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RESEARCH PAPER

Damage Detection of Truss Bridges Using Wavelet Transform of Rotation Signal

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ABSTRACT: Although some researchers indicated that the rotation response of the structure is a proper index to identify the damage location, Wavelet Transform (WT) of rotation response of the bridge is not yet used to detect the damage location in the previous studies. In this regard, a numerical model of a truss bridge was developed using finite element method. The static response of the model under one point-load and a six-axle locomotive was calculated. The response obtained from the model was compared with that of the literature to investigate the validity of the model developed in this study. The WT coefficients of horizontal, vertical and rotation responses of each member of the bridge were obtained based on the Gaus2 wavelet basis function. Several damage scenarios were considered for the bridge to investigate the effectiveness of WT of rotation response of the bridge to detect the damage location. The obtained results show that the WT of horizontal displacement is not a proper index to detect the damage in the bridge members. A Comparison between WT coefficients of vertical displacement and rotation for all members indicates that the rotation response is a proper index to identify the damage and loading locations. In some cases, while the damage causes a significant jump on the WT coefficients of rotations of the members, the WT coefficients of vertical displacement of these members are not influenced by the damage.

Keywords: Rotation Response, Structural Health Monitoring, Truss Bridge, Wavelet Transform.

1. Introduction

Structural health monitoring and damage detection are known as the most important areas in the repair and maintenance of the strategic structures such as bridge, tunnel and dam (Rezaifar and Doostmohammadi, 2016; Martinez et al., 2019; Moreu et al. 2017; Khajehdezfuly et al., 2023; Labibzadeh et al., 2019, 2022; Poorveis et al., 2023). In this regard, several methods have been developed in the literature to detect the damage location in these types of structures. The methods detect the damage

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location in the structures based on the different parameters such as natural frequencies, mode shapes or vibration responses of the structures (Sadeghi and Fathali, 2007; Tiboni et al., 2022; Rizzo and Enshaeian, 2022; Huang et al., 2020; Sadeghi and Hashemi Rezvani, 2013).

According to the literature, the Wavelet Transform (WT) is a useful approach to detect the damage location in the structures. For instance, Hou et al. (2013) detected the damage location by the wavelet transform of noisy response of the one degree of freedom structure. Some researchers applied Continuous Wavelet Transform (CWT) on the mode shapes of the beams to find the damage location (Miao et al., 2020; Janeliukstis et al., 2017). It should be noted that the bridge is usually subjected to the moving loads and the WT approach is implemented on the dynamic response of the bridge.

Moreover, it is not easy to measure the pseudo-static rotational responses under moving loads in practice. In fact, the time history of the response of the bridges and beams were usually measured in the previous studies. However, a review of the literature shows that the image processing approach is a practical method to measure the static and dynamic responses (deflection and rotation) of the beams and bridges under stationary and moving loads, respectively (Ma et al., 2021; Silva et al., 2012, 2014; Andreausa et al., 2017).

Accordingly, in several previous experimental studies, some researchers measured the static response of the damaged structure using the image processing approach in the laboratory and then adopted the WT approach to detect the damage location, respectively (Ma et al., 2021; Silva et al., 2012, 2014; Andreausa et al., 2017). For instance, Ovanesova and Suárez (2004) used WT of static and dynamic responses of the beams and frames to detect the damage. They indicated that the type of wavelet function has a significant effect on the results obtained from the WT.

Douka et al. (2003) applied CWT on the mode shapes of a cantilever beam to find the damage magnitude and its location. Several researchers detected the damage location on the plates and beams using CWT of plate response and mode shapes (Kumar and Singh, 2021; Khiem et al., 2021). Sun and Chang (2002) used WT technique in conjunction with neural network to identify the damage location in the structure.

There are many types of bridges around the world and numerous researchers have used WT and developed several wavelet-based approaches to detect the damage location in the bridge (Silik et al., 2021; Andrea et al., 2016; Kankanamge et al., 2020). For instance, Taha et al. (2004) detected damage location in the bridge using Discretized Wavelet Transform (DWT) approach in conjunction with neural network. Zhong and Oyadiji (2008) and Barone et al. (2008) simulated a bridge as simple beam under static loads and applied WT on the response of the beam in order to identify the crack location in the bridge.

Zhu and Law (2006) modeled a bridge under constant moving loads and used WT of noisy vertical displacement of the bridge mid-point to detect the location of the damage with high severity. The accuracy of their approach was significantly decreased when the load enters/leaves the bridge. Also, when the number of moving loads was increased, the results obtained from their approach had noisy content. Hester and Gonzalez (2011) improved the method developed by Zhu and Law (2006). They modeled a vehicle/bridge interaction model and used WT of vertical acceleration signal of bridge in order to detect the damage location in the bridge. In their work, the vehicle axles entering and leaving the bridge generated some high wavelet coefficients. In fact, their WT approach was not able to detect the damage near the supports.

Cantero and Basu (2014) identified the damage location in the bridge using WT of vertical acceleration of vehicle axles. Several studies have been conducted to find the proper wavelet basis functions and scale factor in order to detect the damage location in the bridge (Serra and Lopez, 2017; Ghanbari Mardasi et al., 2018).

A brief review of the literature indicates that the WT approaches developed in the previous researches have several limitations. Firstly, most of WT approaches detect the damage with high severity. Secondly, as the number of vehicle axles loads increases, the accuracy of the results obtained from the WT approaches decreases. Thirdly, WT approaches are unable to identify the damage locations near the bridge supports. Although few researchers proved that the rotation response of the structure is a proper index to identify the damage location (Hester et al., 2020). WT of rotation response of the bridge is not used to detect the damage location in the previous studies. A review of the available studies shows that there is a need to assess the effectives of rotational response of structure on the damage detection using WT approach.

The rotational response of structure is divided into two main categorizes including pseudo-static and dynamic ones. This research is carried out to identify the damage location in the bridge using WT of pseudo-static rotation response. In this regard, a numerical model of truss bridge is developed using finite element method. The response of the model under different patterns of concentrated static load is obtained. Several damage scenarios are considered for the bridge to investigate the effectiveness of WT of rotation response of the bridge to identify the damage location.

2. Development of Damaged Bridge Numerical Model

In this study, both the excitation and structural response of the bridge are static. In other word, the static concentrated loads are applied to the bridge and then, the static response of the bridge components (including horizontal displacement, rotational deflection and vertical displacement) are calculated through a static analysis. A review of the previous studies shows that the image processing approach is a practical method to measure the static responses (deflection and rotation) of the beams and bridges under the static loads (Ma et al., 2021; Silva et al., 2012, 2014; Andreausa et al., 2017). Accordingly, in several previous experimental studies, some researchers measured the static response of the damaged structures using image processing approach in the laboratory and then adopted the WT to detect the damage location (Ma et al., 2021; Silva et al., 2012, 2014; Andreausa et al., 2017).

