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# *A new weighted BPR model for travel time estimation sensitive to "on the road elements"*

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***Article history:***
*Received: 2021-08-26, Revise: 2023-01-08, Accepted: 2023-04-03*

***ABSTRACT***

*Numerous variables, including vehicle volume, road width, on-street parking, and border parking, can all affect the amount of time a vehicle travels on a given route. While these factors have varying effects, they are typically considered invariant across all segments of a route. None of the previous studies considered an area where these factors could exert influence. In this paper, a new model named "weighted BPR" is proposed that considers an effective area in for each factor where the vehicle's speed changes. These areas are incorporated in the model using some weights. True travel times across many streets are then used to estimate the model weights. Provided that we know which factors are present in a route, we can, then, estimate how long it would take for a vehicle to travel through the route. In addition to estimating the travel time, such model can be used to guide city planners how new structures affect the travel time in cities. Due to the large number of such factors, this research examines only three: bus stops, crosswalks, and speed bumps. To estimate the weight The prototype was evaluated on 37 distinct sections of Tehran's streets. The results indicated that the estimated travel time using the developed model is more precise than that computed using conventional methods. Experiments demonstrated that the technique developed in this paper can estimate the travel time with an accuracy of 2.93 to 3.27 minutes while this value computed by the model developed by national traffic experts ranges between 7.8 to 23.09 minutes.*

***KEYWORDS***

*Travel time,*

*BPR model,*

*on the road factors*

## *1. Introduction*

*Travel time estimation in urban streets is a very important task that is used in several applications including optimization of driving sign locations, road design, development of intelligent transportation systems and traffic management plans. Different models are available in the literature, most important of which are Neural Network Based (Fu et.al., 2020, Ye et.al, 2022, Jin et.al, 2022, Fang et.al 2022), multi-task learning model (Xu et.al., 2020), Lagrangian-Space based (Yang 2020), feed-forward network based (Sun et.al., 2021), copula-based (Chen et.al., 2019), tensor-based bayesian probabilistic model (Tang et.al., 2018), incomplete traffic data method (Tani et.al., 2020), cellular network signaling (Gundlegård & Karlsson 2020), artificial neural network (Amita et.al., 2015), maximum likelihood (Leurent et.al., 2020), consideration of link geometry (Yoon 2021), deep learning (Tran et.al., 2020), linear (Friesz et.al., 1993), logarithmic (Kemal 2004), exponential (Henry et.al., 1975), polynomial (Silverman 1986), and Bureau of Public Road (BPR) (Akcelik 1991 ; Bureau of Public Roads 1964)*

*Many factors influence the vehicle's travel time over a given route including volume, street width, and on-street parking. Qi (Qi et.al., 2020) created a novel prediction*

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model for bus inter-stop travel times (BISTTs) that uses five input variables: stop distance, historical inter-stop travel times, number of intersections, intersection traffic volumes, and intersection signal timing schemes. A test was performed in this technique to check for autocorrelation in the model residuals. Based on field data, cases of two bus routes in Harbin (China) have been studied. The results showed that the mean average errors for both bus routes were less than 8%, indicating a high level of prediction accuracy in predicting BISTTs.

Kelsey Maass (Maass et.al., 2020) proposed a method for estimating street-level travel times in a given metropolitan area using coarse-grained and aggregated travel time data. Several techniques, including graph-based routing, trip sampling, graph sparsification, and least-squares optimization, were used in that study to estimate street-level travel times. Constrained least-squares problems are iteratively solved using sampled trips and weighted shortest-path routing to obtain travel time estimates. Ma (Ma et.al., 2019) proposed a novel segment-based method for predicting bus travel times that uses a combination of real-time taxi and bus datasets to automatically divide bus routes into dwelling and transit segments. By combining distinct effect traffic parameters, two models were created to predict them individually. The results of the experiments show that this method enhances the accuracy of bus travel time prediction, particularly under unusual traffic conditions. The deep network was created by Rong Cheng (Cheng & Feng 2020) using a stacked denoising autoencoder. The method could directly estimate the travel time of any path and solve the long path estimation error accumulation problem. The taxi trajectory data set's experimental results revealed that the stack noise reduction self-encoder-based travel time prediction method has good prediction accuracy and stability.

