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An Improvement in the Efficiency of Radial Basis Functions based on the Optimization of its Parameters using Particle Swarm Optimization

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ABSTRACT

One of the most widely used interpolation methods is the models based on radial basis functions (RBFs) to achieve global and exact surfaces. RBFNN model has been used as an interpolation method in this research to deal with problems in the RBF interpolation method such as the surface fluctuations between sample points and high complexity. Therefore, the centers, radii, and weights of the RBFs were optimized using the PSO algorithm. The centers and radii of RBFs have been initialized using K-means clustering and the K-Nearest Neighbor method respectively. Moreover, the weights of RBFs have been calculated using a pseudo-inverse method. To evaluate the effectiveness of the proposed algorithm the interpolation process has been done on three sets of points with the irregular distribution with different elevation patterns. The results show that MQ and IQ functions provide better accuracy than the GA function in flat areas with low elevation changes as well as areas with average elevation changes. Furthermore, the MQ function could be more accurate than IQ and GA functions in areas with large elevation changes.

KEYWORDS

RBF
Neural Network
Interpolation
PSO
K-means
Clustering
Pseudo Inverse Matrix

1. **Introduction**

Interpolation algorithms are used to create a digital elevation model (DEM) in which a mathematical surface is created with the maximum similarity to the actual level of the land. An interpolation method estimates the value of a feature (elevation) in a point using carried-out measurements at the points around this point (sample points) (Li et al. 2004).

There are different aspects of classification for interpolation methods. One of which is dividing them into exact and inexact methods (Li and Heap 2008). In the exact method, the value calculated by the interpolation method is equal to the actual value in sample points and the surface produced passes through these points. In the inexact method, the value calculated by the interpolation method has minor differences from the actual value in sample points, but the total error has an acceptable value. Among other aspects of the division of interpolation methods are dividing them into local and global categories (Soycan and Soycan 2009).

One of the most widely used general and accurate methods of interpolation is using RBFs to determine a smooth surface by a set of discrete and irregular sample points (Morse et al. 2005). The surface determined by the RBF interpolation method has the following features (Bishop 1995):

- *Smoothest obtained surface among interpolation methods*
- *With a minimum dependency on the distribution of sample points*

- *Containing points with a higher elevation than the maximum elevation and lower elevation than the minimum elevation of sample points*

In the RBF interpolation method, the interpolation function is a linear combination of symmetric RBFs. In other words, for each sample point, a RBF is centered on this point. Moreover, the value for a determined radius is calculated which is the same for all RBFs (González et al. 2003). Finally, the weights of RBFs are calculated by solving a linear system while there is one equation for each sample point. Determining the inverse matrix of coefficients to solve this system has high computational complexity with respect to the equality of the number of RBFs with the number of sample points (Pouderoux et al. 2004). Since the RBF interpolation method is exact, the determined surface will fluctuate in areas between sample points in case of random error in these points (Schwenker et al. 2001). Thus, the approximation is used to resolve these problems by decreasing the number of RBFs. Moreover, by changing the interpolation method from exact to inexact, the problem is converted to an RBFNN problem.

The center and radius of each RBF as well as the weights used in linear combination must be determined in the design of RBFNN which has a significant impact on the accuracy of neural network output. Despite the RBF interpolation method, the centers of RBFs are not considered to be the same as sample points in RBFNN and there are different methods to calculate them. The random assignment of values, selecting from sample points and clustering algorithms can be mentioned among the most important of these methods (Esmaeili and Mozayani 2009).

Another important parameter in the design of the network is the radius of each RBF, which its optimum value should be specified. Unlike the interpolation method where all functions have identical radii, each RBF has a unique radius in the neural network (Fornberg and Zuev 2007). Finally, the network is designed by calculation of the weights of RBFs using analytic methods. One of the methods of determining the optimal values of RBFNN parameters is the use of metaheuristic methods. These methods determine suitable values of neural network parameters by minimizing the overall network error (Gao et al. 2006). Concerning the application of metaheuristic methods in determining the parameters of RBFNN, extensive research has been undertaken in this area.