The approach implemented in this study is same as the method proposed in Ma et al. (2021) and Andreausa et al. (2017). A truss bridge is modeled in this study using two-dimensional finite element method. A review of the previous studies shows that when all components of the lower chord of the truss bridge are subjected to the stationary or moving vertical external loads, the moment, shear and axial internal forces are induced in them and consequently, they are usually simulated using the frame elements (Kordi and Mahmoudi, 2022; Wan et al., 2022). As no vertical external force is applied to other members of the bridge truss, only axial force is induced in them and consequently, they usually are modeled using truss elements (Kordi and Mahmoudi, 2022; Wan et al., 2022). In this study to simplify the simulation process of the truss bridge, all members of the truss bridge are modeled using frame elements. It should be noted that, the internal shear and moment of all members of truss bridge except those of the lower chord are nearly zero because no vertical external force is applied to them.

A schematic view of the model is presented in Figure 1. Frame element is used to simulate the model (Figure 2) (Przemieniecki, 1985). Each frame element has two nodes and each node has three degrees of freedom in the local normal-tangential coordinate system (rotation about normal axis of plane (θ), displacement in normal direction (u) and displacement in tangential direction (v)).

The local (normal-tangential) and global coordinate systems of the frame element are shown in Figure 2. The equilibrium equation of the element eth in the local coordinate system is presented in Eq. (1) (Przemieniecki, 1985).

$$\begin{Bmatrix} F_1 \\ V_1 \\ M_1 \\ F_2 \\ V_2 \\ M_2 \end{Bmatrix}_e = [\boldsymbol{K_e}]_{6\times6} \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_3 \end{Bmatrix}_e \tag{1}$$

where F_i, V_i, M_i, u_i, v_i and θ_i: are axial force, shear force, moment, displacement in tangential (axial) direction, displacement in normal displacement and, rotation about normal axis for ith node of the eth element, respectively. Also, $[\boldsymbol{K_e}]_{6\times6}$ is the frame element stiffness matrix and presented in Eq. (2).

$$[\boldsymbol{K_e}]_{6\times6} = \frac{1}{L}\begin{bmatrix} AE & 0 & 0 & -AE & 0 & 0 \\ 0 & \frac{12EI}{L^2} & \frac{6EI}{L} & 0 & -\frac{12EI}{L^2} & \frac{6EI}{L} \\ 0 & \frac{6EI}{L} & 4EI & 0 & -\frac{6EI}{L} & 2EI \\ -AE & 0 & 0 & AE & 0 & 0 \\ 0 & -\frac{12EI}{L^2} & -\frac{6EI}{L} & 0 & \frac{12EI}{L^2} & \frac{6EI}{L} \\ 0 & \frac{6EI}{L} & 2EI & 0 & -\frac{6EI}{L} & 4EI \end{bmatrix} \tag{2}$$

where A, E, I and L: denote cross-section area, modulus of elasticity, second moment of inertia and, length of the element, respectively.

The force and displacement vectors and, stiffness matrix of the element are converted from local coordinate system to global coordinate system using transformation matrix (Przemieniecki, 1985). The transformation matrix for each element is obtained from Eq. (3) in which, β: is the angle between axial element direction and horizontal direction.

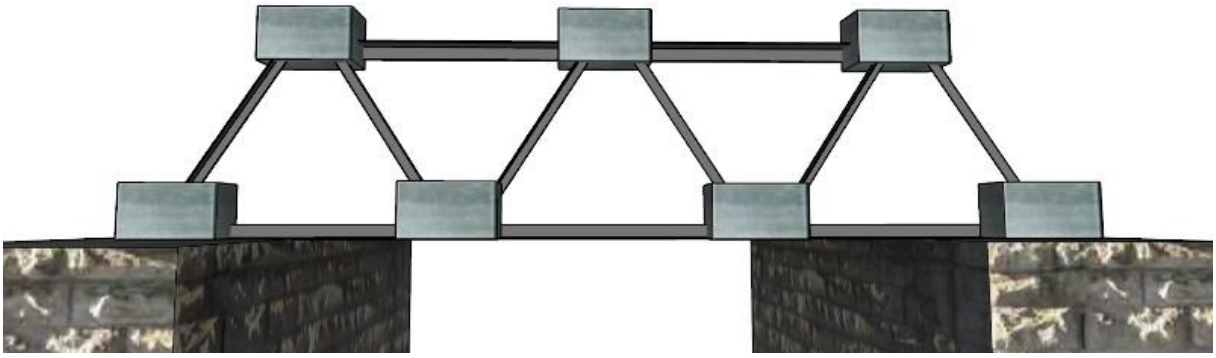


Fig. 1. Schematic view of truss bridge

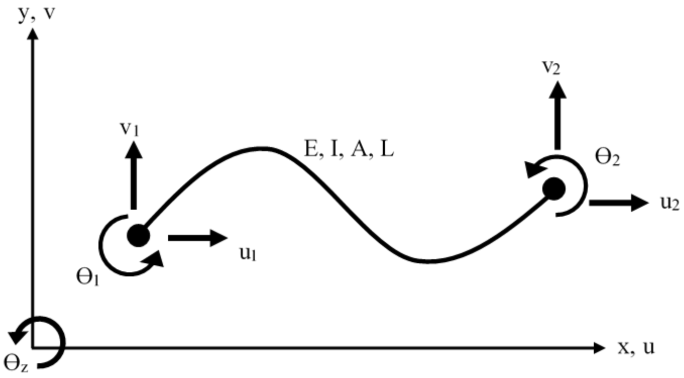


Fig. 2. Local and global coordinate system of frame element

$$[\boldsymbol{T}] = \begin{bmatrix} \cos(\beta) & \sin(\beta) & 0 & 0 & 0 & 0 \\ -\sin(\beta) & \cos(\beta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\beta) & \sin(\beta) & 0 \\ 0 & 0 & 0 & -\sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