In very recent years a lot of attention has been paid to the use of deep neural networks to improve the accuracy of travel time estimation in cities. Wang et.al. (2022) proposed a meta learning based framework to continuously provide accurate travel time estimation over time by leveraging well-designed deep neural network mode. In another study, Rajagopal et.al. (2022) developed a hybrid deep learning model that extracts the optimal feature points from the existing dataset using a stacked autoencoder is presented. Handcrafted feature points are fed into the hybrid deep neural network to predict the travel path and travel time between two geographic points.

In most of the above studies, to calculate the travel time, the influencing factors are mostly assumed to be constant throughout the street. In addition, many researchers estimate travel time taking no account for local effects of small on the road factors. This assumption, however, is not always correct, and can, thus, lead to an inaccurate travel time estimation. Thus in this model, we propose a model that takes into account the local effects of on the road factors like bus stops, speed bumps, and zebra lines,. These factors are many.

Based on the above, the main contribution of this paper is the development of a new mathematical model that in contrast to the current models (which use only volume and capacity) involves additional on the road factors like bus stops and zebra lines to estimate the travel time. These factors are introduced by adding weights to basic models used in current travel time estimation models.

In the following section, the principle of the proposed weighted model s presented first.. Then the methodology of estimating the weights is also described. In Section 3 several tests carried out to evaluate the model are presented and the results discussed. Finally, in Section 4, the conclusions are made and suggestions for future studies are presented.

## 2. The proposed model and methodology of estimating its parameters

### 2.1 Principle of the proposed model

In this section the principle of the proposed model along with the methodology of estimating its parameters are described. In general, this model is uses real-time traffic data. In California, for example, several tests were conducted, and based on the results, it was determined that the Akcelik model (Dowling 2006) was best suited to predict travel time. According to the findings of a comprehensive study conducted in Tehran, the BPR model (Friesz et.al., 1993), was found to be the best model due to its simplicity and integrity The BPR model is based on three parameters (Qi et.al, 2020): volume (the number of vehicles passing through a point of lane in one hour), capacity (the amount of time it takes for a vehicle to travel one kilometer of an empty street) and free flow travel timeThe time it takes for a vehicle to travel one kilometer on a street under ideal conditions, i.e. without any other vehicles . The BRP model is as follow:

$$t = t_0[1 + a(v/c)^b] \qquad (1)$$

Where t is the actual travel time [s], c is the capacity; v is volume and $t_o$ [s] is the free flow travel time. The constants a and b are fitted to data actually obtained from the traffic within a city (these constants were estimated to be a = 0.15 and b = 4 for Tehran. These values were defined by Tehran Traffic and Transportation Company (TTTC) in 1995 (TTTC, 1995).

The standard BPR model, the area of influence for the factors is not considered in the BPR model (Equation 1), but rather the overall effects of different factors are considered based on volume and capacity. Nevertheless, as we know each factor only affects a specific zone, which can decrease the car speed only in that zone. A commercial building, for example, can cause a change in vehicle speed and therefore increase traffic volume within approximately fifty meters before and after the building. A similar effect occurs when a *bus stops in a bus stop. Indeed, this situation causes a lane to be banded or reduced in width, resulting in a reduction in street capacity and an increase in vehicle travel time. As a result, the model should take into account the area surrounding the factors that influence travel time. Considering the variety of factors of such, in this paper we develop our model to be based to only on those that have the most significant effects on travel time. For this, we conducted a comprehensive study in various streets of Tehran to identify the most effective factors. The study suggested that bus stops, speed bumps, and zebra lines, were the most influential parameters. We call them from now on "on the road factors", based on which we will expand the BPR model.*

