



Predicting the fluctuation of travel time reliability as a result of congestion variations by bagging-based regressors

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Abstract:

Travel time reliability affects the behavior of passengers in private or public transportation and can be seen as an important factor in the context of freight transportation. The main cause of travel time oscillation, known as travel time reliability, is congestion. Congestion is classified into two categories: recurring and nonrecurring. Recurring congestion, which is the topic of this study, is formed when supply surpasses capacity. Peak periods are good examples of recurring congestion. In this paper, by utilizing different bagging regressor methods, the effect of speed flow reduction, compared to free flow speed (FFS) in terms of congestion was studied on the planning time index (PTI) on a section of Interstate 64 in the United States (US). Then, by analyzing PTI changes based on congestion variation, it was revealed that when speed reduction surpasses 10%, travel time leaves its reliability. Also, when the congestion is somewhere around 0.7 to 0.75, the unreliability becomes severe. These findings were directly extracted from scatter plots drawn by bagging and bootstrapping samples which were used to improve the accuracy of PTI prediction.

Keywords: Travel time reliability, Planning time index (PTI), Congestion, Bagging regressor, Peak period, Machine learning

1. Introduction

The term travel time reliability (TTR) refers to the travel time fluctuations for the same trip from day to day. The same trip is implied on a trip that is done for the same purpose, the same origin, and destination, within the same time of day, and by the same mode and route. Large variability implies that travel time is unreliable, and this unpredictability causes travelers and shippers to have a challenge with planning their travel. The main cause of unreliability in travel time is congestion. With the occurrence of congestion, it could be expected that travel times become more variable, hence less reliable (National Academies of Sciences and Medicine, 2013).

Congestion can be studied in two categories: recurring and non-recurring. Recurring congestion is predictable and occurs when supply surpasses capacity, whereas the latter refers to conditions where an unexpected event occurs, such as crashes, inclement weather, work zones, and so on (Mahmassani et al., 2014).

Studies have revealed that the average congestion level is continuing to grow in cities and regarding the points mentioned earlier, TTR is a key part of the congestion problem. Travelers try to lessen the negative effect of their tardiness by assigning additional time beyond typical travel time to ensure they arrive on time. Unfortunately, this extra time is associated with extra costs, which has not been considered in previous transportation analyses (Zegeer et al., 2014). Due to its importance, reliability has been the subject of many studies. Different researchers have attempted to depict the context of reliability by utilizing different methods, including linear regression, simulation, machine learning (ML) methods, and so on. This study is an attempt to provide further insight into the effect of recurring congestion (morning and evening peak periods) on TTR measurement, that is planning time index (PTI). What has made this study unique, is utilizing bagging regressors. The advantage of these regressors will be described in detail in the methodology section. The body of the paper is organized as follows:

the rest of this section analyzes conducted studies in the field of TTR from initial points, dating back to 1968, to the present studies. In the second section, the material and methods will be described and further information on dataset will be represented. Section three discusses modeling procedure and by utilizing scatter plots, depicts how congestion can affect PTI. Finally, the conclusion in section 4 is represented.

The initial step of research in the field of congestion and travel time reliability started with Gaver's study in 1968 at Berkeley University. His study looked at the policy choices that occur when both tardiness and an undesirable early departure are penalized (Gaver Jr, 1968). Connectivity reliability and travel time reliability were then introduced by Iida in 1999 to address core ideas, unsolved challenges, and future prospects of road network reliability analysis (Iida, 1999). The fact that reducing travel time uncertainty is as important as saving time, was pointed out by Chen and Recker's study in 2002 (Chen et al., 2002).

Later on, researchers attempted to calculate and assess travel time reliability and its effect on different aspects of transportation systems' behavior. The importance of travel time reliability as a decisive factor affecting travelers' route choice decisions was studied by Liu et al. (2004). (Liu et al., 2004). Emam and Al-Deek, 2006, created a novel approach for calculating travel time reliability using real-world traffic data from Florida's I-4 corridor (Emam and Al-Deek, 2006). Lyman and Bertini predicted the reliability of travel time on a specific corridor in 2008 by the use of archived intelligent transportation system (ITS) data and investigating the use of measured travel time reliability indices for enhancing real-time transportation (Lyman and Bertini, 2008). To examine travel time reliability in New York City, three travel time reliability metrics were used by Yazici et al to assess the influence of New York City's urban grid network on travel time (Yazici et al., 2012). Wang et al. in 2017 established a system for estimating highway vehicle travel time reliability for transportation planning utilizing probe GPS data (Wang et al., 2017). Also, Zheng et al. concluded that travelers' route and departure time

decisions are influenced by the expected travel time and its reliability (Zheng et al., 2018). In addition to the 2004 study by Liu et al., Li et al., 2019, investigated travel time reliability as a critical element influencing passenger behaviors. They used the Lempel-Ziv algorithm to make their study unique (Li et al., 2019). The study of Moghaddam et al. looked at how travelers perceive and respond to travel time information and its reliability in terms of route choice behavior, as measured by a driving simulator and a stated preference (SP) survey (Moghaddam et al., 2019).

As another wave of modeling in the field of travel time reliability, researchers focused on congestion and factors that could cause it. Hojati et al. defined the extra buffer time index (EBTI) to quantify traffic incidents' effects on motorways TTR. The type of incident and the time it takes for travelers to arrive at their destination were factors that might also impact EBTI (Hojati et al., 2016). The study of Samal and Das in 2020 intended to investigate and assess the possibility of modeling congestion metrics under diverse traffic scenarios in the Patia region. (Samal and Das, 2020). Gu et al. performed a review of studies on transportation network performance under perturbations to address reliability, vulnerability, and resilience in networks. They determined that although these notions differ in terms of focus, measurement, and application, their outcomes are not different (Gu et al., 2020). The paper of Zhang and Chen developed an integrated data mining framework based on decision tree and quantile regression approaches to identify periods with varying traffic characteristics and evaluate the impact of rain and snow events on both congestion and system reliability (Zhang and Chen, 2019).

As the importance of TTR became more and more, many studies attempted to utilize novel approaches to address previous concerns and problems. Relying on the Cornish-Fisher expansion, the paper of Zang et al. used the travel time percentile function and provided a closed-form, adaptable, and high-quality technique that was sufficiently adaptive to predict the

percentile function of various travel time distributions (TTDs) (Zang et al., 2018). Ghader et al. in 2019 utilized Cumulative Prospect Theory (CPT) to study how travel mode choice is affected by travel time reliability. Their main focus was on mode choice, but their model could be extended to other choice dimensions (Ghader et al., 2019). Chen and Fan researched to provide a systematic framework for assessing TTR on highway segments in Charlotte, North Carolina. The numerical findings clearly demonstrated that TTR patterns in each case were unique, as well as on various days of the week and weather conditions (Chen and Fan, 2019). The principal focus of Saedi et al. research was to enhance the estimation of network travel time reliability by utilizing network partitioning (Saedi et al., 2020). The results of Chen and Fan's research are noteworthy because they provided a systematic framework to evaluate TTD on various types of highway segments throughout a corridor. They realized that the Burr distribution could give the highest acceptance rate when different times of day (TOD) and days of week (DOW) were considered (Chen and Fan, 2020). Zhu et al. provided numerous categories of perceived information based on a generalized Bayesian traffic model to simulate travelers' daily route choice behavior in terms of travel time reliability (Zhu et al., 2021). The paper of Zhang, X., et al. investigated statistical approaches for clustering Cumulative Distribution Functions (CDFs) of travel times at the segment level into an optimum number of homogenous clusters that could include all essential information about distributions (Zhang et al., 2021). The study of Hoseinzadeh et al. combined crowdsourced data from Waze to develop an algorithm for the hourly measurement of level of service (LOS) (Hoseinzadeh et al., 2021). Afandizadeh et al. evaluated the effect of recurrent congestion on travel time reliability on a 1.467-mile section of the I-64 highway in Virginia. The authors proposed grey models (GM) and random forest regression (RFR) as evaluation tools (Zargari et al., 2022). Chen et al. developed a Collaborative Intelligent Transportation System (CITS) in 2022 to estimate present and future travel times. The findings indicated that the K-nearest neighbor (KNN)

model could deliver the most accurate short-term forecasts (Chen et al., 2022). Also, Udayanga et al. recommended using crowdsourced travel time data from Google distance matrix Application Programming Interface (API) as a feasible approach to combine traffic congestion monitoring in their study (UDAYANGA et al., 2022). The research of Meng et al. investigated the performance of the Support Vector Machine (SVM) in predicting short-term travel times (Meng et al., 2022).

Delving deep into the chronological trajectory of the literature review reveals that travel time reliability was just a simple notion in the beginning, but as time went on, researchers concluded that TTR affects various parts of the transportation system. The tools for addressing these concerns were basic but became more complicated later.

