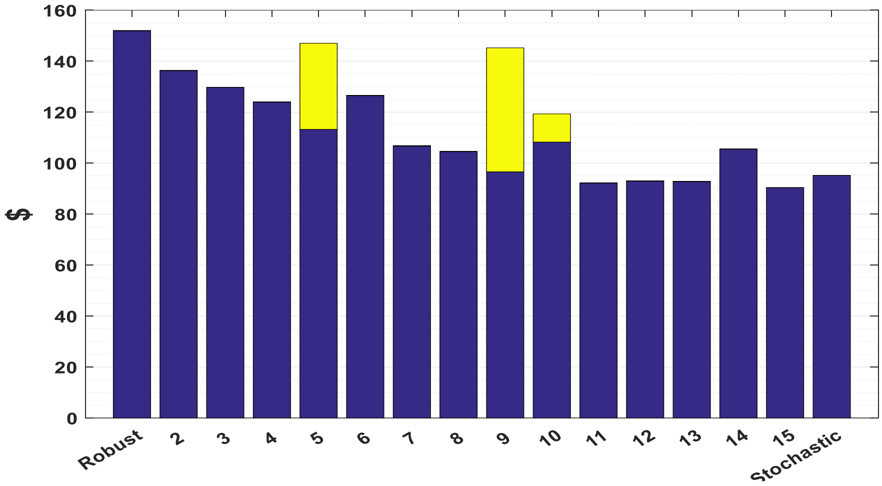

Fig. 6. Total penalty including both power penalty (blue) and gas penalty (yellow) for all HRSO cases ($)

The total power generation in each hour for selected HRSO cases is illustrated in Figure 7. The black, blue, and red plots are robust, 7-cluster, and stochastic cases, respectively. Accordingly, total power generation in robust and stochastic cases are different from each other and from other cases. For instance, regarding the robust case, total power generation is meaningfully less than in other cases (excluding stochastic case) in some hours (8,16-20,22, 23), and meaningfully more than in other hours (7,10,21,24). This difference at most amounts to about 40 MW. Generally speaking, the difference occurs more in middle load rather than peak and base loads. Comparing stochastic and robust cases, the most gaps are considered in 8,10,17, and 19-23 hours (middle plot). Eventually, there are differences in power generation between stochastic and middle-cluster cases in 3,5,7-10,12-15,17-19, 21, and 22 hours.

Total gas generation on a daily-hour basis in selected HRSO cases is depicted in Figure 8. Concerning the robust and middle-cluster cases (or cluster-wise cases), total gas generation is meaningfully different in 1,2,9,10,15,17, and 21-23 hours. It seems this difference escalates by comparing the robust and stochastic cases in middle hours, accounting for 1,2,7,8,10-16,18,21 and 23 hours. As far as the comparison of the stochastic and the middle-cluster cases is concerned, considerable gaps are observed in 8,10-12,14,17,18, and 22 hours. This difference almost ranges from around 10 to 40 kcf/h.

Table 4. Rankings Of Costs And Penalties For All Hrso Cases

Cluster Number	Cost	Power Penalty	Gas Penalty	Total Penalty
Robust	1	1	5	1
2	2	2	6	4
3	4	3	11	5
4	5	5	12	7
5	6	6	2	2
6	3	4	10	6
7	7	8	4	9
8	8	10	14	11
9	11	11	1	3
10	10	7	3	8
11	14	15	8	15
12	12	13	7	13
13	13	14	13	14
14	9	9	9	10
15	15	16	15	16
Stochastic	16	12	16	12

