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# 4-total mean cordial labeling of spider graph

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**ABSTRACT**

Let $G$ be a graph. Let $f : V(G) \to \{0, 1, 2, \ldots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge $uv$, assign the label $f(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. $f$ is called a $k$-total mean cordial labeling of $G$ if $|t_{mf}(i) - t_{mf}(j)| \leq 1$, for all $i, j \in \{0, 1, 2, \ldots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with $x$, $x \in \{0, 1, 2, \ldots, k-1\}$. A graph with admit a $k$-total mean cordial labeling is called $k$-total mean cordial graph. In this paper we investigate the 4-total mean cordial labeling behaviour of some spider graph.

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## 1 Introduction

In this paper we consider simple, finite and undirected graphs only. Cordial labeling was introduced by Cahit [1]. The notion of $k$-total mean cordial labeling has been introduced in [5]. The 4-total mean cordial labeling behaviour of several graphs like cycle, complete

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graph, star, bistar, comb and crown have been studied in [5, 6, 7, 8, 9, 10, 11, 12, 13]. In this paper we investigate the 4- total mean cordial labeling of spider graph. Let $x$ be any real number. Then $\lceil x \rceil$ stands for the smallest integer greater than or equal to $x$. Terms are not defined here follow from Harary[3] and Gallian[2]. .

# 2 $k$-total mean cordial graph

**Definition 1.** Let $G$ be a graph. Let $f : V(G) \rightarrow \{0, 1, 2, \ldots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge $uv$, assign the label $f(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. $f$ is called a $k$-total mean cordial labeling of $G$ if $|t_{mf}(i) - t_{mf}(j)| \leq 1$, for all $i, j \in \{0, 1, 2, \ldots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with $x$, $x \in \{0, 1, 2, \ldots, k-1\}$. A graph with admit a $k$-total mean cordial labeling is called $k$-total mean cordial graph.

# 3 Preliminaries

**Definition 2.** [3] A connected acyclic graph is called a *tree*.

**Definition 3.** [4] A tree is called a *spider graph* if it has a centre vertex $u$ of degree $> 1$ and all the other vertex is either degree 1 or degree 2. Thus the spider is an amalgamation of $k$ paths with various lengths. If it has $x_1^{'}$ of length $a_1$, $x_2^{'}$ of length $a_2$, ..., $x_m^{'}$ of length $a_m$. then it is denoted by $SP\left(a_1^{x_1}, a_2^{x_2}, \ldots, a_m^{x_m}\right)$ where $a_1 < a_2 < \ldots a_m$.

# 4 MAIN RESULTS

**Theorem 4.** The spider graph $SP\left(1^m, 2^n\right)$ is 4-total mean cordial for all values of $m, n \geq 1$.

*Proof.* Let $V\left(SP\left(1^m, 2^n\right)\right) = \{u, u_i, v_j, w_j : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E\left(SP\left(1^m, 2^n\right)\right) = \{uu_i : 1 \leq i \leq m\} \cup \{uv_j, v_jw_j : 1 \leq j \leq n\}$.
Note that $\left|V\left(SP\left(1^m, 2^n\right)\right)\right| + \left|E\left(SP\left(1^m, 2^n\right)\right)\right| = 2m + 4n + 1$.
Assign the label 2 to the vertex $u$. Now we assign the label 0 to the $n$ vertices $v_1$, $v_2$, ..., $v_n$. Next we assign the label 3 to the $n$ vertices $w_1$, $w_2$, ..., $w_n$.

**Case 1.** $m \equiv 0 \pmod 4$.
Let $m = 4r$, $r \geq 1$. Assign the label 0 to the $2r$ vertices $u_1$, $u_2$, ..., $u_{2r}$. Now we assign the label 2 to the $r$ vertices $u_{2r+1}$, $u_{2r+2}$, ..., $u_{3r}$. Next we assign the label 3 to the $r$

vertices $u_{3r+1}$, $u_{3r+2}$, ..., $u_{4r}$.

**Case 2.** $m \equiv 1 \pmod 4$.
Let $m = 4r+1$, $r \geq 0$. Assign the label to the vertices $u_i$ $(1 \leq i \leq 4r)$ as in case 1. Next we assign the label 0 to the vertex $u_{4r+1}$.