Schwenker et al. (2001) described a variety of methods of training RBFNN and classified these training methods into three categories of one-Phase, two-phase, and three-phase learning method. In the two-phase method, which is one of the most commonly used methods, initially, the first layer containing the centers and radii of RBFs are trained and then the weights of the output layer are determined. The results of this research show that the efficiency and precision of the RBFNN model are improved in the application of classification by converting the two-phase method into a three-phase method. The centers and radii of RBFs and the weights of the output layer are simultaneously trained in the three-phase method. In this method, the dependency of the weights of the output layer on the centers and radii of RBFs has been disregarded.

An integration of PSO and RBFNN algorithms has been used to predict the electrical power system load in research presented by Lu and Zhou (2009). The centers and radii of RBFs have been calculated using a subtractive clustering algorithm (Sarimveis et al. 2003). Moreover, the PSO algorithm is used to determine the weights of RBFs concerning its ability to solve nonlinear optimization problems. The lack of considering the center and radius parameters in the optimization process and the lack of calculating the weights of RBFs using analytical methods are among the disadvantages of this method. The weights of functions can be determined precisely by optimizing the parameters of RBFs due to the dependence of the weights on these parameters.

Esmaeili and Mozayani (2009) presented research in which the centers and radii of the RBFs in the RBFNN model have been optimized using the PSO algorithm. Furthermore, the singular value decomposition (SVD) method has been used to determine the optimal values for the output layer. The results obtained from this research show that the proposed method has less RMSE than Simple-PSO and newrb algorithms. The lack of a reliable evaluation case study and implementation of only one type of RBF could be considered the major drawbacks of this research.

The value of a function has been approximated by a series of input and known output data as in research presented by Awad (2010). RBFNN has been combined with a genetic algorithm for this purpose. In addition, the weights of RBFs have been calculated using the SVD method. The initial values of the centers and radii of functions have been determined, respectively, by enhanced clustering function approximation (ECFA) (Pomares et al. 2012) and k nearest neighbor (KNN) (Larose 2005) methods to be used in the genetic algorithm. The results show that the approximation of functions by RBFNN optimized by the genetic algorithm has less RMSE than other neural network methods. High complexity in determining the initial values of the centers is among the downsides of the mentioned method.

In this research, a surface with a reasonable approximation of the actual level of the land has been fitted to a set of scattered points. The RBFNN model has been used instead of the RBF interpolation method for this purpose. Optimal values of the centers and radii of RBFs as well as the output layer weights must be determined to use RBFNN in interpolation problem-solving. Thus, the PSO algorithm is used to calculate optimal values of the centers and radii of RBFs considering characteristics such as high speed of convergence, the use of storage to store appropriate solutions, and sharing information between particles. Then, the output layer weights proportional to the center and radius of each function were determined by calculating the pseudo inverse matrix of coefficients in the optimization process. Finally, the effect of different RBFs on the accuracy of interpolation is evaluated based on the RMSE.

2. Radial Basis Functions (RBFs)

One of the most widely used methods in solving interpolation problems is the use of radial basis functions which is an exact and global method. Thus, the RBF interpolation method is used to determine the multivariate function of F while $F: R^m \rightarrow R^d$ *for the sample point set of* (X_R, Z_R) *when equation 1 is established (Orr 1996).*

$$F(X_R) = Z_R \quad R=1,2,\ldots,n \tag{1}$$

where n is the number of sample points, X_R *denotes horizontal coordinates of sample points and* Z_R *is their elevations. The RBF of F can be written in form of a linear combination of RBFs similar to equation 2 (Orr 1996).*

$$F(X) = \sum_{i=1}^{n} w_i f(\|X - X_i\|) + \sum_{j=1}^{q} a_j P_j(X) \tag{2}$$

In equation 2, $\|.\|$ *is Euclidean distance and* w_i *and f are real values. In addition, the second part of equation 2 is a low-grade and pre-determined polynomial function. Thus, the coefficients of radial basis* (w_i) *and polynomial* (a_j) *functions must be determined in a way so that equation 1 and equation 3 are established (Orr 1996).*

$$\sum_{i=1}^{n} w_i P_j(X_R) = 0 \quad j=1, 2, \ldots, q \tag{3}$$

Moreover, the polynomial part in equation 2 can be removed in some cases (Schwenker et al. 2001). As a result, this equation is written in the form of equation 4.