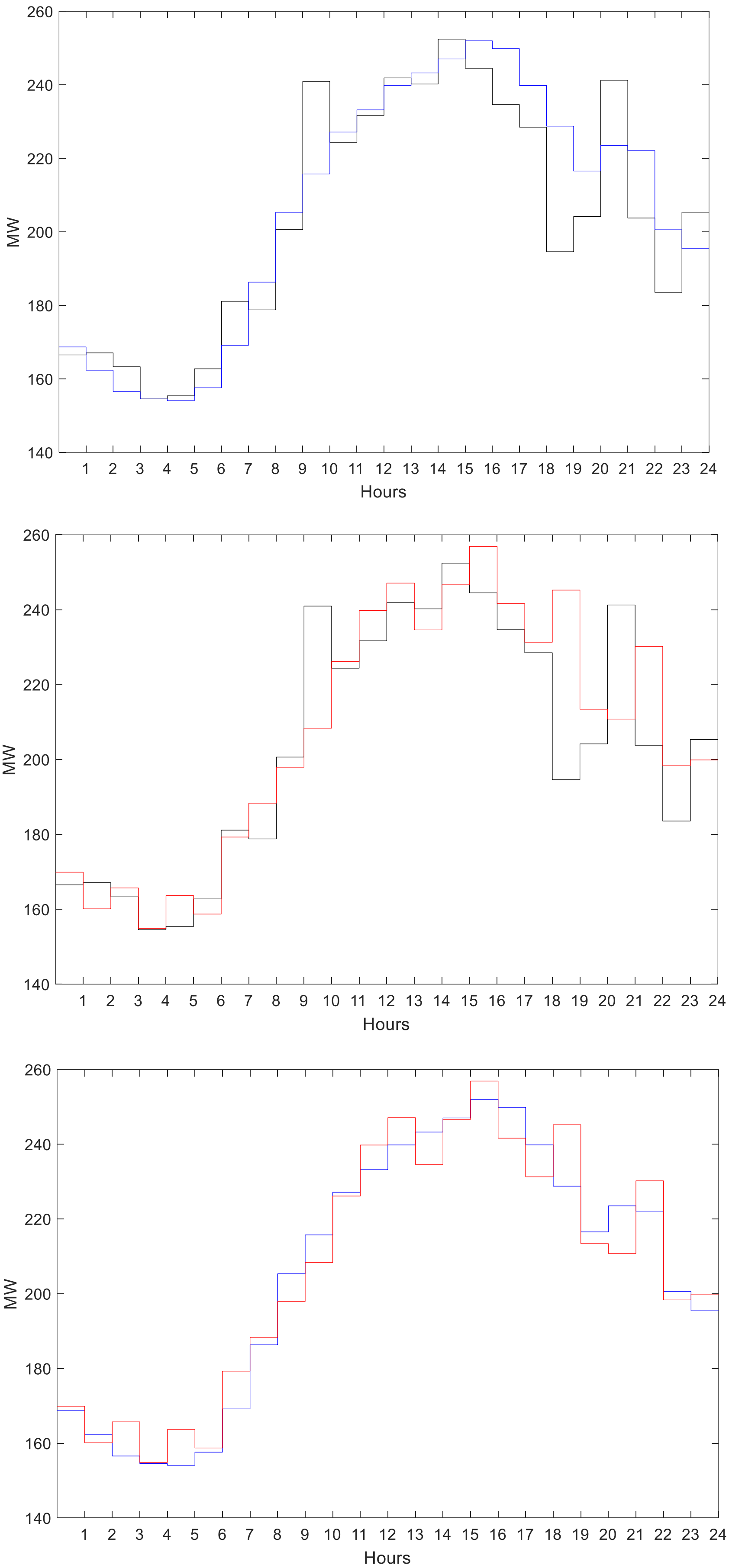

Fig. 7. Total power generation on an hourly basis for selected HRSO cases (MW)