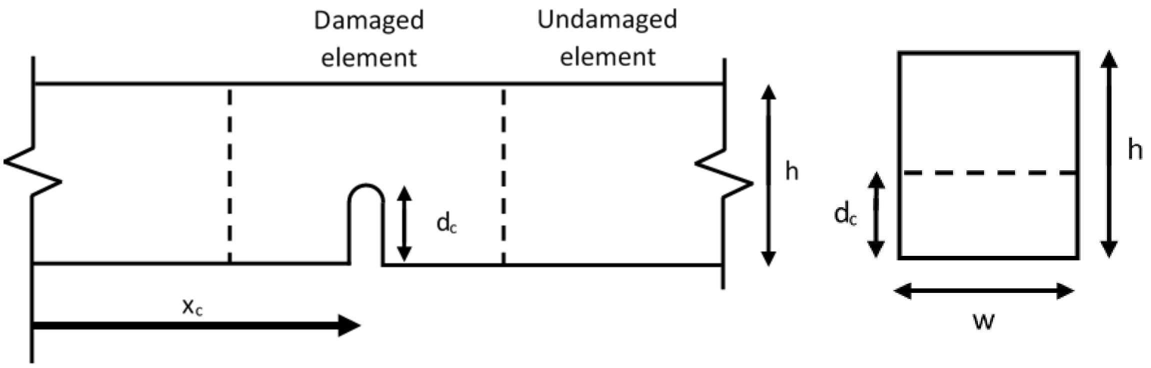
In order to consider the defect (or damage) at a special location of the bridge component (damaged component), a crack with depth of d_c is considered at that location (Figure 3). The flexural rigidity of the element at that location is decreased because of the crack (damaged element). The reduced flexural rigidity of the damaged element is obtained using Eq. (4) (Christides and Barr, 1999).

$$EI_c = \frac{EI}{1 + \frac{I}{I + I_c} e^{\left(\frac{-1.334|x - x_c|}{h}\right)}} \tag{4}$$

where EI_c: stands for flexural rigidity of the damaged element, EI: is flexural rigidity of undamaged element, I_c: is second moment inertia of the damaged element, h: is depth of cross-section, x_c: stands for damage location at the component and, x: is the length of the component. For a rectangular cross-section with depth h and width w, I_c is derived using Eq. (5), as follows:

$$I_d = \frac{1}{12} w(h - d_c)^3 \tag{5}$$

For instance, Figure 4 shows the effects of three crack depths located at $x_c/l = 0.2$ on the variation of flexural rigidity of a component with length of l. As illustrated in this figure, the flexural rigidity of the component is reduced by the crack considered at $x_c/l = 0.2$ location. The reduction of flexural rigidity depends on the crack depth.



(a) **(b)**

Fig. 3. Crack considered at the component: a) Longitudinal section; and b) Cross section

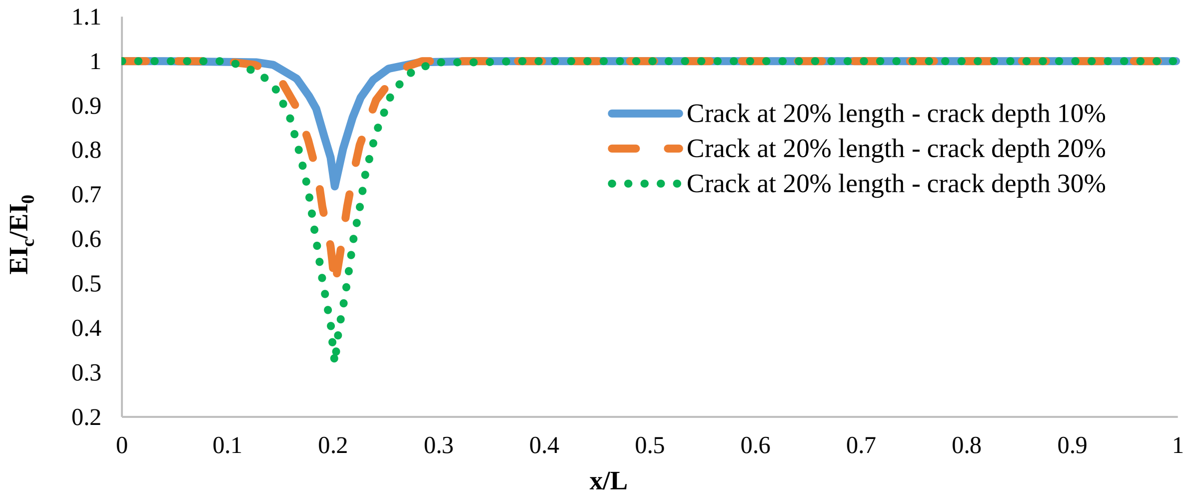


Fig. 4. Variation of flexural rigidity on the damaged component

The bridge is discretized by the frame elements. The force vectors and stiffness matrices of the elements in the global coordinate system are assembled to obtain the force vector ($\{F\}_T$) and stiffness matrix ($[\boldsymbol{K_T}]$) of the model. The displacement vector ($\{\Delta\}_T$) of the model in global coordinate system is derived using Eq. (6).

$$\{\Delta\}_T = [\boldsymbol{K_T}]^{-1}\{F\}_T \tag{6}$$

3. Validation of the Model

A comparison is made between the results obtained from the model developed in this study and those of presented in the literature in order to investigate the validation of the model. Mahato and Harish (2015) experimentally investigated the responses of the simply supported and cantilever beams under concentrated load. The bridge model is simplified to simulate the simply supported and cantilever beams under concentrated load in order to make a comparison between the results obtained from the simplified model and those measured by Mahato and Harish (2015). The simply supported and cantilever beams simulated in this section are shown in Figure 5.

Mahato and Harish (2015) measured the maximum vertical displacement of the cantilever beam under different magnitudes of point load located at the mid-span and end-edge locations. Also, they investigate the maximum vertical displacement of the simply supported beam under mid-span concentrated load with different magnitudes. Figure 6 presents the results obtained from the simplified model developed in this study and those measured by Mahato and Harish (2015). As illustrated in Figure 6, the difference between the results is negligible.