**Case 3.** $m \equiv 2 \pmod 4$.
Let $m = 4r+2$, $r \geq 0$. Label the vertices $u_i$ $(1 \leq i \leq 4r+1)$ as in Case 2. Now we assign the labels 3 to the vertex $u_{4r+2}$.

**Case 4.** $m \equiv 3 \pmod 4$.
Let $m = 4r+3$, $r \geq 0$. As in case 3, we assign the label to the vertices $u_i$ $(1 \leq i \leq 4r+2)$. Finally we assign the label 0 to the vertex $u_{4r+3}$.

Thus this vertex labeling $f$ is a 4-total mean cordial labeling follows from the Table 1.

| $m$ | $t_{mf}(0)$ | $t_{mf}(1)$ | $t_{mf}(2)$ | $t_{mf}(3)$ |
|---|---|---|---|---|
| $m = 4r$ | $2r+n$ | $2r+n$ | $2r+n+1$ | $2r+n$ |
| $m = 4r+1$ | $2r+n+1$ | $2r+n+1$ | $2r+n+1$ | $2r+n$ |
| $m = 4r+2$ | $2r+n+1$ | $2r+n+1$ | $2r+n+1$ | $2r+n+2$ |
| $m = 4r+3$ | $2r+n+2$ | $2r+n+2$ | $2r+n+1$ | $2r+n+2$ |

Table 1:

□

**Theorem 5.** The spider graph $SP(1^m, 3^n)$ is 4-total mean cordial for all values of $m, n \geq 1$.

*Proof.* Let $V(SP(1^m, 3^n)) = \{u, u_i, x_j, y_j, z_j : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(SP(1^m, 3^n)) = \{uu_i : 1 \leq i \leq m\} \cup \{ux_j, x_jy_j, y_jz_j : 1 \leq j \leq n\}$.
Clearly $|V(SP(1^m, 3^n))| + |E(SP(1^m, 3^n))| = 2m + 6n + 1$.
Assign the label 1 to the vertex $u$.
**Case 1.** $m \equiv 1 \pmod 2$.
Let $m = 2t+1$, $t \geq 0$. Now we assign the label 3 to the $t+1$ vertices $u_1$, $u_2$, ..., $u_{t+1}$. Next we assign the label 0 to the $t$ vertices $u_{t+2}$, $u_{t+3}$, ..., $u_{2t+1}$.

**Subcase 1.** $n \equiv 0 \pmod 4$.
Let $n = 4r$, $r \geq 1$. Assign the label 0 to the $2r$ vertices $x_1$, $x_2$, ..., $x_{2r}$. Now we assign the label 3 to the $2r$ vertices $x_{2r+1}$, $x_{2r+2}$, ..., $x_{4r}$. Next we assign the label 0 to the $r$ vertices $y_1$, $y_2$, ..., $y_r$. We now assign the label 1 to the $r$ vertices $y_{r+1}$, $y_{r+2}$, ..., $y_{2r}$. Next we assign the label 2 to the $2r$ vertices $y_{2r+1}$, $y_{2r+2}$, ..., $y_{4r}$. Now we assign the label

0 to the $r$ vertices $z_1$, $z_2$, ..., $z_r$. We now assign the label 1 to the $r$ vertices $z_{r+1}$, $z_{r+2}$, ..., $z_{2r}$. Now we assign the label 1 to the $r$ vertices $z_{2r+1}$, $z_{2r+2}$, ..., $z_{3r}$. Next we assign the label 3 to the $r$ vertices $z_{3r+1}$, $z_{3r+2}$, ..., $z_{4r}$.

**Subcase 2.** $n \equiv 1 \pmod 4$.
Let $n = 4r+1$, $r \geq 0$. Assign the label to the vertices $x_j$, $y_j$, $z_j$ $(1 \leq j \leq 4r)$ as in Subcase 1. Next we assign the labels 0, 0, 3 to the vertices $x_{4r+1}$, $y_{4r+1}$, $z_{4r+1}$.