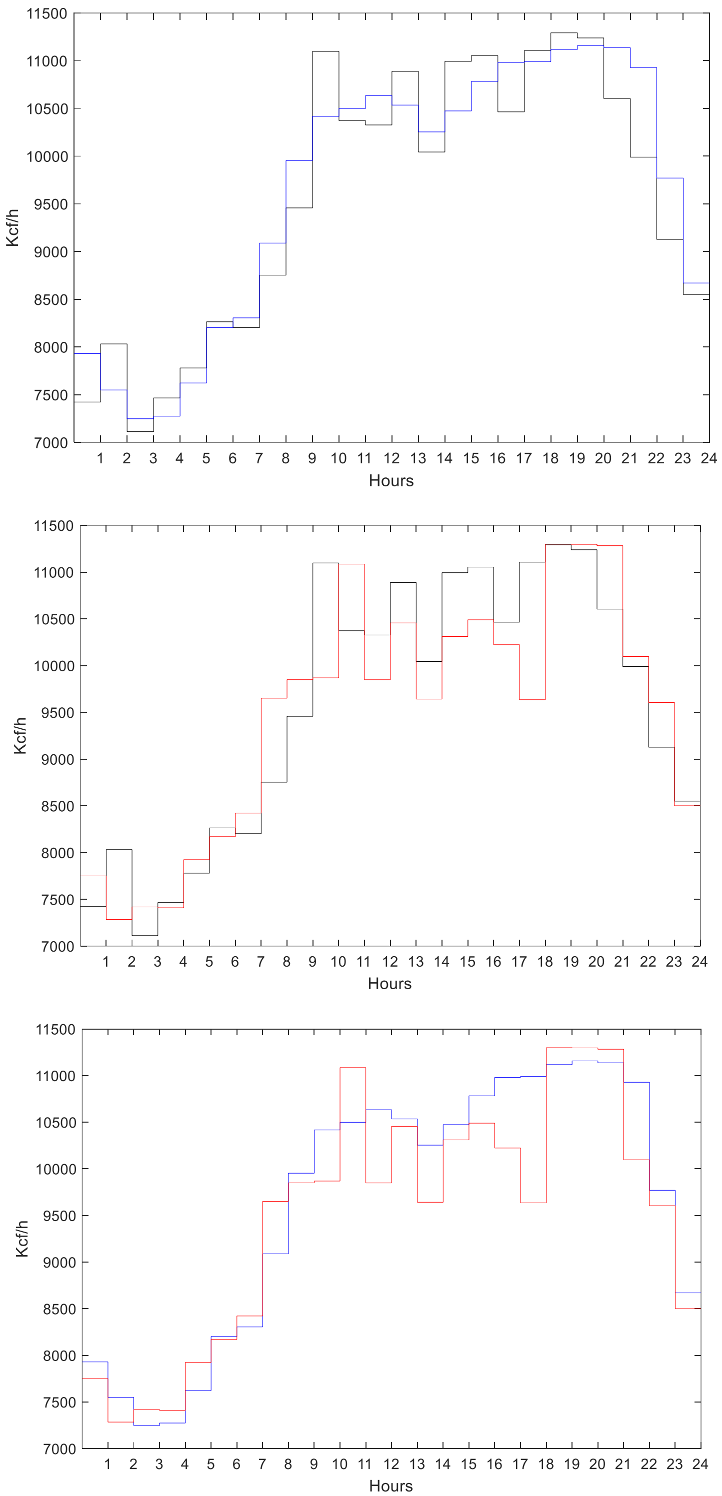

Fig. 8. Total gas generation on an hourly basis for selected HRSO cases (kcf/h)

Total power and gas generation are demonstrated in Figure 9. Total power generation takes the least amount in the robust case, with 9804.5 MW. The stochastic case has the largest total power generation among other cases, accounting for 9937.5 MW. The more the cluster number, the less the total power is generated with the exception of 6-cluster, 10-cluster, 13-cluster, and 14-cluster. Regarding total gas generation, 12-cluster HRSO has the largest amount, accounting for 463310 kcf/h, while the stochastic case has the least amount, with 455040 kcf/h, and 13-cluster in 11^{th} rank, with about 9920 kcf/h. Moreover, no special relationship is observed between the number of clusters and total gas generation.

The power generation unit 1 is in an "on" state at all hours for all HRSO cases. The power generation unit 3 is in the "off" state in the first seven hours, turning "on" afterward for all HRSO cases. However, the on-off pattern of power generation unit 2 is different among HRSO cases, where that unit is off for all HRSO cases for the first seven hours and remains on for all HRSO cases from hour 10 to hour 22. The on-off status of power generation unit 2 in all hours, except three hours (8,9, and 23), is the same in all HRSO cases. This power generation unit is on in hour eight in 5-cluster, 6-cluster, 12-cluster, 13-cluster, and stochastic cases while it is off for the rest. In hour 9, the power generation unit is on except for the robust and 2-cluster cases. Regarding hour 23, the unit is on in all cases but 5-cluster, 6-cluster, 10-cluster, and robust cases.

Hence, it is evident that as we move from robust case, in which there is a single cluster including all scenarios, to increasing cluster-wise cases and eventually stochastic case (which is a case in which every scenario is itself a single-point cluster) more units are in on status. While 13 power generation units are on in robust case, 14 units are on in 2-cluster and 10-cluster, 15 units are on in 3-, 4-, 5-, 6-, 7-, 8-, 9-, 11-, 14-, and 15-cluster cases, and the most units in on status (16 units) are in 12-cluster, 13-cluster, and robust cases.