2. Materials and Methods

To talk about bagging regressors, first, a background about ensemble methods is needed, then different branches of ensemble methods will be briefly discussed. Afterwards, bagging methods will be illustrated and finally, supplementary explanations about bagging regressor will be represented.

Ensemble methods aim to increase generalizability/robustness over a single estimator by combining the predictions of numerous base estimators created using a specific learning methodology. Typically, two groups of ensemble approaches are distinguished: averaging and boosting methods. The core argument behind averaging approaches is to create numerous independent estimators, then average their estimations. Because its variance is decreased, the composite estimator is generally better than the individual single-base estimators. Unlike the first method, the latter produces sequential base estimators, and the composite estimator's bias is reduced. This method merges numerous weak models into a powerful ensemble. Examples of averaging methods are bagging methods and forest of randomized trees. Also, AdaBoost

and gradient tree boosting are examples of boosting methods. The interested reader is referred to the cited references for a detailed description of the methods (Zhou, 2012)

Bagging methods are a class of algorithms in which numerous samples of black-box estimators are built on random subsets of the original training set, and then their individual estimations are aggregated to generate a final prediction. These techniques are used to lessen the variation of a base estimator by including randomization into its development mechanism and then constructing an ensemble from it. Under many circumstances, bagging methods are a fairly straightforward approach to improve compared to a single model without changing the underlying base algorithm. Bagging approaches perform best with strong and complicated models because they reduce overfitting. Please read this reference for further information on this method (Kadiyala and Kumar, 2018)

As an ensemble estimator, a bagging regressor fits base regressors of the main database and then aggregates their individual forecasts (through voting or averaging) to generate a final prediction. A meta-estimator of this type is often used to minimize the variance of a black-box estimator by incorporating randomization into its building mechanism and then constructing an ensemble from it (Pedregosa et al., 2011). The parameters of the bagging regressor are as follows: base estimator (the base estimator that fits on subsets of the dataset which are created randomly), number of estimators (number of base estimators in the ensemble), maximum samples (number of samples which are drawn to train base estimators), maximum features (number of features to train base estimator), bootstrap (how samples are drawn, with or without replacement), bootstrap features (if features are extracted with replacement.), out-of-bag score (determines whether you utilize out-of-bag samples for estimating the generalization error), warm start, number of jobs, random state, and verbose. The utilized methods in this study are as follows. It should be noted that suggested studies and articles are for the future reference of interested readers.

- bagging regressor with Stochastic Gradient Descent (SGD) base estimator (Mazloumi et al., 2022)
- Bagging regressor with Passive Aggressive base estimator (Mastelini et al., 2022)
- Bagging regressor with Ridge base estimator (Abdulhafedh, 2022)
- Bagging regressor with linear base estimator (Shabbir et al., 2022)
- Bagging regressor with Support Vector Regression (SVR) (Kernel= radial basis function (RBF)) base estimator (Ara et al., 2020)
- bagging regressor with Support Vector Regression (SVR), (Kernel= polynomial (Poly)) base estimator (Ara et al., 2020)
- Bagging regressor with RANdom SAmples Consensus (RANSAC) base estimator (Almejr et al., 2022)
- Bagging regressor with Decision Tree (DT) base estimator (Abdulhafedh, 2022)
- Bagging regressor with Theil-Sen base estimator (Szafranski and Duan, 2022)
- Bagging regressor with Gradient Boosting (GB) base estimator (Khan et al., 2022)
- bagging regressor with random forest (RF) base estimator (Zhan et al., 2021)
- bagging regressor with polynomial (Poly) base estimator (Adhastian and Wibowo, 2022)
- bagging regressor with support vector regression (SVR), (Kernel= linear) base estimator (Sarang, 2023)
- bagging regressor with Bayesian ridge base estimator (Gacto et al., 2019)
- bagging regressor with Quantile base estimator (Kang and Hansen, 2021)

For more information, you can also refer to “sklearn ensemble module” in Python. In support vector regression (SVR) models, the type of kernel function has been written in parentheses.

The dataset of this paper is composed of two elements, namely the travel time reliability (TTR) metric and congestion indices. As the dependent variable, congestion is defined as the ratio of traffic speed over a one-hour period to the free flow speed. This definition for congestion was

directly extracted from INRIX. Also, INRIX was used to obtain the TTR and congestion statistics.

Every day, billions of data points are used by INRIX to gather anonymized data on traffic congestion, traffic incidents, and weather-related road conditions. The data has multiple sources, including connected cars and mobile devices, cameras and sensors on the road, major events that are expected to impact traffic, and other sources. This analysis is conducted by the company to comprehend mobility trends. To put it another way, INRIX offers cutting-edge solutions for real-time traffic. The interested reader can refer to INRIX's website¹ for further information.

To calculate congestion, INRIX uses a multi-step process. First, the Space Mean Speed (SMS) is calculated for the desired segment. In fact, SMS is the mean speed of all cars crossing a specific segment of road over a given period. Then, this speed is divided by free flow speed to calculate the congestion of that segment for the specific period.

Also, the planning time index (PTI), defined as the ratio of the 95th percentile of travel time to the free-flow travel time, is the independent variable (Lyman and Bertini, 2008). To extract PTI, INRIX builds a statistical distribution by the travel time data of vehicles passing a specific segment. Then, the 95th percentile of this distribution will be divided to travel time of free flow, which can be easily calculated, and PTI will be extracted.

The data collection period ranges from February 1, 2018, to October 31, 2018, for 273 days, and only considers workdays. Every day is split into 24 equal sections. Analyzing the trend of mean congestion values using the two-tailed comparing mean has proven that there are two peak periods in a day, namely morning peak and evening peak. The morning peak is from 7 a.m. until 9 a.m. and the evening peak starts at 15 and ends at 18. Each observation represents the average amount of congestion (the ratio of flow speed in one hour to free flow speed) and

¹ <https://inrix.com/>

PTI of vehicles that have passed through the 1.467-mile segment during one hour. Also, it should be pointed out that all passing vehicles, regardless of their type (the information is gathered anonymously from cars, trucks, and many other types of vehicles) were considered. Furthermore, the number of observations (samples) that have been analyzed in this research for various days are as follows (The numbers in the parenthesis represent morning and evening peaks respectively): Monday (78,117), Tuesday (74,111), Wednesday (76,114), Thursday (78,117), Friday (76,114). **Table 1** summarizes the statistical features of morning and evening peaks. As a case study, in this paper, a road segment along the I-64 freeway in Chesapeake, Virginia was analyzed, which is the same as Afandizadeh et al dataset in their research (Zargari et al., 2022). This segment contains 3 sections whose lengths are 1.467, 0.036, and 0.777 miles respectively. The focus is on the first segment, which is shown in **Fig 1**.

Table 1: Summary of PTI and congestion statistics

Metrics	Peaks	Time periods	Statistics					
			Min	Q1	Median	Q3	Max	Average
PTI	Morning	7:00-8:00	1.0	1.1	2.1	3.4	20.3	2.9
		8:00-9:00	1.0	1	1.4	2.8	20.3	2.4
	Evening	15:00-16:00	1.0	1.4	2.3	2.7	8.7	2.2
		16:00-17:00	1.0	2.2	2.8	3.4	12.2	3.0
		17:00-18:00	1.0	1.3	2.4	3.2	20.3	2.6
Congestion *100	Morning	7:00-8:00	10.1	44.1	76	98.4	100.0	69.8
		8:00-9:00	9.0	61.1	92.9	100	100.0	79.0
	Evening	15:00-16:00	16.3	55.2	71.7	95	100.0	72.2
		16:00-17:00	16.2	39.1	46.9	69.3	100.0	54.6
		17:00-18:00	7.1	43.5	60.6	95.3	100.0	65.5