$$F(X) = \sum_{i=1}^{n} w_i f(\|X - X_i\|) \tag{4}$$

The interpolation function is determined by calculating the weights (w_i) *and in other words, solving the linear system of equation 5 for all of the sample points (Orr 1996).*

$$H_{n\times n} W_{n\times 1} = L_{n\times 1} \tag{5}$$

where H, W, and L are matrices of coefficients, weights, and observations, respectively. These matrices are defined in the form of equations 6, 7, and 8.

$$H_{n\times n} = \begin{bmatrix} h_{11} & \cdots & h_{1n} \\ \vdots & \ddots & \vdots \\ h_{n1} & \cdots & h_{nn} \end{bmatrix},$$

$$h_{ij} = f(\|X_{Ri} - X_{Rj}\|) \quad i,j=1,2,\ldots,n \tag{6}$$

and

$$W_{n\times 1} = [w_1 \ldots w_n]T \tag{7}$$

and

$$L_{n\times 1} = [Z_{R1} \ldots Z_{Rn}]T \tag{8}$$

The elements of the observation matrix are the elevation of sample points. Moreover, the type of RBFs h_{ij} *must be determined for the formation of the coefficients matrix. A variety of RBFs are used in the RBF interpolation method of which the following are among the most used:*

- *Gaussian (GA) function (equation 9)*

$$h_{ij} = f(\|X_{Ri} - X_{Rj}\|) = e^{\frac{-\|X_{Ri}-X_{Rj}\|^2}{\sigma i^2}} \tag{9}$$

- *Multi Quadratic (MQ) function (equation 10)*

$$h_{ij} = f(\|X_{Ri} - X_{Rj}\|) = \sqrt{\frac{\|X_{Ri}-X_{Rj}\|^2}{\sigma i^2} + 1} \tag{10}$$

- *Inverse Quadratic (IQ) function (equation 11)*

$$h_{ij} = f(\|X_{Ri} - X_{Rj}\|) = \frac{1}{\left(\frac{\|X_{Ri}-X_{Rj}\|^2}{\sigma i^2}+1\right)^{0.5}} \tag{11}$$

The center parameter is the position of the sample points in the RBF interpolation method and the radius is considered the same for all functions. Computing the inverse of a square coefficient matrix has high complexity and requires cost and time in determining the coefficients of the linear system. In addition, since the RBF interpolation method is an exact method, the fitted surface will fluctuate between these points in case of existing errors between sample points. To resolve these drawbacks the problem can be converted from interpolation to the RBFNN model.

Broomhead and Lowe (1988) proposed the RBFNN model by reducing the number of RBFs in the RBF interpolation method. This neural network model is formed by three input, hidden and output layers according to Figure 1 (Park and Sandberg 1991).

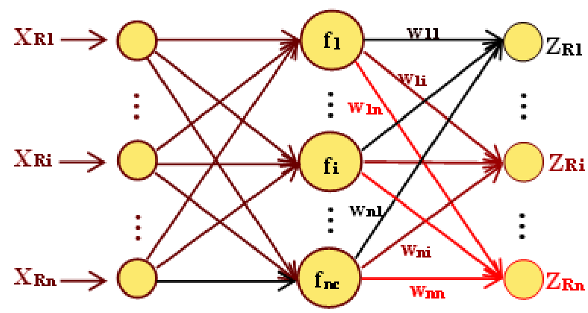

Figure 1. Structure of RBF Neural Network

The input layer consists of input vectors (sample points) which are connected to the hidden layer by unit weights. The hidden layer contains the RBFs and each function has a unique center and radius. Furthermore, the number of these functions is not necessarily equal to the number of interpolation sample points and is determined based on its profound impact on the accuracy of the interpolation method by default (Bai and Zhang 2002). The third layer is the output layer which is a linear combination of neurons (RBFs). Thus, equation 2 is written in the RBFNN model in form of equation 12.