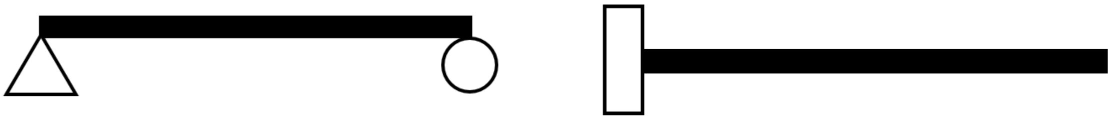

(a) (b)

Fig. 5. Simplified beam model: a) Simply supported; and b) Cantilever

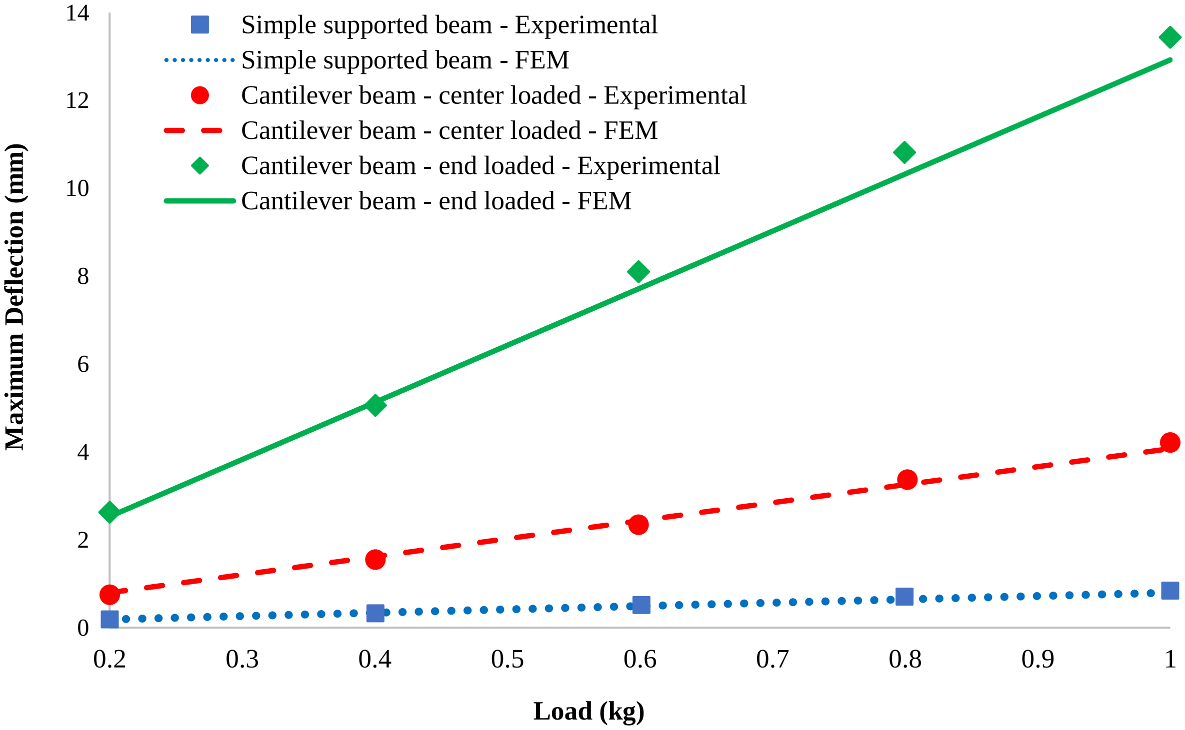


Fig. 6. Comparison between the results obtained from present study and previous study

4. Parametric Study

The effectiveness of wavelet transform of rotation signal of the bridge on the damage detection of truss bridge was investigated during the parametric study. In this regard, two scenarios were considered for the loading of the bridge. In the first scenario, one point-load was applied to the bridge. In the second scenario, a locomotive was stationed on the bridge. In a real condition, measurement noises are included in the identification results. However, the results obtained from the model developed in this study have no noise.

A review of the literature shows that in some cases, before implementation of WT on the responses of the damaged structure measured in the test, some filtration approaches were used to denoise the measured data (Ma et al., 2021). In this regard, a pre-filtration process was carried out for denoising the measured response of the structure. In this study, it was assumed that, the measured response of the bridge was denoised using a pre-filtration process and accordingly, the WT was implemented on the denoised data in all scenarios. The details are as follows.

4.1. Bridge under one Point-Load

The bridge presented in Figure 7 was simulated in this section. As illustrated in this figure, the truss bridge includes 11 members. The properties of bridge members are presented in Table 1. As shown in Figure 7, a point load with magnitude of 500 N is applied on the mid-span of 6th member.

Table 1. Properties of bridge members

Properties	Magnitude
Young modulus ($\mathrm{N/m^2}$)	2×10^{11}
Second moment of inertia ($\mathrm{m^4}$)	8.33×10^{-10}
Length (m)	1
Height (m)	0.01
Width (m)	0.01

According to Figure 7, three damages with different severities are considered at the different locations of members 4, 7 and 10. Three cracks with depths of 45, 35 and 25% were considered at members 4, 7 and 10, respectively. The location of the crack at member 4, 7 and 10 was $x_c/l = 0.8$, $x_c/l = 0.2$ and $x_c/l = 0.8$, respectively.

The bridge was modeled using 50 frame elements. The vertical, horizontal and rotation of each node were calculated. The undeformed and deformed shapes of the bridge are shown in Figure 8.

The wavelet coefficients of horizontal, vertical and rotation of each member of the bridge were calculated based on the Gaus2 wavelet basis function (Figures 9 and 10). As illustrated in Figures 9 and 10, wavelet coefficients of horizontal displacement of all members are not affected by the damages. In other words, the WT of horizontal displacement is not a proper index to detect the damage in the bridge members.