*Equations 2 and 3 depict the standard BPR model extended to include the mentioned zones of influence. Here,* $t_i$ *represents the travel time associated with the each factor while* $t_{Free}$ *denotes the travel time when none of the road factors but volume and capacity (see below) are considered.*

$$t_{Total} = \sum_{i=1}^{n} t_i + t_{Free} \tag{2}$$

$$t_i = t_0 \times \left(1 + \gamma_i \times a \times \left(\frac{v}{c}\right)^b\right) \times L_i \tag{3}$$

*As shown in Equation 3, for each factor i, the coefficients* $\gamma_i$ *and* $L_i$ *are defined as the weight and the zone of influence for each factor respectively. The zone of influence is measured in meters and is the distance before and after the on the road factor that affects the travel time of a vehicle. In the following the methodology of estimating the travel time using the proposed model is described.*

#### *2.2 Methodology of estimating the parameters of the proposed model*

*To solve the Equation 3 for different* $\gamma_i$ *and* $L_i$*, we need to have the travel time (*$t_i$*), the volume (v), and the capacity (c) for some streets. The first two, are collected via field observations while the last one is estimated using an existing model used by traffic experts. In Iran, this is the HCM model. Since this model is developed for Australian streets, we use a calibration coefficient to improve its accuracy.*

*Figure ... shows how these values are used to estimate the parameters of the proposed model. At first the values for v, c, and travel time are defined for some train streets. At his stage also an initial value for the calibration coefficient is defined. The value of this coefficient in the denominator is between 0 and 1. The initial value for this coefficient is 0.001. Then, using the train data, the observation equations are formed and solved to estimate the weight and zone of influence of each on-the-road factor. The estimated model is then evaluated by estimating the travel time for some test streets. The process is repeated with different calibration coefficient values, with increments of 0.001. In the end, the model giving the best accuracy for the travel time of test streets, is presented as the final model. These steps are further explained in the following subsections.*