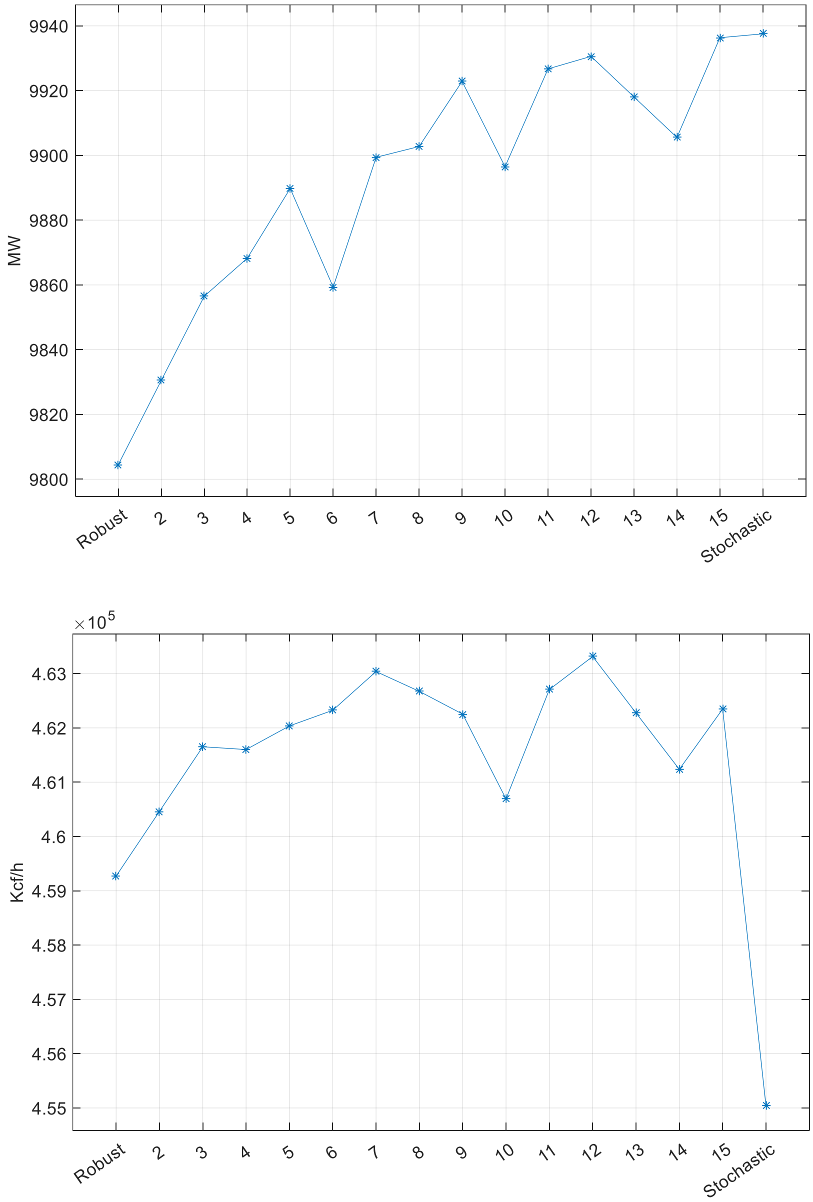

Fig. 9. Total power and gas generation for selected HRSO cases (kcf/h)

Table 5. Unit Commitment Of Power Generation Unit 2 In Hrso Cases

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Robust	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
2	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
3	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
4	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
5	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
6	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
7	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
8	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
9	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
10	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
11	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
12	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
13	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
14	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
15	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
Stochastic	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

Conclusion

This paper proposes a scenario-wise hybrid robust and stochastic optimization approach for coordinated scheduling of natural gas-power generation systems, considering uncertainty in natural gas and power demands. Uncertainty is represented by the discretization of the stochastic space of the problem through two-variate stochastic scenarios; 50 stochastic scenarios are generated through the Latin Hypercube Sampling method. A mixed integer linear programming model is applied, considering stochastic scenarios. In order to employ a hybrid robust stochastic optimization approach to solve the model, generated scenarios are partitioned into a different number of clusters, together with no clustering (robust case) and each cluster containing exactly one scenario (stochastic case) providing 17 different cases for the HRSO approach. In trying to answer the question of which number of clusters of scenarios is best, the two most common clustering indexes, Dunn and Davies-Bouldin, are employed and their value for each number of clusters of scenarios is obtained. Since the results of the mentioned clustering indexes are not enough to reach a conclusion, the TOPSIS method as a multi-criteria decision-making tool of clustering is applied to deduce the values of two indexes. The result indicates that the best number of clusters for stochastic scenarios is 13. The results of the simulation of the model for all cases of HRSO demonstrate that the main difference among all cases is seen between stochastic and robust cases while there is no major difference among others. As expected, there is almost a decreasing trend in penalties from robust cases to stochastic ones. It implies that as the number of clusters increases, the amount of objective function decreases.