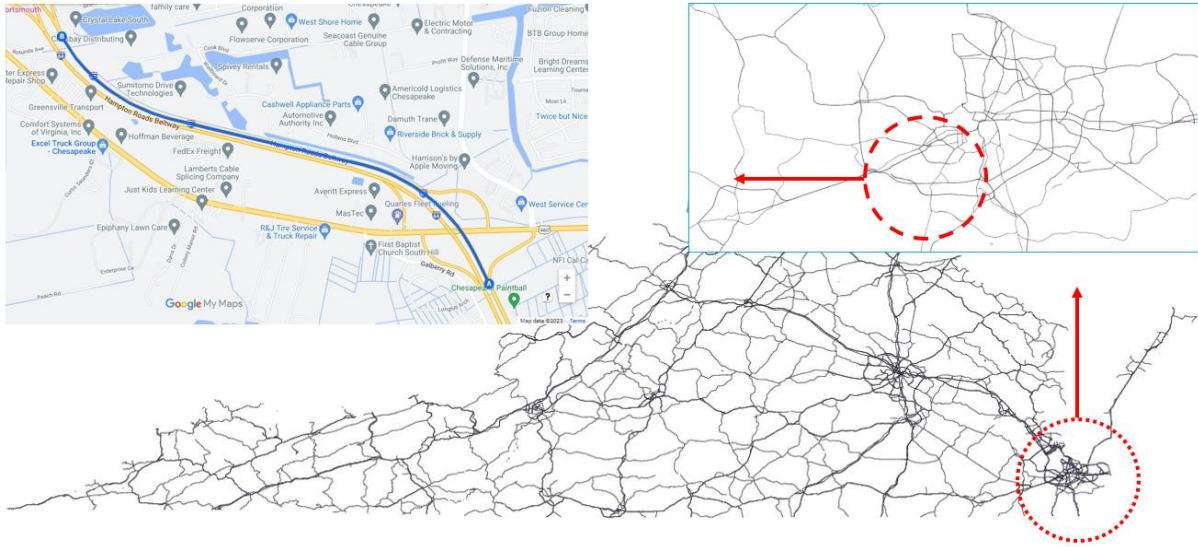


Fig 1: Location of the sample segment (Source: Google Maps& Virginia shape files)

3. Results and Discussion

To better explain the advantages of using bagging regressors, the results of modeling, including coefficient of determination, Mean Squared Error (MSE), and the stability ratio, are represented in **tables 2 to 6**. Coefficient of determination, R^2 , explains the variability of factors that would be caused by its relationship to another factor, and MSE assesses the average squared difference between the observed and predicted values. Coefficient of determination and MSE (consequently RMSE) are among well-known measures for quantifying the quality of an estimator and numerous studies have taken the advantage of such measures, like the study of Nohekhan et al. (Nohekhan et al., 2022). Stability ratio, or simply, ratio, is the ratio of train set coefficient of determination to test set coefficient of determination and is a numerical criterion to show whether the model has overfitting or underfitting. The ideal value for this ratio is one. When the ratio equals one, it means that the performance of the train set and test set is the same, in terms of coefficient of determination. When overfitting occurs, this ratio becomes bigger than one, and in the case of underfitting, this ratio is less than one. The interested reader is referred to these publications for an in-depth review of the mentioned metrics: (Arias-Castro, 2022). To opt for the prior model, three metrics will be used: coefficient of determination,

MSE, and ratio. The closer the coefficient of determination is to one, the better the model is (in both training set and test set). Also, the prior model has less error, so models with less MSE should be considered. Furthermore, overfitting and underfitting, which are common issues in machine learning models, are monitored through the stability ratio. Akaike information criterion (AIC), and Bayesian information criterion (BIC) are two statistical metrics that could be utilized to estimate the quality of each model, relative to each of the other models. AIC is simply used as a metric to compare the performance of models. The model with the lowest AIC offers the best fit. The absolute value of the AIC value is not important. For further information about these two metrics, please read the following references: (Hastie et al., 2009). As explicitly stated by Chakrabarti and Ghosh, “The Bayesian Information Criterion (BIC) is more useful in selecting a correct model while the AIC is more appropriate in finding the best model for predicting future observations” since the purpose of this study is to predict fluctuations of PTI as congestion changes, AIC will be used for selecting prior model (Chakrabarti and Ghosh, 2011). For Mondays, as the results of **tables 2(a)** and **2(b)** reveal, AIC suggests that the bagging regressor with the gradient boosting as the base estimator is the prior model in both morning and evening peaks.

Table 2 (a): Modeling results for Mondays-morning peak

Models	Training			Validation			Stability
	R ²	MSE	AIC	R ²	MSE	AIC	
SVR (kernel=RBF)	0.55	2.46	-100.2	0.72	0.81	-27.09	0.76
SVR (kernel=linear)	0.53	2.54	-100.2	0.69	0.91	-27.09	0.77
RANSAC Regressor	0.6	2.19	-104.2	0.77	0.69	-31.09	0.78
Decision Tree Regressor	0.59	2.21	-100.2	0.76	0.71	-27.09	0.78
TheilSen Regressor	0.62	2.07	-110.2	0.78	0.64	-37.09	0.79
Passive Aggressive Regressor	0.64	1.96	-94.2	0.8	0.6	-21.09	0.8
SGD Regressor	0.58	2.28	-84.2	0.71	0.84	-11.09	0.81
Ridge	0.62	2.06	-104.2	0.76	0.7	-31.09	0.82
Linear Regression	0.68	1.73	-94.2	0.83	0.49	-21.09	0.82
Quantile Regressor	0.66	1.87	-96.2	0.8	0.6	-23.09	0.82
Bayesian Ridge	0.65	1.88	-112.2	0.79	0.61	-39.09	0.83
SVR (kernel=poly, degree=2)	0.79	1.12	-88.2	0.86	0.41	-15.09	0.92

Polynomial Regression	0.88	0.64	-100.2	0.87	0.39	-27.09	1.02
Gradient Boosting Regressor	0.87	0.73	-112.2	0.84	0.48	-39.09	1.03
Random Forest Regressor	0.92	0.42	-80.2	0.87	0.4	-7.09	1.07

Table 2 (b): Modeling results for Mondays-evening peak

Models	Training			Validation			Stability
	R ²	MSE	AIC	R ²	MSE	AIC	
Quantile Regressor	0.7	0.42	-187.77	0.76	0.28	-62.91	0.91
Passive Aggressive Regressor	0.69	0.44	-185.77	0.74	0.3	-60.91	0.93
Ridge	0.67	0.45	-195.77	0.72	0.33	-70.91	0.94
Bayesian Ridge	0.69	0.43	-203.77	0.74	0.3	-78.91	0.94
TheilSen Regressor	0.66	0.47	-201.77	0.68	0.37	-76.91	0.98
RANSAC Regressor	0.66	0.47	-195.77	0.67	0.37	-70.91	0.98
Decision Tree Regressor	0.66	0.48	-191.77	0.66	0.39	-66.91	0.99
SVR (kernel=linear)	0.59	0.56	-191.77	0.58	0.48	-66.91	1.02
SVR (kernel=RBF)	0.63	0.52	-191.77	0.62	0.44	-66.91	1.02
Linear Regression	0.75	0.35	-185.77	0.72	0.32	-60.91	1.04
Gradient Boosting Regressor	0.88	0.17	-203.77	0.82	0.21	-78.91	1.08
SGD Regressor	0.44	0.77	-175.77	0.4	0.69	-50.91	1.11
SVR (kernel=poly, degree=2)	0.89	0.15	-179.77	0.79	0.24	-54.91	1.12
Random Forest Regressor	0.95	0.07	-171.77	0.79	0.24	-46.91	1.2
Polynomial Regression	0.96	0.05	-191.77	0.75	0.29	-66.91	1.28

The results of modeling for Tuesdays are shown in **tables 3(a)** and **3(b)**. As it can be seen, in both morning and evening peaks, gradient boosting regressor has the best set of metrics and is opted for the modeling process.

Table 3 (a): Modeling results for Tuesdays-morning peak

Models	Training			Validation			Stability
	R ²	MSE	AIC	R ²	MSE	AIC	
SVR (kernel=linear)	0.52	2.62	-95.5	0.66	1.45	-19.46	0.8
SGD Regressor	0.62	2.11	-79.5	0.71	1.21	-3.46	0.86
SVR (kernel=RBF)	0.55	2.46	-95.5	0.64	1.53	-19.46	0.87
Ridge	0.66	1.87	-99.5	0.76	1.01	-23.46	0.87
Passive Aggressive Regressor	0.66	1.85	-89.5	0.76	1	-13.46	0.87
TheilSen Regressor	0.64	1.98	-105.5	0.74	1.11	-29.46	0.87
RANSAC Regressor	0.62	2.08	-99.5	0.72	1.2	-23.46	0.87
Quantile Regressor	0.69	1.73	-91.5	0.78	0.93	-15.46	0.88
Bayesian Ridge	0.68	1.77	-107.5	0.76	1	-31.46	0.89
Decision Tree Regressor	0.71	1.58	-95.5	0.78	0.92	-19.46	0.91
Linear Regression	0.84	0.91	-89.5	0.82	0.76	-13.46	1.02
Gradient Boosting Regressor	0.97	0.18	-107.5	0.79	0.88	-31.46	1.22

SVR (kernel=poly, degree=2)	0.96	0.21	-83.5	0.78	0.91	-7.46	1.23
Random Forest Regressor	0.99	0.06	-75.5	0.77	0.97	0.54	1.29
Polynomial Regression	0.98	0.1	-95.5	0.76	1	-19.46	1.29