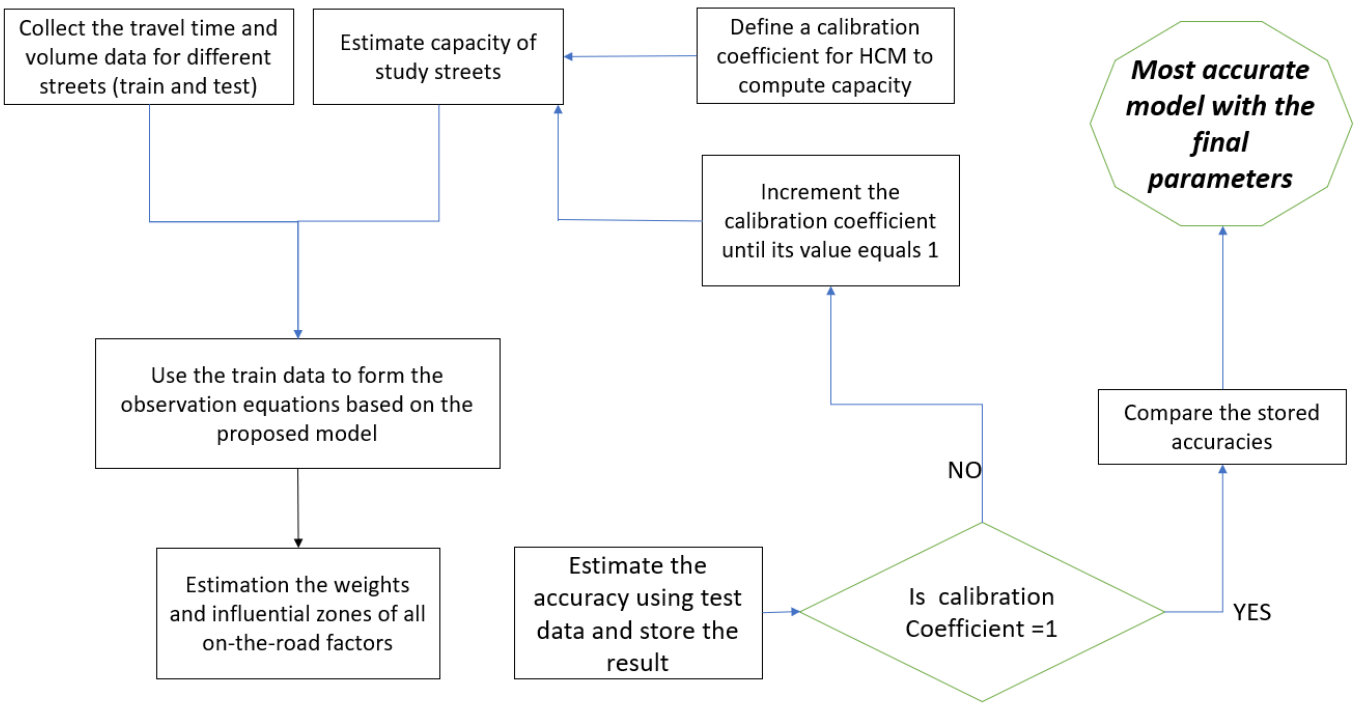


*Figure **1**. The flowchart of estimating the parameters of the proposed model*

#### *3.2. Data collection*

*As mentioned, at this stage the data about travel time, volume and capacity of some train and test streets are gathered.*

*Travel time: Human observation, closed circuit cameras, radar equipment, and license plate detection are all methods for calculating actual travel time (Shahi 2003 ; Li et.al., 2020 ). In this paper, the license plate detection method is used. This technique calculates travel time by* *recording the time it takes vehicles to cross a segment with a known length of a specified route. The license plate and time for each vehicle are recorded at the beginning and end of the segment, and the average time for all vehicles is used for that segment.*

*Volume: There are two methods for calculating traffic volume: mathematical and statistical models. In the first technique, a mathematical model is used to predict the volume of vehicles passing through a given cross section of a route. This technique is simple, but it can predict outcomes that are not consistent with reality. As a result, the more reliable statistical-based approach is used in this paper, in which the number of vehicles passing through a given cross section is counted and considered as the volume. It should be noted that the type of vehicle has an impact on the volume of traffic. In other words, a car takes up the space of several motorcycles; thus, the car takes up a larger portion of the street. In addition, a taxi stops several times along the route, causing more congestion than a private car. To consider vehicle size, regular vehicles are used as a reference with a scaling coefficient of one, and the following coefficients are used for other types of vehicles: 2.5 for a truck, 0.5 for a motorcycle or bicycle, 2.5 for a bus or minibus, 3 for a city bus, and 2 for a taxi.*

*Capacity: Statistical or numerical methods can be used to estimate traffic capacity. Usually, the traffic volume and travel time for many streets of the same type (for example, Arterial– 1) are taken into account in statistical methods, and an average value for that street type is calculated using linear interpolation. Such methods can't properly model "on-the-road factors" because an average value represents the overall effects of a street type. In numerical methods, each effect is estimated separately before the total effect is calculated. The capacity is computed using these methods by taking into account the contribution of each traffic factor, such as traffic regulations used in the United States (HCM 2000 ; AlKheder 2020), Australia (Akcelik 1981 ; Prassas & Roess 2020), and the United Kingdom (Transport and Road Research Laboratory 1963 ; Rolison 2020). Using a specific mathematical model, the impact of each traffic factor on capacity is calculated. According to Norozi (2004), of the three models, the American model (HCM 2000) is the most suitable for Tehran streets (Norozi 2004). He was able to calculate the delay at a signalized intersection in Tehran with accuracy (Norozi 2004). The proposed model, which is used to compute the traffic capacity, is also based the HCM model. These models must be calibrated for the city of Tehran. For this we start with some initial calibration coefficient fc. To obtain this coefficient, we set it to 0.001 and change it with increments of 0.001. The value giving the most accurate model is chosen as the final coefficient*

*The calibration process produces a coefficient for computing the actual capacity "c" (equation 4). In this equation, fc is the calibration coefficient and c' is the approximate capacity at the model of a route far from intersection. This coefficient is calculated using equation 5.*

$$c = c' \times f_c \qquad (4)$$

$$c' = s_0 \times N \times f_w \times f_a \times f_g \times f_{lu} \times f_p \times PHF \qquad (5)$$