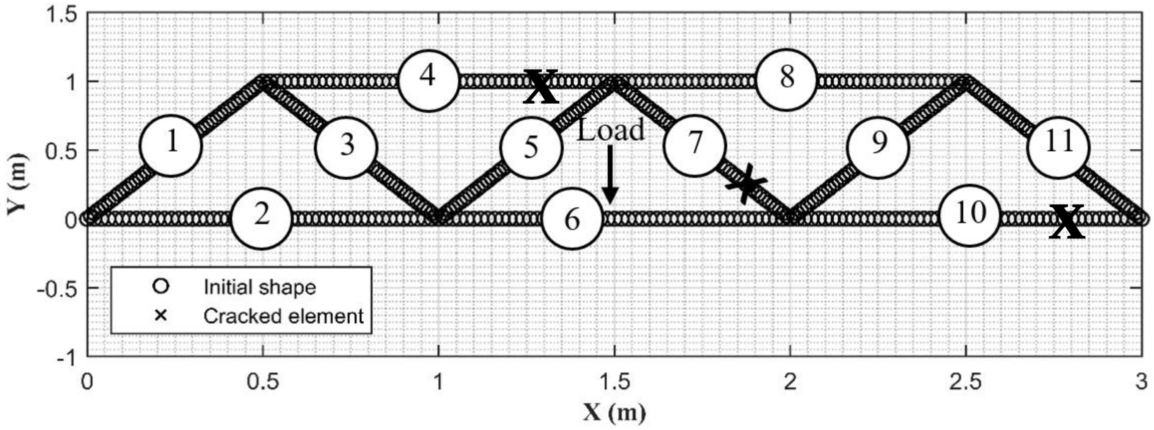


Fig. 7. The bridge model in the first scenario

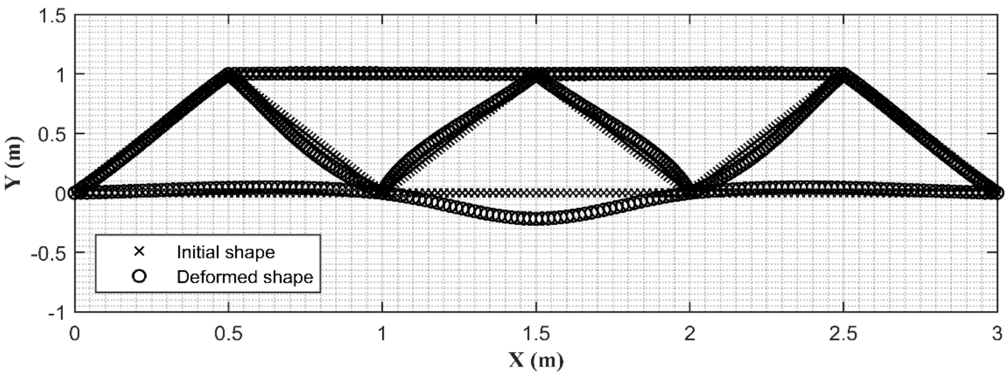
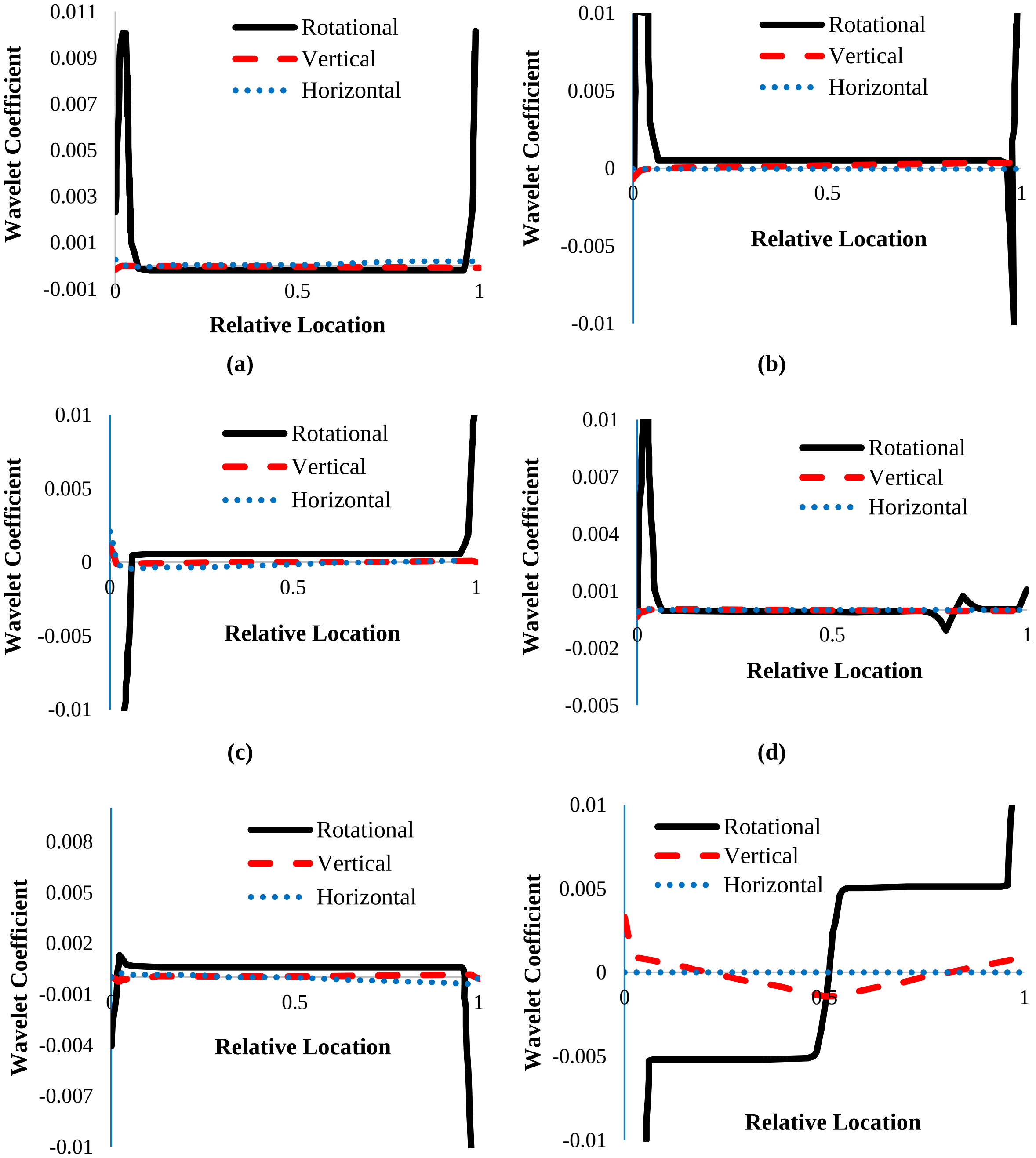


Fig. 8. Undeformed and deformed shapes of the bridge in the first scenario



(a) **(b)**

(c) **(d)**

(e) **(f)**

Fig. 9. Wavelet coefficients of members 1 to 6 for first scenario: a) Member 1; b) Member 2; c) Member 3; d) Member 4; e) Member 5; and f) Member 6