Future work could be focused on how many clusters should be chosen for studying and analyzing the objective functions so that these number of clusters are offered to different risk-averse attributes. In better words, various individuals with corresponding risk behaviors could choose their appropriate number of clusters, without needing to examine all possible numbers of clusters.

Nomenclature

Indices, parameters, sets, and functions

ω, Ω	Index and set of the stochastic scenarios. ω=0 means the day-ahead forecast scenario
t, T	Index and set of time periods
Ne, NE	Index and set of electricity network nodes
el, EL	Index and set of electricity transmission lines

Ng, NG	Index and set of natural gas network nodes
pl, PL	Index and set of pipelines
k, K	Index and set of piecewise linear function pieces
Nep(Ne)	Subset of generating units
Nel(Ne)	Subset of electricity demand nodes
Ngp(Ng)	Subset of refineries nodes
Ngl(Ng)	Subset of natural gas demand nodes
APL(pl)	Subset of active pipelines
PPL(pl)	Subset of passive pipelines
$gload_{Ng,t}$	Natural gas demand parameter
C_{pl}	Pipeline constant parameter
$m_{pl,k}$	Slope parameter in piecewise linear function
$b_{pl,k}$	Intercept parameter in piecewise linear function
$\underline{gfl}_{pl,k}$	Gas flow lower bound
$\overline{gfl}_{pl,k}$	Gas flow upper bound
pr_{Ng}^{max}	Gas pressure upper bound
pr_{Ng}^{min}	Gas pressure lower bound
β_{Ng}^{max}	Active pipeline output pressure ratio upper bound
β_{Ng}^{min}	Active pipeline output pressure ratio lower bound
GP_{Ng}^{max}	Refinery upper production bound
GP_{Ng}^{min}	Refinery lower production bound
$\mathrm{C}_{\mathrm{Ng}}^{\mathrm{GLS}}$	Natural gas load shedding penalty cost
$eload_{Ne,t}$	Electricity demand parameter
PTDF	Electricity transmission matrix
PSDF	Phase shifter distribution matrix
DCDF	DC distribution matrix
α_{el}^{max}	Phase shifter angle upper bound
α_{el}^{min}	Phase shifter angle lower bound
pf_{el}^{max}	Electricity flow upper bound
pf_{el}^{min}	Electricity flow lower bound
$maxg_{Ne}$	Power plant upper generation bound
$ming_{Ne}$	Power plant lower generation bound
UR_{Ne}	Ramp-up rate of power plant
DR_{Ne}	Ramp-down rate of power plant
$minut_{Ne}$	Minimum-up time of power plant
$mindt_{Ne}$	Minimum-down time of power plant
C_{Ne}^{SU}	Power plant start-up cost
C_{Ne}^{SD}	Power plant shut-down cost
C_{Ne}^{FC}	Natural gas cost for power plant
C_{Ne}^{PLS}	Electricity load shedding cost
$F_{Ne,t,\omega}$	Power plant fuel usage function
bf_{Ne} , cf_{Ne}	Coefficient of the piecewise linear fuel usage cost function of power plants
$gfl_{Ng,t,\omega,k}$	Natural gas flow variable in piecewise linear function
$o_{pl,k}$	Binary variable in piecewise linear function

A. Variables

$gp_{Ng,t,\omega}$	Refinery natural gas production
$gel_{Ng,t,\omega}$	Natural gas demand (power plants)
$gnl_{Ng,t,\omega}$	Natural gas demand (other than power plants)
$gls_{Ng,t,\omega}$	Natural gas load shedding
$gf_{Ng\rightarrow Ng',t,\omega}$	Natural gas flow
$\pi_{Ng,t,\omega}$	Square natural gas pressure
$P_{Ne,t,\omega}$	Electricity nodal injection
$pls_{Ne,t,\omega}$	Electricity load shedding
$pf_{el,t,\omega}$	Electricity flow
$\alpha_{el,t,\omega}$	Phase shifter angle variable

$I_{Ne,t}$	Power plant situation (On, Off) binary variable
$\mathrm{su}_{\mathrm{Ne,t}}$	Power plant start-up binary variable
$\mathrm{sd}_{\mathrm{Ne,t}}$	Power plant shut-down binary variable
TC	Total cost

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