*Where So is the "base saturation flow rate" which in this paper is equal to 1900 for arterial streets (Shahi 2003); N is the number of traffic lanes; fw is the road width adjustment factor; fa is area adjustment factor; fg is the lane grad coefficient; flu is the lane utilization coefficient; fp is the parking activity coefficient and PHF is the peak hour factor.*

### 2.3. Estimating the weights and influential zones of all on-the-road factors

*Once the data is collected, a system of equations in which the number of equations is more than unknowns is constructed for the proposed model. These equations are formed using the training data. The data includes volume, travel time and the capacity. The capacity is estimated using the HCM model calibrated using an initial coefficient of described in previous section. The initial value of this coefficient is equal to 0.001.*

*The general form this equations is L = F(X) where x is the unknown vector (i.e. the weights and influential zones of the on-the-road factors) and L is the vector of observation (i.e. the travel time, volume and capacity). This system is solved using least squares method of weighted residuals to obtain an equivalent L = AX system. The unknown coefficients are then computed (Vanicek 1986) by:*

$$X = (A^T PA)^{-1}(A^T PL) \qquad (6)$$

*Where P is a weight matrix that accounts for differences in travel time on observations; in this case, it is unity because the observations were made uniformly and with the same level of accuracy for each of the streets used to calibrate the model.*

*Once the unknowns are computed, the HCM calibration coefficient is incremented by 0.001 and the adjustment process is repeated. This is done until the coefficient value is equal to 1. At each stage, the travel time is computed for the test data using the resulting weighted model. The travel times thus computed are sorted and the model giving the minimum travel time is considered as the final model.*

*In the following, tests carried out to evaluate the proposed model in the city of Tehran are reported.*

## 3. Evaluations

### 3.1. Data

*In this study, to estimate the weights and influential zones of the proposed factors, the data for 37 sections of 22 streets in Tehran were used. For travel time, license plate detection* *was used, and for traffic volume, a statistical method was used. The traffic influence factors for each of the streets were then identified. The number of traffic lanes, lane width, and area type were all considered. DTM was also used to determine the slopes of the streets. Furthermore, as previously stated, the initial saturation flow rate per lane was set at 1900 vehicles per hour. Field visits and GPS localization were used to determine the location and number of each traffic influence factor.*

*Furthermore, to simplify this model, the streets chosen were not in commercial zones or in the city center. The volume and traffic time data were classified into two types of city streets: type one and type two. This allowed one to obtain the coefficients based on the type of street. The model*

*was calibrated using data from 18 arterial-1 streets and 11 arterial-2 streets. Arterial-1 streets refer to highway and freeways, while Arteiral-2 streets include other main urban streets. Figure 1 depicts all the streets used in this study.*