**Subcase 3.** $n \equiv 2 \pmod 4$.
Let $n = 4r+2$, $r \geq 0$. Label the vertices $x_j$, $y_j$, $z_j$ $(1 \leq j \leq 4r+1)$ as in Subcase 2. Now we assign the labels 3, 2, 0 to the vertices $x_{4r+2}$, $y_{4r+2}$, $z_{4r+2}$.

**Subcase 4.** $n \equiv 3 \pmod 4$.
Let $n = 4r+3$, $r \geq 0$. As in Subcase 3, we assign the label to the vertices $x_j$, $y_j$, $z_j$ $(1 \leq j \leq 4r+1)$. Finally we assign the labels 0, 2, 3 to the vertices $x_{4r+3}$, $y_{4r+3}$, $z_{4r+3}$.

**Case 2.** $m \equiv 0 \pmod 2$.
Let $m = 2t$, $t \geq 1$. Assign the label 3 to the $t$ vertices $u_1$, $u_2$, ..., $u_t$. Now we assign the label 0 to the $t$ vertices $u_{t+1}$, $u_{t+2}$, ..., $u_{2t}$.

**Subcase 1.** $n \equiv 0 \pmod 4$.
Let $n = 4r$, $r \geq 1$. Label the vertices as in Subcase 1 of Case 1.

**Subcase 2.** $n \equiv 1 \pmod 4$.
Let $m = 4r+1$, $r \geq 0$. Assign the label to the vertices $x_j$, $y_j$, $z_j$ $(1 \leq j \leq 4r)$ as in Subcase 1 of Case 2. Now we assign the labels 0, 3, 0 to the vertices $x_{4r+1}$, $y_{4r+1}$, $z_{4r+1}$.

**Subcase 3.** $n \equiv 2 \pmod 4$.
Let $n = 4r+2$, $r \geq 0$. Label the vertices $x_j$, $y_j$, $z_j$ $(1 \leq j \leq 4r+1)$ as in Subcase 2 of Case 2. Next we assign the labels 3, 2, 0 to the vertices $x_{4r+2}$, $y_{4r+2}$, $z_{4r+2}$.

**Subcase 4.** $n \equiv 3 \pmod 4$.
Let $n = 4r+3$, $r \geq 0$. Now we assign the label to the vertices $x_j$, $y_j$, $z_j$ $(1 \leq j \leq 4r+1)$ as in Subcase 3 of Case 2. Finally we assign the label 0, 2, 3 to the vertices $x_{4r+3}$, $y_{4r+3}$, $z_{4r+3}$.

Thus this vertex labeling $f$ is a 4-total mean cordial labeling follows from the Table 2.

□

**Theorem 6.** The Spider graph $SP(1^m, 4^n)$ is a 4-total mean cordial for all values of $m, n \geq 1$.

| $m$ | $n$ | $t_{mf}(0)$ | $t_{mf}(1)$ | $t_{mf}(2)$ | $t_{mf}(3)$ |
|---|---|---|---|---|---|
| $m=2t+1$ | $n=4r$ | $t+6r$ | $t+6r+1$ | $t+6r+1$ | $t+6r+1$ |
| $m=2t+1$ | $n=4r+1$ | $t+6r+3$ | $t+6r+2$ | $t+6r+2$ | $t+6r+2$ |
| $m=2t+1$ | $n=4r+2$ | $t+6r+4$ | $t+6r+3$ | $t+6r+4$ | $t+6r+4$ |
| $m=2t+1$ | $n=4r+3$ | $t+6r+5$ | $t+6r+5$ | $t+6r+5$ | $t+6r+6$ |
| $m=2t$ | $n=4r$ | $t+6r$ | $t+6r+1$ | $t+6r$ | $t+6r$ |
| $m=2t$ | $n=4r+1$ | $t+6r+2$ | $t+6r+2$ | $t+6r+2$ | $t+6r+1$ |
| $m=2t$ | $n=4r+2$ | $t+6r+3$ | $t+6r+3$ | $t+6r+4$ | $t+6r+3$ |
| $m=2t$ | $n=4r+3$ | $t+6r+4$ | $t+6r+5$ | $t+6r+5$ | $t+6r+5$ |

Table 2:

*Proof.* Let $V\left(SP\left(1^m,4^n\right)\right)=\{u,u_i,w_j,x_j,y_j,z_j:1\le i\le m,1\le j\le n\}$ and $E\left(SP\left(1^m,4^n\right)\right)=\{uu_i:1\le i\le m\}\cup\{uw_j,w_jx_j,x_jy_j,y_jz_j:1\le j\le n\}$.
Obviously $\left|V\left(SP\left(1^m,4^n\right)\right)\right|+\left|E\left(SP\left(1^m,4^n\right)\right)\right|=2m+8n+1$.