Table 3 (b): Modeling results for Tuesdays-evening peak

Models	Training			Validation			Stability
	R ²	MSE	AIC	R ²	MSE	AIC	
Bayesian Ridge	0.66	0.64	-203.04	0.53	1.23	-71.54	1.25
Quantile Regressor	0.66	0.63	-187.04	0.52	1.24	-55.54	1.26
Random Forest Regressor	0.97	0.05	-171.04	0.77	0.6	-39.54	1.27
Ridge	0.64	0.68	-195.04	0.5	1.32	-63.54	1.28
Linear Regression	0.78	0.41	-185.04	0.6	1.05	-53.54	1.31
SVR (kernel=linear)	0.49	0.96	-191.04	0.37	1.64	-59.54	1.32
Gradient Boosting Regressor	0.96	0.08	-203.04	0.72	0.73	-71.54	1.32
SVR (kernel=poly, degree=2)	0.96	0.07	-179.04	0.72	0.73	-47.54	1.34
TheilSen Regressor	0.6	0.76	-201.04	0.44	1.45	-69.54	1.34
RANSAC Regressor	0.6	0.76	-195.04	0.44	1.46	-63.54	1.35
SGD Regressor	0.5	0.94	-175.04	0.37	1.65	-43.54	1.35
SVR (kernel=RBF)	0.57	0.81	-191.04	0.42	1.53	-59.54	1.37
Decision Tree Regressor	0.82	0.34	-191.04	0.59	1.06	-59.54	1.38
Polynomial Regression	0.97	0.05	-191.04	0.7	0.79	-59.54	1.4
Passive Aggressive Regressor	0.5	0.95	-185.04	0.34	1.72	-53.54	1.45

The same story goes true for morning and evening peaks of Wednesdays. Gradient boosting has shown the most satisfying performance metrics for both peaks and was chosen as the prior model of this day.

Table 4 (a): Modeling results for Wednesdays-morning peak

Models	Training			Validation			Stability
	R ²	MSE	AIC	R ²	MSE	AIC	
SVR (kernel=linear)	0.35	6.2	-93.94	0.74	0.6	-20.46	0.48
SVR (kernel=RBF)	0.41	5.68	-93.94	0.79	0.48	-20.46	0.51
RANSAC Regressor	0.45	5.24	-97.94	0.83	0.39	-24.46	0.55
Decision Tree Regressor	0.49	4.88	-93.94	0.86	0.32	-20.46	0.57
TheilSen Regressor	0.49	4.94	-103.94	0.84	0.37	-30.46	0.58
Passive Aggressive Regressor	0.52	4.63	-87.94	0.76	0.55	-14.46	0.68
SGD Regressor	0.55	4.32	-77.94	0.74	0.59	-4.46	0.74
Ridge	0.58	4	-97.94	0.55	1.03	-24.46	1.06
Quantile Regressor	0.59	3.95	-89.94	0.54	1.05	-16.46	1.09
Random Forest Regressor	0.92	0.75	-73.94	0.82	0.41	-0.46	1.12
Gradient Boosting Regressor	0.93	0.66	-105.94	0.81	0.43	-32.46	1.15

Polynomial Regression	0.96	0.35	-93.94	0.81	0.43	-20.46	1.19
SVR (kernel=poly, degree=2)	0.88	1.12	-81.94	0.72	0.64	-8.46	1.23
Bayesian Ridge	0.59	3.95	-105.94	0.47	1.2	-32.46	1.24
Linear Regression	0.81	1.87	-87.94	0.77	0.31	-31.46	1.02

Table 4 (b): Modeling results for Wednesdays-evening peak

Models	Training			Validation			Stability
	R ²	MSE	AIC	R ²	MSE	AIC	
SVR (kernel=linear)	0.55	0.73	-184.96	0.67	0.44	-55.99	0.83
SGD Regressor	0.5	0.82	-168.96	0.59	0.54	-39.99	0.84
Ridge	0.64	0.58	-188.96	0.76	0.32	-59.99	0.85
SVR (kernel=RBF)	0.59	0.67	-184.96	0.69	0.41	-55.99	0.85
TheilSen Regressor	0.63	0.61	-194.96	0.74	0.35	-65.99	0.85
RANSAC Regressor	0.62	0.62	-188.96	0.73	0.36	-59.99	0.85
Passive Aggressive Regressor	0.61	0.64	-178.96	0.71	0.39	-49.99	0.86
Quantile Regressor	0.68	0.53	-180.96	0.78	0.29	-51.99	0.87
Bayesian Ridge	0.68	0.53	-196.96	0.78	0.29	-67.99	0.87
Decision Tree Regressor	0.72	0.46	-184.96	0.82	0.24	-55.99	0.88
Linear Regression	0.78	0.35	-178.96	0.74	0.34	-49.99	1.05
SVR (kernel=poly, degree=2)	0.95	0.09	-172.96	0.74	0.35	-43.99	1.28
Random Forest Regressor	0.97	0.05	-164.96	0.74	0.35	-35.99	1.32
Gradient Boosting Regressor	0.95	0.07	-196.96	0.71	0.38	-67.99	1.34
Polynomial Regression	0.96	0.06	-184.96	0.68	0.42	-55.99	1.41

For Thursdays, as the results of **tables 5(a)** and **5(b)** reveal, gradient boosting is the prior choice for both peaks and will be included in the sensitivity analysis stage.

Table 5 (a): Modeling results for Thursdays-morning peak

Models	Training			Validation			Stability
	R ²	MSE	AIC	R ²	MSE	AIC	
SVR (kernel=linear)	0.33	7.02	-96.45	0.47	3.04	-17.14	0.7
TheilSen Regressor	0.45	5.78	-106.45	0.55	2.57	-27.14	0.82
RANSAC Regressor	0.43	5.97	-100.45	0.52	2.71	-21.14	0.82
SVR (kernel=RBF))	0.39	6.37	-96.45	0.47	2.99	-17.14	0.82
Passive Aggressive Regressor	0.51	5.12	-90.45	0.61	2.24	-11.14	0.84
SGD Regressor	0.54	4.79	-80.45	0.64	2.06	-1.14	0.85
Ridge	0.57	4.49	-100.45	0.63	2.08	-21.14	0.9
Decision Tree Regressor	0.65	3.66	-96.45	0.69	1.75	-17.14	0.94
Bayesian Ridge	0.6	4.22	-108.45	0.59	2.32	-29.14	1.01
SVR (kernel=poly, degree=2)	0.9	1.03	-84.45	0.79	1.18	-5.14	1.14
Random Forest Regressor	0.97	0.34	-76.45	0.83	0.98	2.86	1.17
Quantile Regressor	0.6	4.18	-92.45	0.51	2.77	-13.14	1.17

Gradient Boosting Regressor	0.98	0.22	-108.45	0.8	1.13	-29.14	1.22
Polynomial Regression	0.96	0.41	-96.45	0.77	1.32	-17.14	1.25
Linear Regression	0.8	2.07	-90.45	0.63	2.11	-11.14	1.28

Table 5 (b): Modeling results for Thursdays-evening peak

Models	Training			Validation			Stability
	R ²	MSE	AIC	R ²	MSE	AIC	
SVR (kernel=linear)	0.2	5.11	-193.05	0.39	1.82	-62.21	0.53
SVR (kernel=RBF)	0.28	4.62	-193.05	0.47	1.56	-62.21	0.59
TheilSen Regressor	0.33	4.28	-203.05	0.55	1.32	-72.21	0.6
SGD Regressor	0.36	4.08	-177.05	0.6	1.19	-46.21	0.61
RANSAC Regressor	0.34	4.23	-197.05	0.56	1.31	-66.21	0.61
Passive Aggressive Regressor	0.39	3.89	-187.05	0.63	1.08	-56.21	0.62
Ridge	0.41	3.76	-197.05	0.65	1.03	-66.21	0.64
Decision Tree Regressor	0.56	2.8	-193.05	0.88	0.36	-62.21	0.64
Quantile Regressor	0.44	3.6	-189.05	0.65	1.04	-58.21	0.68
Bayesian Ridge	0.44	3.57	-205.05	0.64	1.08	-74.21	0.7
SVR (kernel=poly, degree=2)	0.89	0.71	-181.05	0.93	0.21	-50.21	0.96
Gradient Boosting Regressor	0.96	0.27	-205.05	0.91	0.25	-74.21	1.05
Random Forest Regressor	0.97	0.18	-173.05	0.92	0.24	-42.21	1.06
Polynomial Regression	0.99	0.08	-193.05	0.9	0.29	-62.21	1.09
Linear Regression	0.72	1.8	-187.05	0.57	1.29	-56.21	1.27

Finally, the gradient boosting regressor is the prior model of both peaks for Fridays.