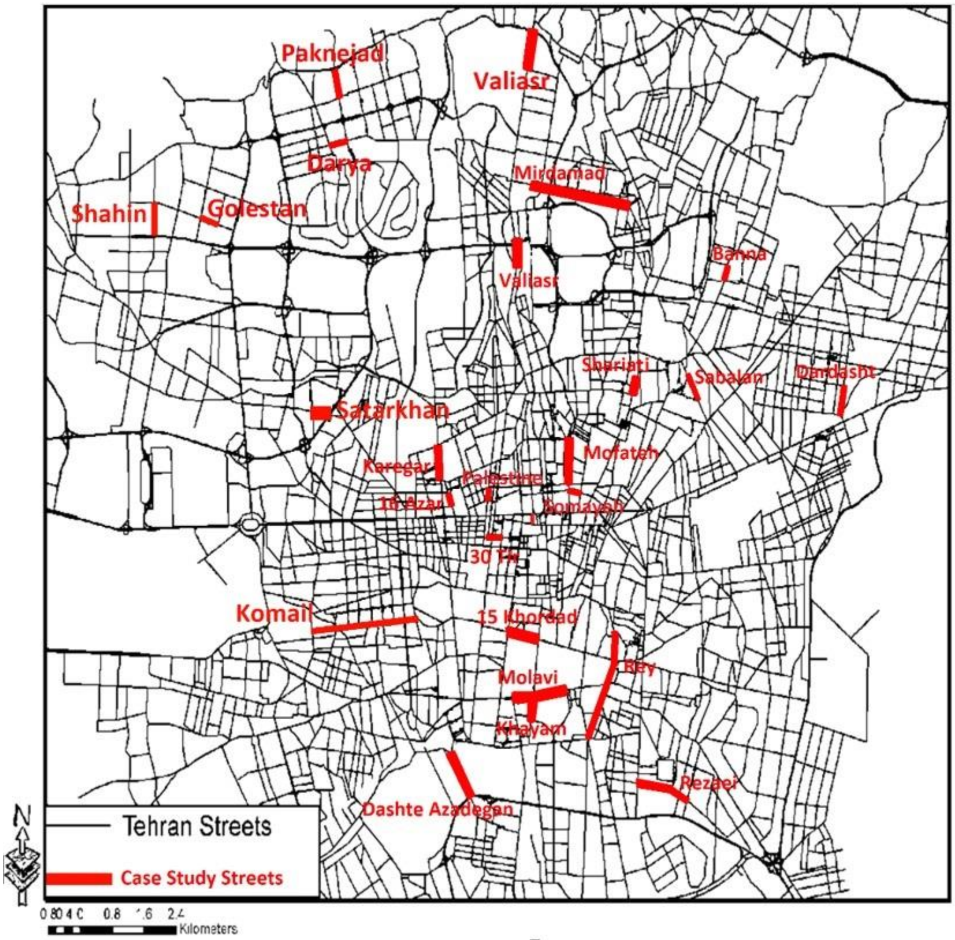


*Figure **2**. The one degree and two degree arterials in Tehran*

### *3.2. Results and Discussions*

#### *3.2.1. Computing zones of influence*

*The first step was to compute the zone of influence of each factor. Zones of influence are influenced by a variety of factors that differ from street to street, making statistical investigations time-consuming and difficult. In this study we did this using least squares, which took into account the minimum and maximum radios of factors' influence ranging from 5 to 100 meters radios at one-meter intervals. In other words, we set the influential zones to e.g., 5m. Then solved the model for the weights. Then the influential zone was increased to 10m, and the process repeated. This was done for all influential zones from 5m to 100m. In the end, we studied the effect how changing from one zone to another affects the travel time. Figure 2 shows the graphs for groups of streets based on zone and degree of influence. The zones having the largest effect on the travel time were chosen as the final value. Details of this can be found in Hosseininaveh Ahmadabadyan, 2007.*

*As illustrated in Figure 2, as the distance between a factor's location and its effect increases, the effect of that factor decreases. When the factors are compared over a constant zone of 100 meters for arterials-2, it was discovered that speed bumps have the greatest effect and zebra lines have the least effect.*

#### *3.2.2. Comparing the accuracy of the proposed model with that used by traffic experts*

*Tehran traffic experts have designed models based on the BPR model to estimate the travel time in this city. As can be seen in table (1), the experts have obtained the capacity in* *the form of a coefficient in the width of the road using the BPR function, which means integrating the impact of all factors affecting the capacity.*

*Table (1). travel time models presented by traffic engineers in Tehran (TTTC, 1995)*

| *Type of road* | *Free speed (km/h)* | *Travel time models* |
|---|---|---|
| *Arterial-1 streets* | 60 | $t = 1 \times \left(1 + 0.15 \times \left(\frac{v}{310 \times w}\right)^4\right)$ |
| *Arterial-2 Streets* | 50 | $t = 1.2 \times \left(1 + 0.15 \times \left(\frac{v}{240 \times w}\right)^4\right)$ |

*Now, the question that exists is which model provides a more accurate travel time. In response to this question, a comparison was made between the model presented by Tehran traffic experts and the BPR model with additional terms presented in this article in terms of accuracy in estimating travel time.*