Assign the label 2 to the vertex $u$. Now we assign the label 3 to the $n$ vertices $w_1$, $w_2$, …, $w_n$. Next we assign the label 0 to the $n$ vertices $x_1$, $x_2$, …, $x_n$. We now assign the label 2 to the $n$ vertices $y_1$, $y_2$, …, $y_n$. Next we assign the label 0 to the $n$ vertices $z_1$, $z_2$, …, $z_n$.

**Case 1.** $m\equiv 0 \pmod 4$.
Let $m=4r$, $r\ge 1$. Assign the label 0 to the $2r$ vertices $u_1$, $u_2$, …, $u_{2r}$. Next we assign the label 2 to the $r$ vertices $u_{2r+1}$, $u_{2r+2}$, …, $u_{3r}$. Now we assign the label 3 to the $r$ vertices $u_{3r+1}$, $u_{3r+2}$, …, $u_{4r}$.

**Case 2.** $m\equiv 1 \pmod 4$.
Let $m=4r+1$, $r\ge 0$. Now we assign the label to the vertices $u_i$ $(1\le i\le 4r)$ as in case 1. Next we assign the label 0 to the vertex $u_{4r+1}$.

**Case 3.** $m\equiv 2 \pmod 4$.
Let $m=4r+2$, $r\ge 0$. Label the vertices $u_i$ $(1\le i\le 4r+1)$ as in Case 2. Now we assign the labels 3 to the vertex $u_{4r+2}$.

**Case 4.** $m\equiv 3 \pmod 4$.
Let $m=4r+3$, $r\ge 0$. As in case 3, we assign the label to the vertices $u_i$ $(1\le i\le 4r+2)$. Finally we assign the label 0 to the vertex $u_{4r+3}$.

Thus this vertex labeling $f$ is a 4-total mean cordial labeling follows from the Table 3.

□

**Theorem 7.** The Spider graph $SP\left(2^m,3^n\right)$ is a 4-total mean cordial for all values of

| $m$ | $t_{mf}(0)$ | $t_{mf}(1)$ | $t_{mf}(2)$ | $t_{mf}(3)$ |
|---|---|---|---|---|
| $m = 4r$ | $2r + 2n$ | $2r + 2n$ | $2r + 2n + 1$ | $2r + 2n$ |
| $m = 4r + 1$ | $2r + 2n + 1$ | $2r + 2n + 1$ | $2r + 2n + 1$ | $2r + 2n$ |
| $m = 4r + 2$ | $2r + 2n + 1$ | $2r + 2n + 1$ | $2r + 2n + 1$ | $2r + 2n + 2$ |
| $m = 4r + 3$ | $2r + 2n + 2$ | $2r + 2n + 2$ | $2r + 2n + 1$ | $2r + 2n + 2$ |

Table 3:

$m, n \geq 1$.

*Proof.* Let $V\left(SP\left(2^m, 3^n\right)\right) = \{u, u_i, v_i, x_j, y_j, z_j : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E\left(SP\left(2^m, 3^n\right)\right) = \{uu_i, u_iv_i : 1 \leq i \leq m\} \cup \{ux_j, x_jy_j, y_jz_j : 1 \leq j \leq n\}$.
Clearly $\left|V\left(SP\left(2^m, 3^n\right)\right)\right| + \left|E\left(SP\left(2^m, 3^n\right)\right)\right| = 4m + 6n + 1$.

Assign the label 1 to the vertex $u$. Next we assign the label 0 to the $m$ vertices $u_1$, $u_2$, …, $u_m$. Now we assign the label 3 to the $m$ vertices $v_1$, $v_2$, …, $v_m$.