Surprisingly, the results show that gradient boosting is the top model of all days in both peaks.

Table 6(a): Modeling results for Fridays-morning peak

Models	Training			Validation			Stability
	R ²	MSE	AIC	R ²	MSE	AIC	
SVR (kernel=linear)	0.27	7.63	-119.87	0.91	0.04	-58.81	0.3
RANSAC Regressor	0.22	8.15	-123.87	0.65	0.14	-62.81	0.34
SVR (kernel=RBF)	0.31	7.3	-119.87	0.82	0.07	-58.81	0.37
Passive Aggressive Regressor	0.43	6.03	-113.87	0.95	0.02	-52.81	0.45
Decision Tree Regressor	0.42	6.15	-119.87	0.87	0.05	-58.81	0.48
Ridge	0.59	4.31	-123.87	0.77	0.09	-62.81	0.77
SGD Regressor	0.61	4.09	-103.87	0.64	0.14	-42.81	0.96
SVR (kernel=poly, degree=2)	0.92	0.89	-107.87	0.93	0.03	-46.81	0.98
Gradient Boosting Regressor	0.97	0.31	-131.87	0.93	0.03	-70.81	1.05
Random Forest Regressor	0.99	0.13	-99.87	0.94	0.02	-38.81	1.05
Polynomial Regression	0.99	0.15	-119.87	0.93	0.03	-58.81	1.06
Linear Regression	0.94	0.68	-113.87	0.44	0.22	-52.81	2.15
TheilSen Regressor	0.67	3.44	-129.87	0.19	0.32	-68.81	3.47

Quantile Regressor	0.65	3.66	-115.87	0.82	0.29	-52.81	0.89
Bayesian Ridge	0.69	3.22	-131.87	0.8	0.33	-52.81	0.92

Table 6(b): Modeling results for Fridays-evening peak

Models	Training			Validation			Stability
	R ²	MSE	AIC	R ²	MSE	AIC	
SVR (kernel=linear)	0.59	0.9	-167.91	0.73	0.44	-46.48	0.8
SVR (kernel=RBF)	0.65	0.76	-167.91	0.78	0.36	-46.48	0.84
SGD Regressor	0.61	0.84	-151.91	0.73	0.44	-30.48	0.84
RANSAC Regressor	0.67	0.72	-171.91	0.8	0.33	-50.48	0.84
TheilSen Regressor	0.69	0.66	-177.91	0.82	0.3	-56.48	0.85
Passive Aggressive Regressor	0.67	0.73	-161.91	0.78	0.36	-40.48	0.85
Ridge	0.71	0.62	-171.91	0.82	0.29	-50.48	0.87
Quantile Regressor	0.73	0.59	-163.91	0.82	0.29	-42.48	0.89
Bayesian Ridge	0.73	0.58	-179.91	0.81	0.3	-58.48	0.9
Linear Regression	0.82	0.39	-161.91	0.85	0.24	-40.48	0.96
Decision Tree Regressor	0.87	0.29	-167.91	0.9	0.16	-46.48	0.97
Gradient Boosting Regressor	0.96	0.09	-179.91	0.9	0.17	-58.48	1.07
SVR (kernel=poly, degree=2)	0.95	0.1	-155.91	0.89	0.18	-34.48	1.08
Random Forest Regressor	0.97	0.06	-147.91	0.85	0.24	-26.48	1.14
Polynomial Regression	0.97	0.08	-167.91	0.84	0.26	-46.48	1.15

As the main method of this study (the prior model) is an ML method, sensitivity plots are very useful tools to visualize how congestion influences PTI. Scatter plots are simple and have made interpretation and usability easy for everyone. After choosing the prior models of different days and peaks, this section is assigned to depict the sensitivity analysis plot. This step will depict how speed reduction (hence congestion reduction) will affect PTI. The intensity of increase is somehow different. This paper uses bagging and bootstrapping to improve the model results and more accurately predict the PTI. To do so, by using different training sets, hundreds of models were randomly produced by bootstrapping from the original dataset to produce these plots.

Figures 2 to 6, in two peaks ((a) for the morning peak, and (b) for the evening peak)) represent the fluctuations in a scatter plot. According to Chen and Fan, it is reliable when PTI is below 1.5. When it surpasses 1.5 but doesn't reach 2.5, it is labeled as moderately to heavily

unreliable, and for the values of PTI bigger than 2.5, it is said to be extremely unreliable.

Equations 1, 2, and 3 show this classification numerically.

$$PTI < 1.5: \text{Reliable} \quad (1)$$

$$1.5 \leq PTI \leq 2.5: \text{Moderate to heavy} \quad (2)$$

$$PTI > 2.5: \text{Extremely unreliable} \quad (3)$$

As **figures 2 to 6** suggest, in both peaks, regardless of the day, when congestion is reduced to 0.9, PTI reaches 1.5. It means that a 10% reduction in speed (compared to free-flow speed), causes a 50% increase in travel time compared to free-flow travel time. In other words, when congestion is in the range of 1 to 0.9, PTI is reliable in both peaks. Somewhere between 0.7 and 0.75 is a point where the PTI reaches 2.5, meaning that PTI is leaving the moderate or heavy unreliable part, and enters an extremely unreliable phase. It is worth noting that, the intensity of PTI increase is slight until congestion is 0.5, then as congestion decreases, the PTI increase will be more severe. This study's findings are compatible with the results of utilizing random forest (RF) regression, which was the paper's main focus by Afandizadeh et al (Zargari et al., 2022).

As the main aim of this study is to analyze on planning level, sub-temporal variations including possible variability between the months were excluded. Undoubtedly, such variations are interesting to study, but could be the subject of later studies, and is out of the scope of this manuscript. Also, the type of road, road geometry, and the type of traffic (modal split) can influence the results but requires more detailed data.