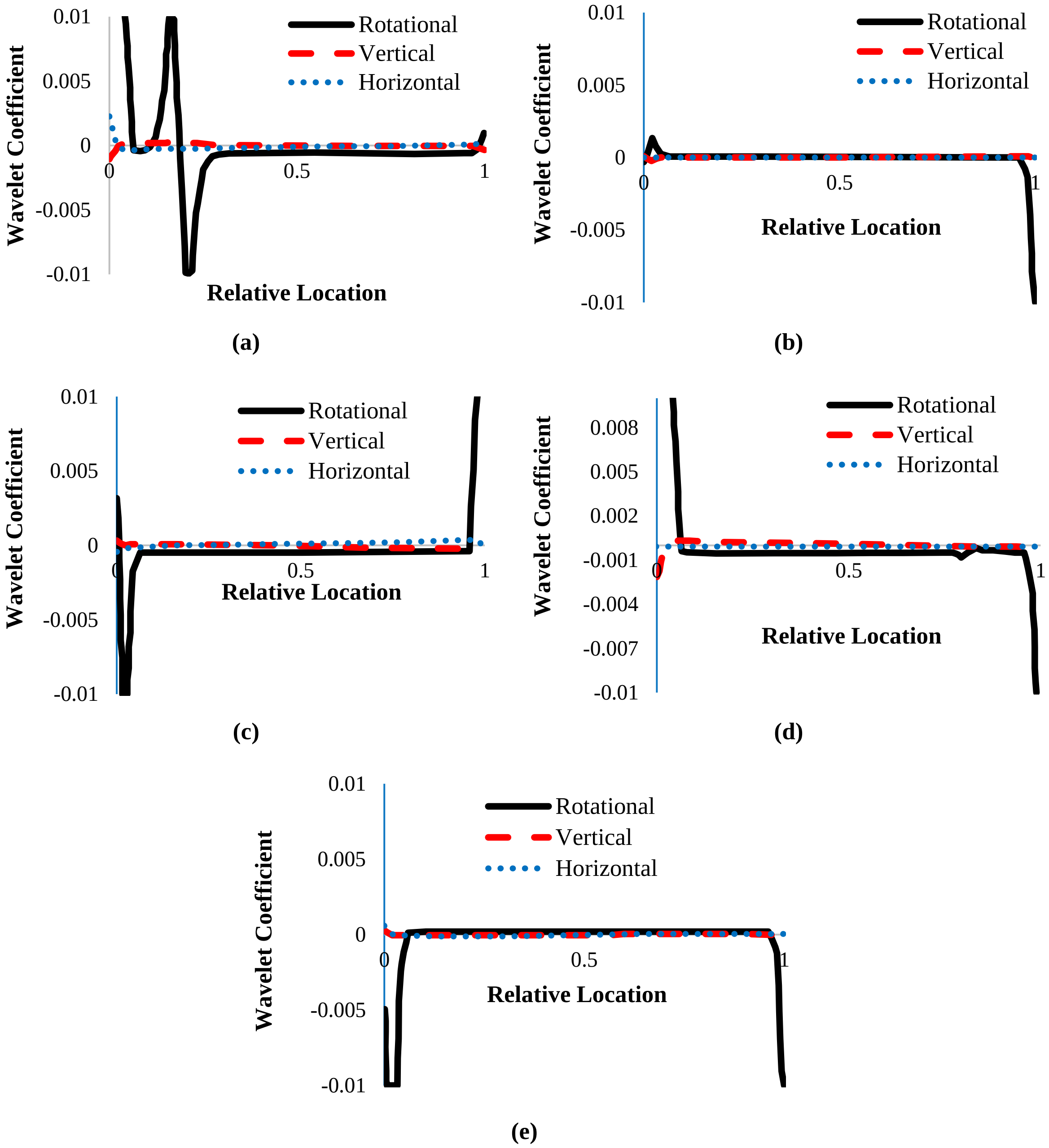


Fig. 10. Wavelet coefficients of members 7 to 11 for first scenario: a) Member 7; b) Member 8; c) Member 9; d) Member 10; and e) Member 11

A Comparison between WT coefficients of vertical displacement and rotation for all members shows that the rotation response is a proper index to identify the damage and loading locations. For instance, the damage location on the 4th and 10th members is illustrated as a jump in the WT coefficient of rotations of these members. However, the WT coefficients of vertical displacements of 4th and 10th members are constant over the members' length and the damage location is not detected. A significant jump is also seen in WT coefficient of rotation response of Member 6. This jump corresponds to the loading location on the 6th member. Moreover, a significant variation is obvious in the WT coefficient of rotation response of the 7th member which corresponds to the damage location on Member 7. The WT coefficient of vertical displacement of Member 7 is not affected considerably by the damage on this member.

4.2. Bridge under one Locomotive

A six-axle locomotive was placed on the

bridge simulated in the previous section. As illustrated in Figure 11, six concentrated loads were applied to Members 2 and 6. The magnitude of each load is 100 N. As shown in Figure 11, three damages with different severity were considered on the 4, 6 and 8th members. Three cracks with depths of 30, 20 and 10% were considered at Members 4, 7 and 10, respectively. The location of the crack at Members 4, 6 and 8 was $x_c/l = 0.8$, $x_c/l = 0.7$ and $x_c/l = 0.5$, respectively.

The bridge was modeled using 50 refined frame elements. The vertical, horizontal and rotation of each node were calculated. The undeformed and deformed shapes of the bridge are shown in Figure 12.

The wavelet coefficients of horizontal, vertical and rotation of each member of the bridge were obtained based on the Gaus2 wavelet basis function (Figures 13 and 14). The trend of the results obtained in this section is same as those presented in the previous section. As shown in these figures, wavelet coefficients of horizontal displacement of all members are not affected by the damages.

Assessment of the WT coefficients of vertical displacement and rotation for all members presented in Figures 13 and 14 prove that the rotation response is a proper index to detect the damage and loading locations. As shown in Figures 13 and 14, three concentrated point loads applied on the 2th and 6th members cause a step change in the WT coefficients of rotations of these members. However, the WT coefficients of vertical displacement of the 2th and 6th members are changed slightly by the concentrated point loads. While the damage causes a significant jump on the WT coefficients of rotations of the 4th, 6th and 8th members, the WT coefficients of vertical displacement of these members are not influenced by the damage.