**Case 1.** $n \equiv 0 \pmod 4$.
Let $n = 4r$, $r \geq 1$. Assign the label 0 to the $2r$ vertices $x_1$, $x_2$, …, $x_{2r}$. Now we assign the label 3 to the $2r$ vertices $x_{2r+1}$, $x_{2r+2}$, …, $x_{4r}$. Next we assign the label 0 to the $r$ vertices $y_1$, $y_2$, …, $y_r$. We now assign the label 1 to the $r$ vertices $y_{r+1}$, $y_{r+2}$, …, $y_{2r}$. Next we assign the label 2 to the $2r$ vertices $y_{2r+1}$, $y_{2r+2}$, …, $y_{4r}$. Now we assign the label 0 to the $r$ vertices $z_1$, $z_2$, …, $z_r$. We now assign the label 1 to the $r$ vertices $z_{r+1}$, $z_{r+2}$, …, $z_{2r}$. Now weassign the label 1 to the $r$ vertices $z_{2r+1}$, $z_{2r+2}$, …, $z_{3r}$. Next we assign the label 3 to the $r$ vertices $z_{3r+1}$, $z_{3r+2}$, …, $z_{4r}$.

**Case 2.** $n \equiv 1 \pmod 4$.
Let $n = 4r + 1$, $r \geq 0$. Assign the label to the vertices $x_j$, $y_j$, $z_j$ $(1 \leq j \leq 4r)$ as in Case 1. Next we assign the labels 3, 2, 0 to the vertices $x_{4r+1}$, $y_{4r+1}$, $z_{4r+1}$.

**Case 3.** $n \equiv 2 \pmod 4$.
Let $n = 4r + 2$, $r \geq 0$. Label the vertices $x_j$, $y_j$, $z_j$ $(1 \leq j \leq 4r + 1)$ as in Case 2. Now we assign the labels 0, 3, 0 to the vertices $x_{4r+2}$, $y_{4r+2}$, $z_{4r+2}$.

**Case 4.** $n \equiv 3 \pmod 4$.
Let $n = 4r + 3$, $r \geq 0$. As in Case 2, we assign the label to the vertices $x_j$, $y_j$, $z_j$ $(1 \leq j \leq 4r + 1)$. Finally we assign the labels 0, 0, 3, 3, 2, 0 to the vertices $x_{4r+2}$, $y_{4r+2}$, $z_{4r+2}$, $x_{4r+3}$, $y_{4r+3}$, $z_{4r+3}$.

Thus this vertex labeling $f$ is a 4-total mean cordial labeling follows from the Table 4.

□

| $n$ | $t_{mf}(0)$ | $t_{mf}(1)$ | $t_{mf}(2)$ | $t_{mf}(3)$ |
|---|---|---|---|---|
| $n = 4r$ | $m + 6r$ | $m + 6r + 1$ | $m + 6r$ | $m + 6r$ |
| $n = 4r + 1$ | $m + 6r + 1$ | $m + 6r + 2$ | $m + 6r + 2$ | $m + 6r + 2$ |
| $n = 4r + 2$ | $m + 6r + 3$ | $m + 6r + 3$ | $m + 6r + 4$ | $m + 6r + 3$ |
| $n = 4r + 3$ | $m + 6r + 5$ | $m + 6r + 4$ | $m + 6r + 5$ | $m + 6r + 5$ |

Table 4:

**Theorem 8.** The Spider graph $SP\left(2^m, 4^n\right)$ is a 4-total mean cordial for all values of $m, n \geq 1$.

*Proof.* Let $V\left(SP\left(2^m, 4^n\right)\right) = \{u, u_i, v_i, w_j, x_j, y_j, z_j : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E\left(SP\left(2^m, 4^n\right)\right) = \{uu_i, u_iv_i : 1 \leq i \leq m\} \cup \{uw_j, w_jx_j, x_jy_j, y_jz_j : 1 \leq j \leq n\}$. Note that $\left|V\left(SP\left(2^m, 4^n\right)\right)\right| + \left|E\left(SP\left(2^m, 4^n\right)\right)\right| = 4m + 8n + 1$.