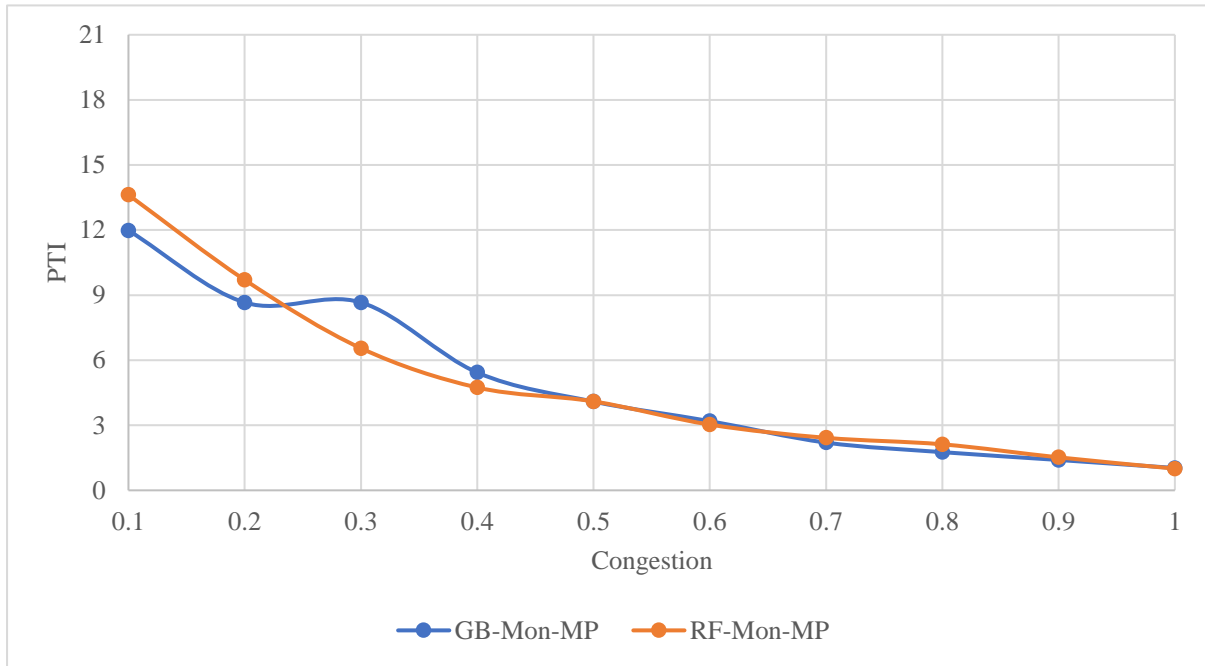


Fig 2 (a): Comparison between GB and RF modeling results- Monday morning peak

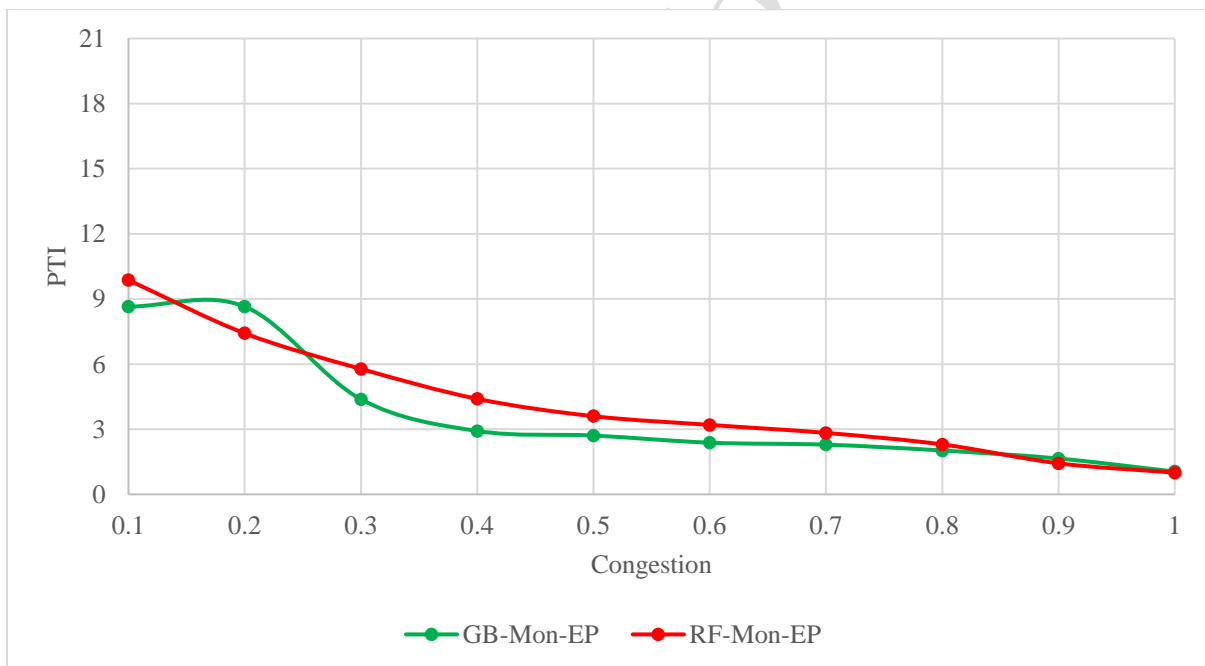


Fig 2 (b): Comparison between GB and RF modeling results- Monday evening peak

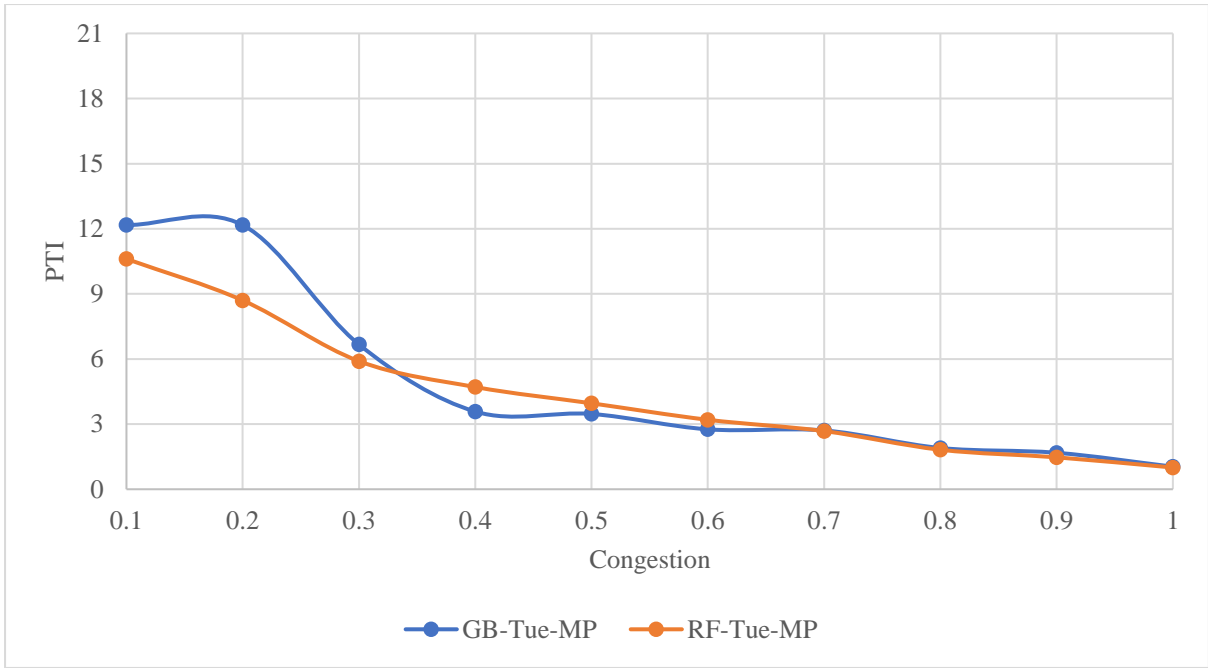


Fig 3 (a): Comparison between GB and RF modeling results- Tuesday morning peak

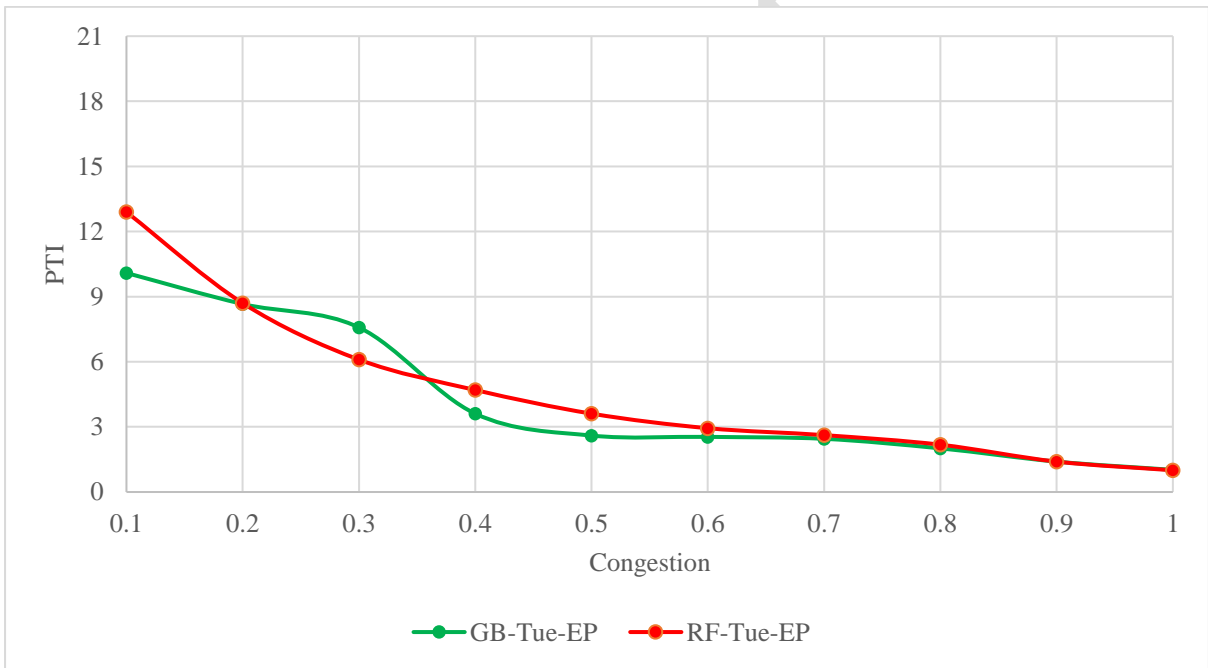


Fig 3 (b): Comparison between GB and RF modeling results- Tuesday evening peak

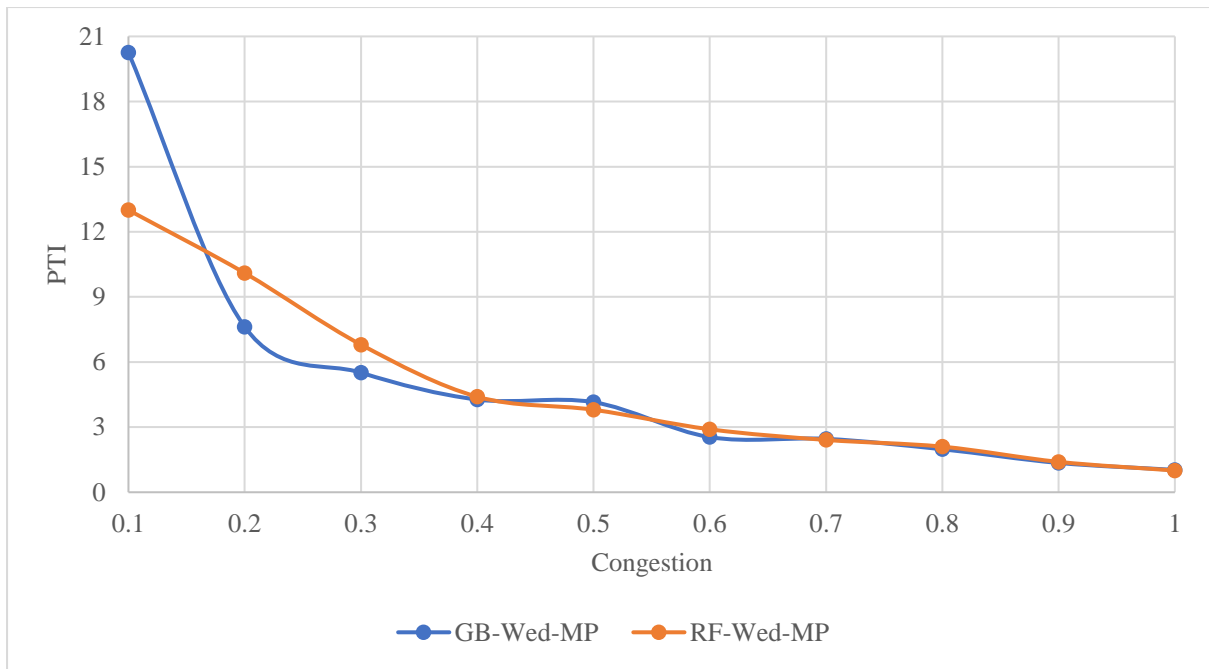


Fig 4 (a): Comparison between GB and RF modeling results- Wednesday morning peak

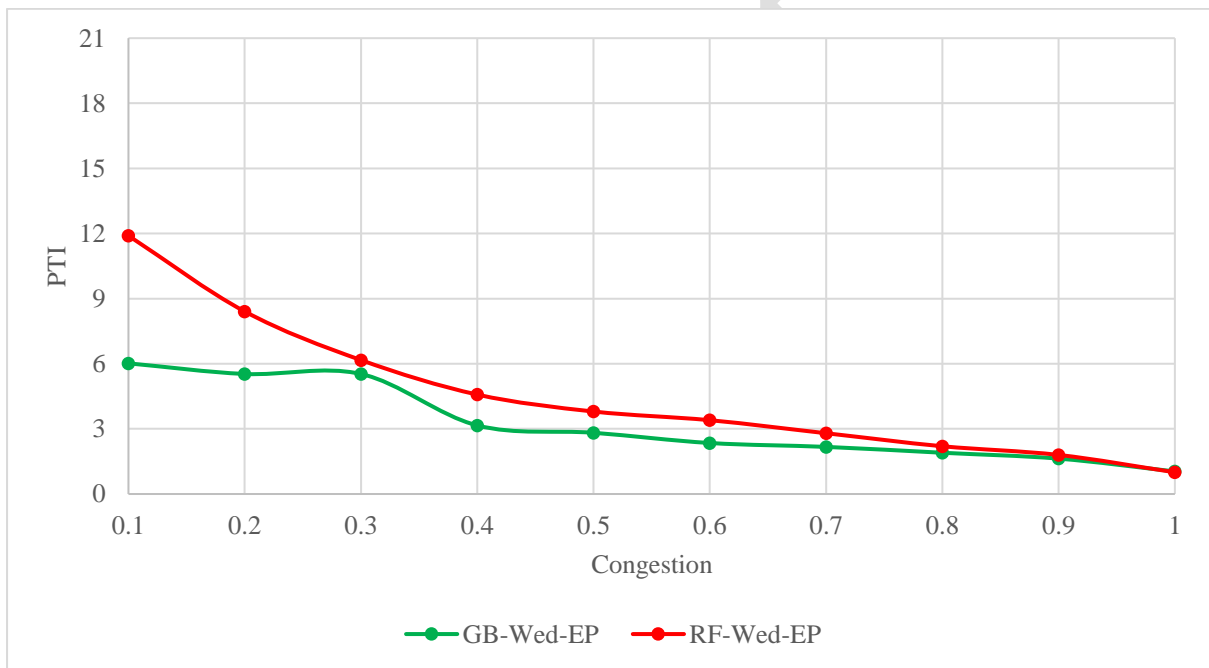


Fig 4 (b): Comparison between GB and RF modeling results- Wednesday evening peak

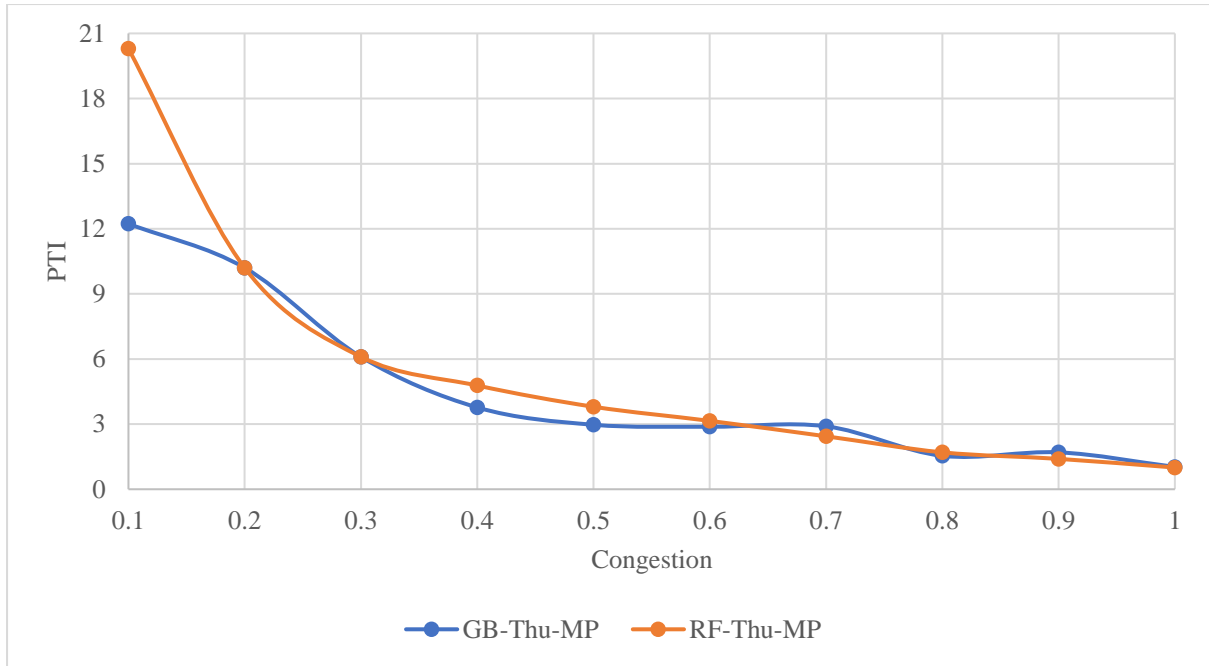


Fig 5 (a): Comparison between GB and RF modeling results- Thursday morning peak

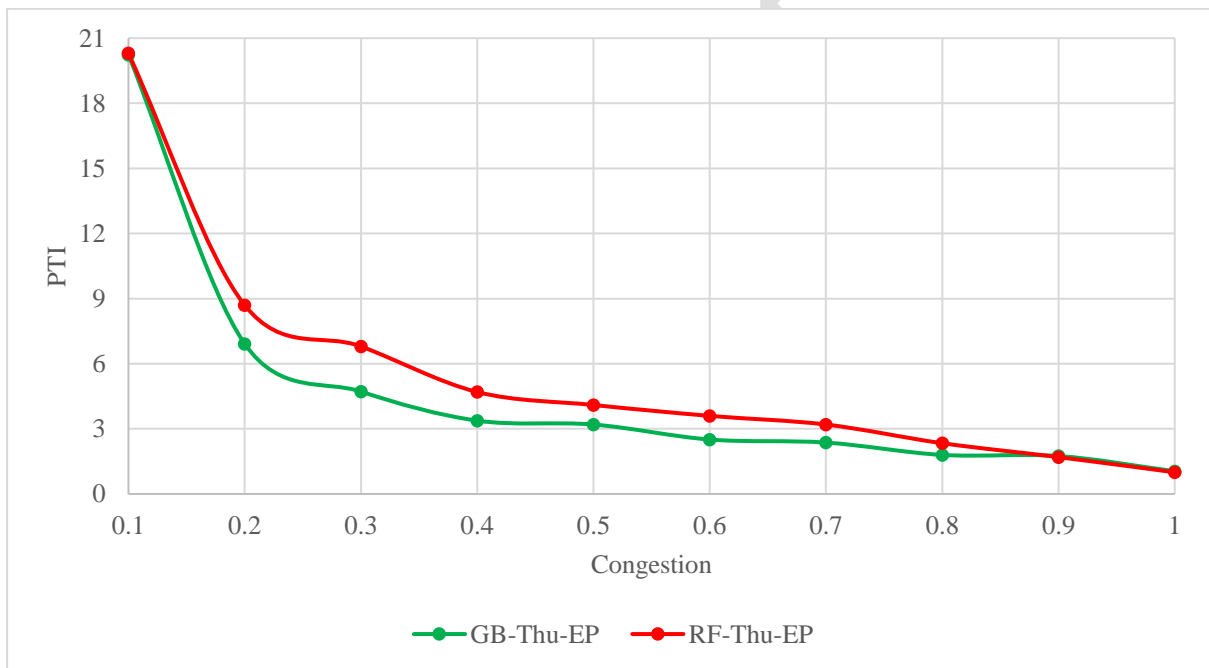


Fig 5 (b): Comparison between GB and RF modeling results- Thursday evening peak

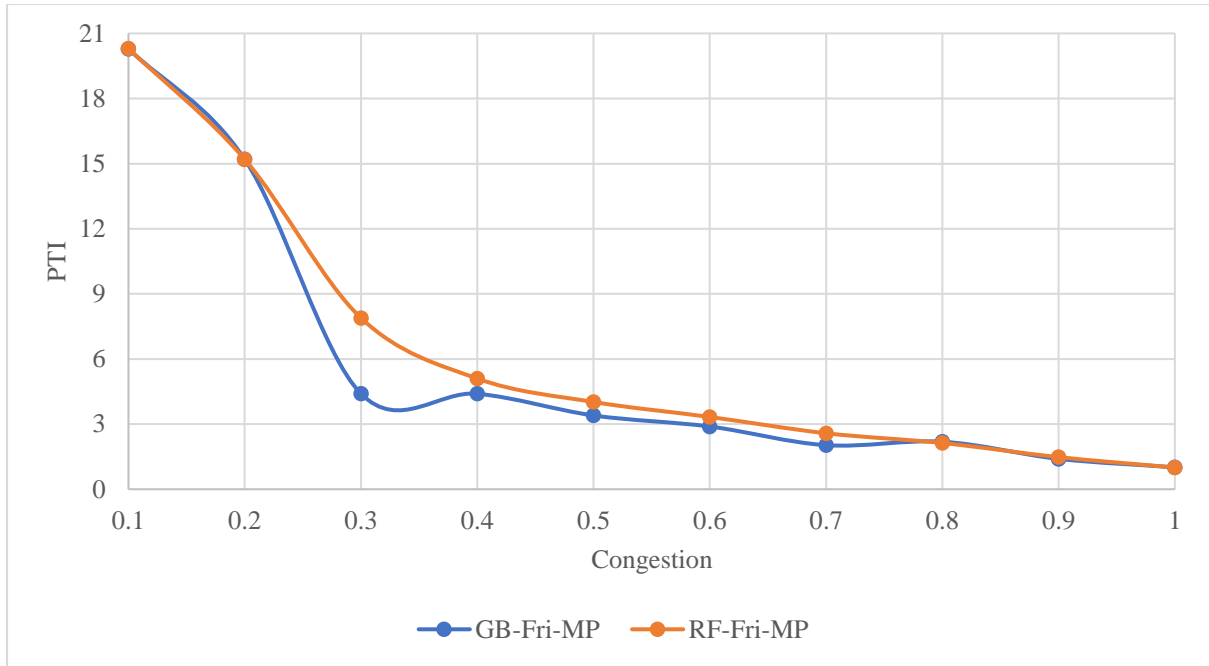


Fig 6 (a): Comparison between GB and RF modeling results- Friday morning peak

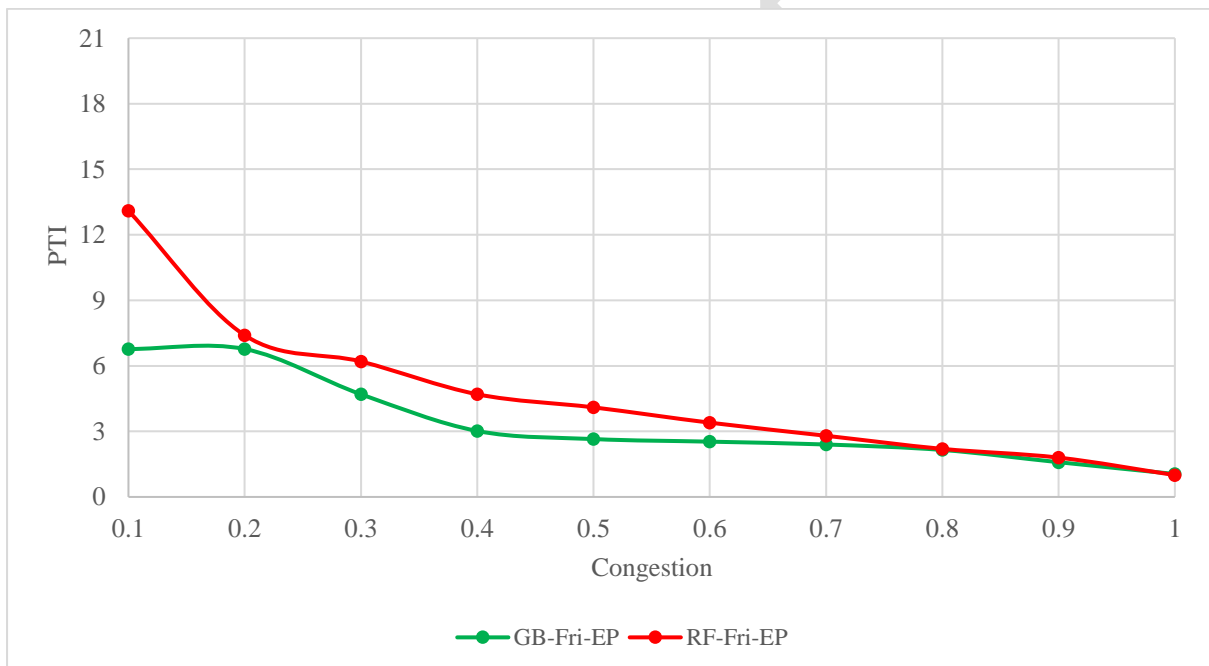


Fig 6 (b): Comparison between GB and RF modeling results- Friday evening peak

4. Conclusion