*For this purpose, by using the data related to the streets that were given in the census, the travel time of those streets was obtained according to each of the mentioned models (models provided by traffic experts and BPR model with additional terms). Then by obtaining the ratio of volume to capacity of these streets, the data related to them were sorted from small to large based on the ratio of volume to capacity. After that, graphs were drawn based on the ratio of volume to capacity, the travel time resulting from the methods and the actual travel time. These diagrams are shown in figure (3).*

*In these diagrams, the horizontal axis represents the ratio of volume to capacity and the vertical axis represents the travel time. As can be seen in the diagram of first-class arterial streets, the travel time obtained from the presented model (BPR model with additional terms marked by continuous line) compared to the travel time obtained from the model presented by traffic experts (dotted line) ) is closer to the actual travel time (indicated by +) for almost every street segment. These results are repeated in the second grade arterial streets similarly. Therefore, it can be said that the travel time obtained from the presented model is more accurate than the travel time obtained from the model provided by traffic experts.*

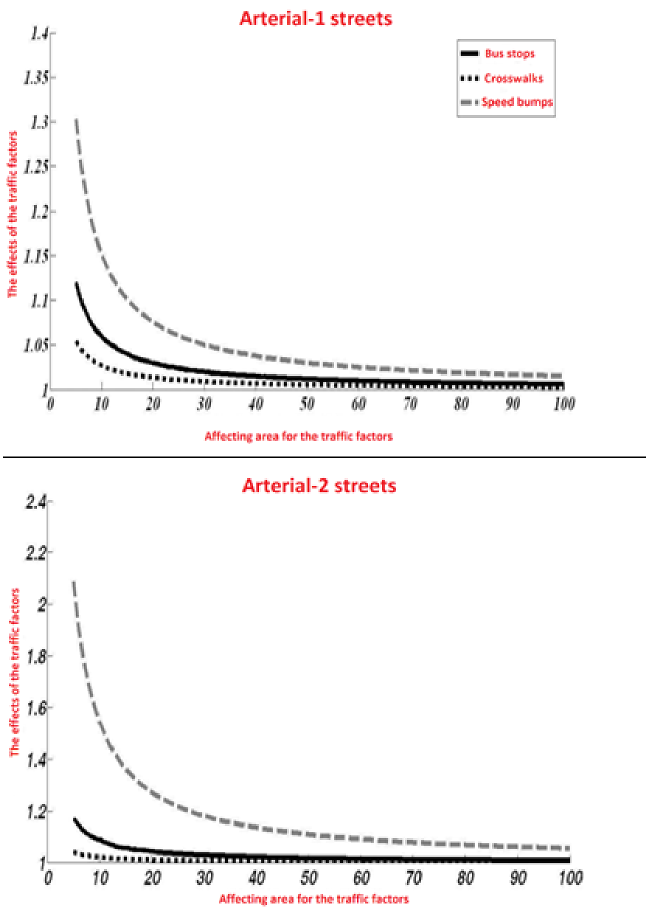


*Figure (3). The coefficients for the effect of the factors*

*In order to determine how many percentages the presented model improves the accuracy in predicting the travel time compared to the models presented by the traffic experts, the standard deviation related to it in predicting the travel time and the standard deviation related to the models presented by the traffic experts. It was obtained according to equation (7) for the streets that participated in the survey.*

*The standard deviation is calculated based on:*

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(t_R - t_M)^2}{n-1}} \qquad (7)$$

*Where $t_R$ denotes the actual travel time and $t_M$ denotes the predicted travel time generated by a model; and n denotes the total number of street segments.*

### *3.3. Comparing the performance of the proposed model against the model used by traffic experts*

*This measure was calculated for both proposed and currently used models; the results are summarized in Table 3.*

*Table (3): RMSE for two kinds of model on arterials*

| In *Seconds* | *Arterial-1 streets* | *Arterial-2 streets* |
|---|---|---|
| *RMSE for BPR models presented by experts* | 7.8158 | 23.0999 |

| | | |
|---|---|---|
| *RMSE for weighted BPR models* | 2.9358 | 3.2688 |
| *Ratio of RMSE for the models* | 2.6622 | 7.0668 |