Assign the label 2 to the vertex $u$. Next we assign the label 0 to the $m$ vertices $u_1$, $u_2$, $\ldots$, $u_m$. Now we assign the label 3 to the $m$ vertices $v_1$, $v_2$, $\ldots$, $v_n$. We now assign the label 3 to the $n$ vertices $w_1$, $w_2$, $\ldots$, $w_n$. Next$x_1$, $x_2$, $\ldots$, $x_n$. We now assign the label 2 to the $n$ vertices $y_1$, $y_2$, $\ldots$, $y_n$. Finally we assign the label 3 to the $n$ vertices $z_1$, $z_2$, $\ldots$, $z_n$. Obviously $t_{mf}\left(0\right) = t_{mf}\left(1\right) = t_{mf}\left(3\right) = m + 2n$; $t_{mf}\left(3\right) = m + 2n + 1$.

□

**Theorem 9.** The Spider graph $SP\left(1^n, 2^n, 3^n\right)$ is 4-total mean cordial for all values of $n \geq 1$.

*Proof.* Let $V\left(SP\left(1^n, 2^n, 3^n\right)\right) = \{u, u_i, v_i, w_i, x_i, y_i, z_i : 1 \leq i \leq n\}$ and $E\left(SP\left(1^n, 2^n, 3^n\right)\right) = \{uu_i, uv_i, v_iw_i, ux_i, x_iy_i, y_iz_i : 1 \leq i \leq n\}$.
Obviously $\left|V\left(SP\left(1^n, 2^n, 3^n\right)\right)\right| + \left|E\left(SP\left(1^n, 2^n, 3^n\right)\right)\right| = 12n + 1$.

Assign the label 1 to the vertex $u$. We now assign the label 0 to the $n$ vertices $u_1$, $u_2$, $\ldots$, $u_n$. Now we assign the label 0 to the $n$ vertices $v_1$, $v_2$, $\ldots$, $v_n$. Next we assign the label 3 to the $n$ vertices $w_1$, $w_2$, $\ldots$, $w_n$. Now we assign the label 0 to the $n$ vertices $x_1$, $x_2$, $\ldots$, $x_n$. We now assign the label 3 to the $n$ vertices $y_1$, $y_2$, $\ldots$, $y_n$. Finally we assign the label 2 to the $n$ vertices $z_1$, $z_2$, $\ldots$, $z_n$.
Clearly $t_{mf}\left(0\right) = t_{mf}\left(2\right) = t_{mf}\left(3\right) = 3n$; $t_{mf}\left(1\right) = 3n + 1$.

□

**Theorem 10.** The Spider graph $SP\left(1^n, 2^n, 3^n, 4^n\right)$ is 4-total mean cordial for all values of $n \geq 1$.

*Proof.* Let $V\left(SP\left(1^n,2^n,3^n,4^n\right)\right)=\{u,u_i,v_i,w_i,x_i,y_i,z_i,p_i,q_i,r_i,s_i:1\leq i\leq n\}$ and $E\left(SP\left(1^n,2^n,3^n,4_n\right)\right)=\{uu_i,uv_i,v_iw_i,ux_i,x_iy_i,y_iz_i,up_i,p_iq_i,q_ir_i,r_is_i:1\leq i\leq n\}$. Obviously $\left|V\left(SP\left(1^n,2^n,3^n,4^n\right)\right)\right|+\left|E\left(SP\left(1^n,2^n,3^n,4^n\right)\right)\right|=20n+1$.

Assign the label 1 to the vertex $u$. We now assign the label 0 to the $4n$ vertices $u_1$, $u_2$, …, $u_n$, $v_1$, $v_2$, …, $v_n$,$w_1$, $w_2$, …, $w_n$,$p_1$, $p_2$, …, $p_n$. Next we assign the label 1 to the $n$ vertices $x_1$, $x_2$, …, $x_n$. Now we assign the label 2 to the $2n$ vertices $y_1$, $y_2$, …, $y_n$,$z_1$, $z_2$, …, $z_n$. Finally we assign the label 3 to the $3n$ vertices $q_1$, $q_2$, …, $q_n$, $r_1$, $r_2$, …, $r_n$,$s_1$, $s_2$, …, $s_n$.
Clearly $t_{mf}\left(0\right)=t_{mf}\left(2\right)=t_{mf}\left(3\right)=5n$; $t_{mf}\left(1\right)=5n+1$.

□

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