To predict how speed reduction (in terms of congestion) can lead to an increase in planning time index (PTI), the authors used 15 bagging-based regressor methods on a 1.467-mile section of I-64 in Virginia, US. The data of congestion, as the independent variable, and PTI as the dependent variable were considered for modeling. The performance of these methods was then

assessed by mean squared error (MSE), goodness of fit (R^2), stability ratio, and Akaike information criterion (AIC). Surprisingly, the Gradient Boosting (GB) regressor could eliminate competitive methods (in terms of minimum AIC and stability ratio). After the model selection process, the results were separately depicted in a scatter plot for morning and evening peaks. The results revealed that when congestion reaches 0.9, PTI goes beyond the reliable area in both peaks. The corresponding congestion for entering the extremely unreliable area is between 0.7 and 0.75 for both peaks. Finally, somewhere between 0.5 and 0.4, the plots have shown an intense increase, meaning that for congestion values less than these values, the increase in PTI is severe. TTR is a crucial component of congested traffic regimes that has not been taken into account traditionally by the Congestion Management Process (CMP). The emphasis on travel time reliability is driven by elements like restrictions on roadway expansion. This research points out potential areas where the CMP could incorporate TTR. A comprehensive knowledge of the regional transportation systems and a toolbox of techniques are produced by incorporating TTR into CMP. A CMP that incorporates reliability will usually intend to take advantage of operational strategies like Advanced Traveler Information Systems (ATIS) rather than capacity improvements, so the utilized methodology in this study will depict a framework that explains the value of TTR incorporation into the CMP. Understanding the relationship between congestion and travel time reliability can improve the transportation system's performance in various ways. It can help in developing congestion mitigation strategies, identifying operation strategies, quantifying the benefits of traffic management, improving safety, and maximizing the use of existing capacity. Focusing on improving travel time reliability can lead to strategies that reduce the impact of congestion on travelers, improve safety, support economic growth, and make better use of existing infrastructure. Transportation agencies can use tools such as the Organizing for Reliability Tools from the Strategic Highway

Research Program 2 (SHRP2) to systematically improve their capabilities in transportation systems management and operations.

5. References

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