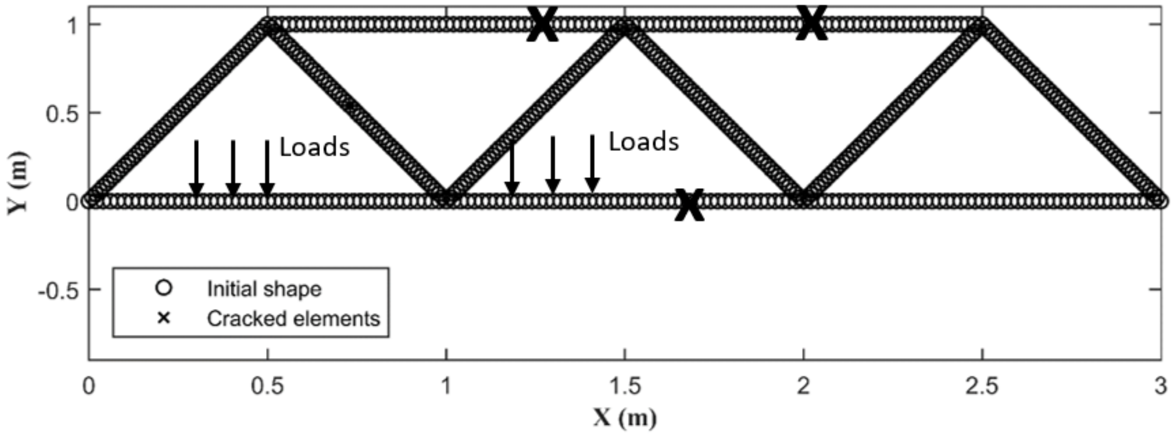


Fig. 11. The bridge model in the second scenario

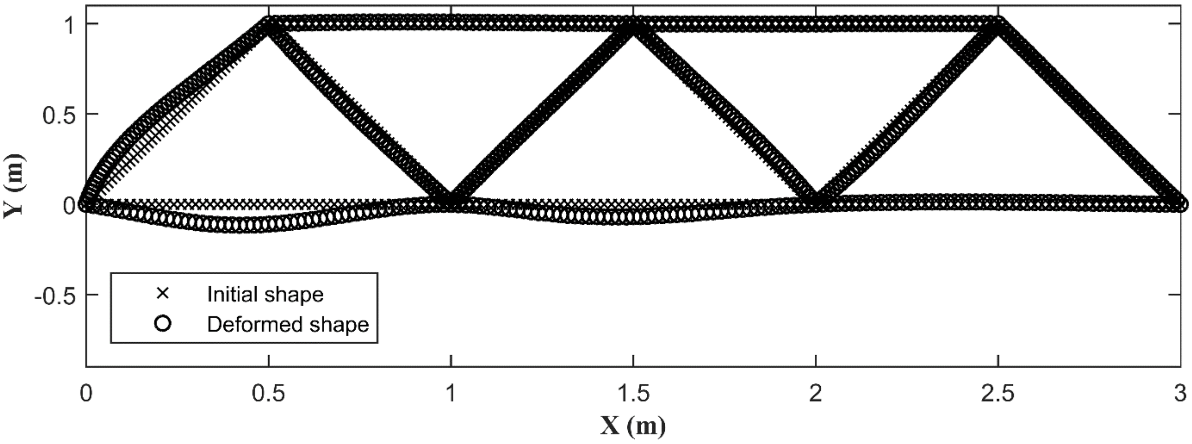
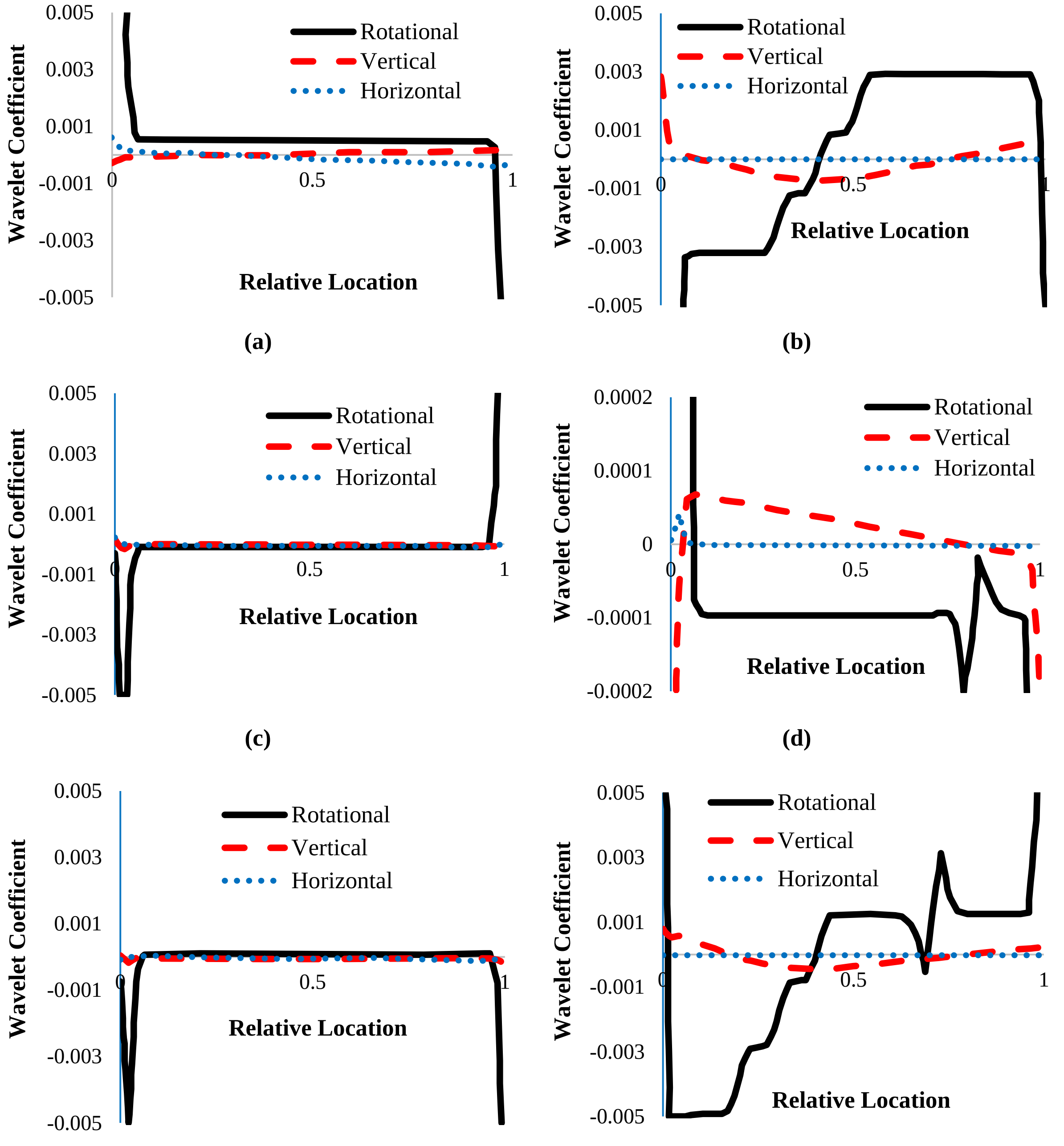