*As shown in this table, the proposed model is approximately three and seven times more accurate than the existing model (BPR models presented by experts) for arterial- 1 and arterial- 2 streets, respectively.*

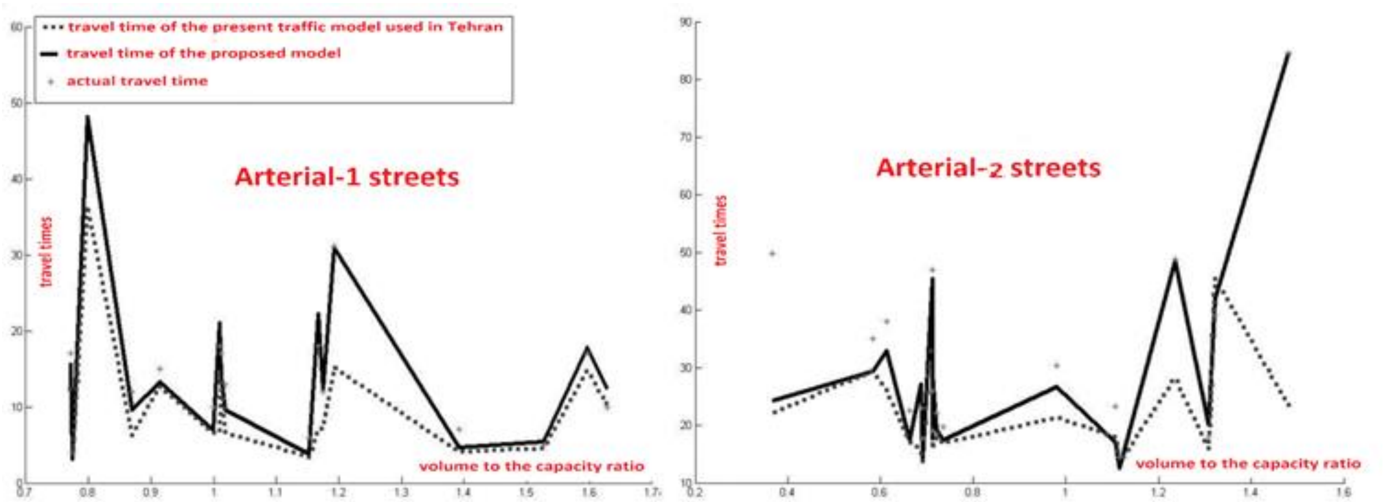


*Figure 4. Comparing Travel time resulted from traffic engineer's model and presented model*

***4.1. Conclusion***

*The paper proposes a weighted BPR model for estimating the effects of route factors on travel time. The model is an extension to the BPR model where in addition to volume and capacity, some on-the-road factors along with their zone of influence are used to estimate the travel time. In this paper, to reduce the complexity of the problem, we added street bumps, zebra lines and bus stops to the basic BPR model. In order to compute the parameters of the weighted model, we gathered a large set of data in train and test groups. The former was used to estimate the unknowns using the Least Squares method, while the latter was involved in evaluating the effectiveness of the presented model in many Tehran streets. The following are the outcomes of applying this model to the city of Tehran:*

*a) The proposed model is three times more accurate than the models used by traffic experts to estimate traffic flow in arterial-1 streets. For arterial-2 streets, this difference in accuracy is sevenfold.*

*b) The weighted BPR model incorporates on-road factors that are not included in currently used models; obviously, including these factors improves model accuracy, and thus should be included and expanded upon when developing more accurate models.*

*One important note was that, even in countries like Iran where traffic rules are not observed properly, still including additional on-the-roads parameters can improve the accuracy of travel time estimation. Nevertheless, the computations would be more reliable and repeatable if the rules are followed more properly by cars and pedestrians.*

*We suggest, studying the effect of other factors like car parks, taxi stations and the like in future studies. Also, as deep learning techniques have evolved greatly in recent years, we suggest exploring their ability in development of a travel estimation approach.*

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