Fig. 12. Undeformed and deformed shapes of the bridge in the second scenario

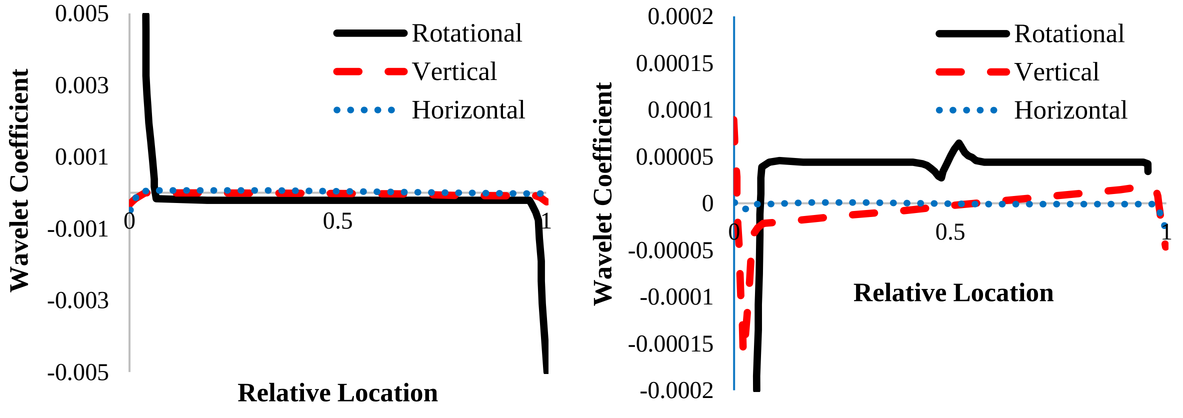


(a) (b)

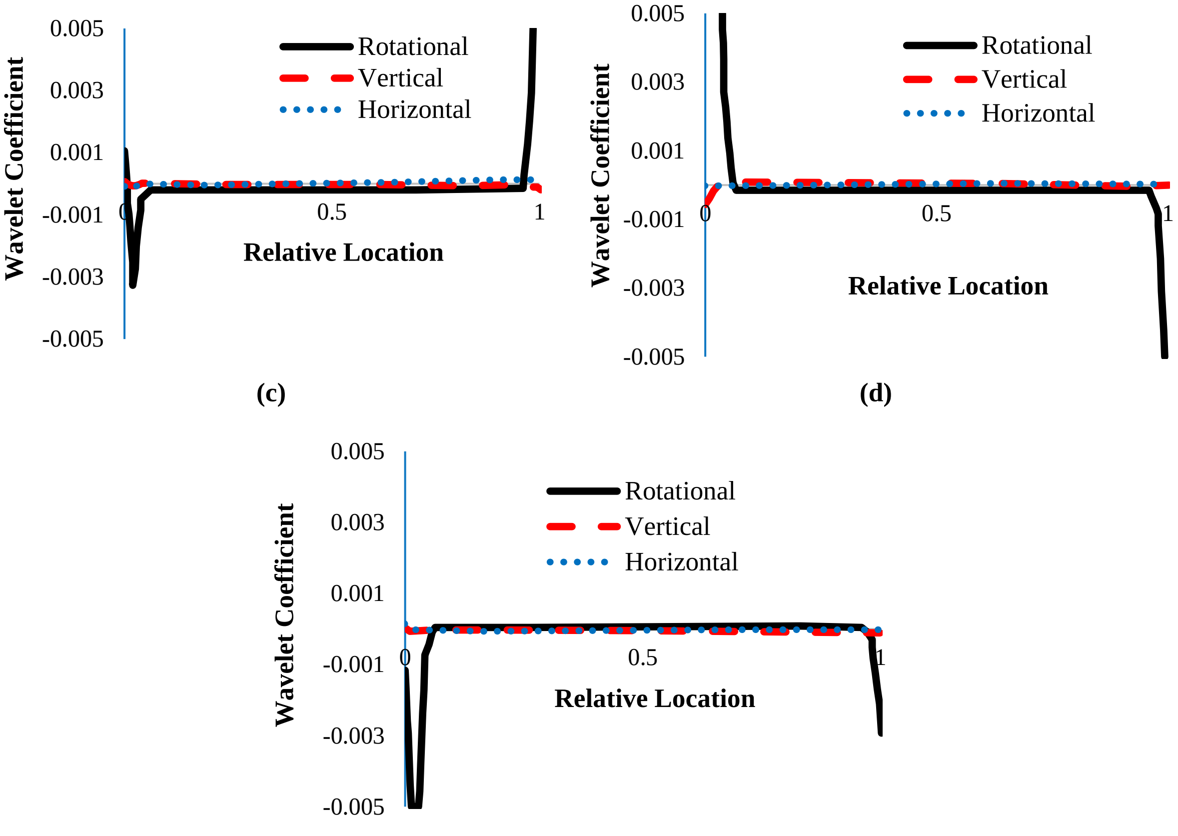
(c) (d)

(e) (f)

Fig. 13. Wavelet coefficients of Members 1 to 6 for the second scenario: a) Member 1; b) Member 2; c) Member 3; d) Member 4; e) Member 5; and f) Member 6



(a) (b)



(c)

(d)

(e)

Fig. 14. Wavelet coefficients of Members 7 to 11 for the second scenario: a) Member 7; b) Member 8; c) Member 9; d) Member 10; and e) Member 11

5. Conclusions

A review of the literature indicated that the rotation response of the structure was a proper index to identify the damage location. On the other hand, the previous studies proved that the Wavelet Transform (WT) is a robust method to detect the damage location in the structure. However, WT of pseudo-static rotation response of the bridge was not used to detect the damage location in the literature. This study was carried out to eliminate this limitation. In this regard, a numerical model of a truss bridge was developed using finite element method. The members of the bridge were modeled using frame element. The static response of the model under one point-load and a six-axle locomotive was obtained. The responses obtained from the model was compared with those of open literature to investigate the validity of the responses. The WT coefficients of horizontal, vertical and rotation of each member of the bridge were obtained based on the Gaus2 wavelet basis function. Several damage scenarios were considered for the bridge to investigate the effectiveness of WT of rotation response of the bridge to detect the damage location.

The results obtained from the study showed whereas the WT of horizontal displacement was not a proper index to detect the damage in the bridge members, a comparison between WT coefficients of vertical displacement and rotation for all members indicated that the rotation response can properly identify the damage and loading locations. In all cases (one- or three-point loads), while the damage causes a significant jump on the WT coefficients of rotations of the members, the WT coefficients of vertical displacement of these members are not influenced by the damage. The results indicated that the WT of pseudo-static rotation response is very effective approach to detect the damage location. In practice, image processing approach was used to measure the pseudo-static rotation response of the structure and then, the WT approach was implemented on the the pseudo-static rotation response. The time history of rotation response of the structure was another type of rotation response (dynamic response) of the structure which usually measured in the tests. The outputs indicated the importance and necessity of investigation of the effectiveness of WT of time history of rotation response of the structure to identify the damage location.

It should be noted that the effect of noise measurement on the response of the structure was neglected in this study. Although this paper was the first one to show that the wavelet transformation of a rotational signal is a proper index to detect the damage location in the bridge, the measurement noises were neglected in the approach implemented in this study. The effect of noise measurement on the dynamic response of structure will be considered in the further study.

6. References

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