$$F^*(X) = \sum_{i=1}^{nc} w_i f(\|X - C_i\|) \tag{12}$$

where C_i *is the center of the i-th RBF,* w_i *is the weight of this function and nc is the number of RBFs. According to equation 12, the centers of RBFs can contain values other than interpolation sample points.*

There are various methods for determining the parameters of the RBFNN model. One of the most common methods according to which the neural network layers are *individually determined is a two-phase method (Schwenker et al. 2001).*

3. Proposed Methodology

3.1. Dataset

As mentioned before, in this research, the RBFNN model has been used to solve the interpolation problem and determine the topography of the area. To evaluate the effectiveness of the proposed algorithm the interpolation process has been done on three sets of points with the irregular distribution. The distribution of sample and checkpoints in the first studied area is according to Figure 2. Checkpoints do not participate in the interpolation process and are used in the calculation of interpolation method accuracy (Kresse and Danko 2012).

The distribution of sample points in the second studied area is shown in Figure 3 and some checkpoints have been selected among them to evaluate the accuracy of the proposed method. Finally, to evaluate the proposed method in the third area, samples and checkpoints have been selected with distribution according to Figure 4. Table 1 shows the number of sample points, the number of checkpoints, the area, and the maximum and minimum elevation for the three studied areas.

Table 1 Properties of three studied areas

Information on Studied Area	***First Study Area***	***Second Study Area***	***Third Study Area***
Number of Sample Points	***782***	***774***	***367***
Number of Check Points	***98***	***87***	***163***
Area (Hectare)	***60***	***60***	***33***
Minimum Elevation	***980***	***880***	***1958***
Maximum Elevation	***1140***	***1300***	***1993***

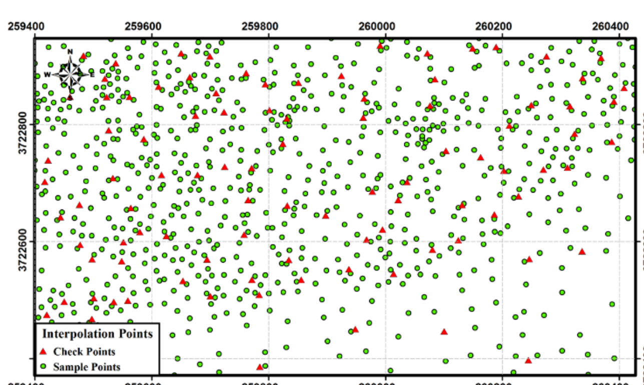

Figure 2 Distribution of sample and check points in the first studied area (The circles and triangles are sample points and check points respectively)

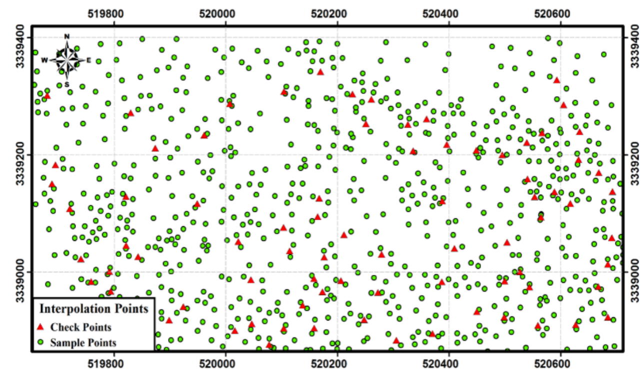

Figure 3 Distribution of sample and check points in the second study area (The circles and triangles are sample points and check points respectively)

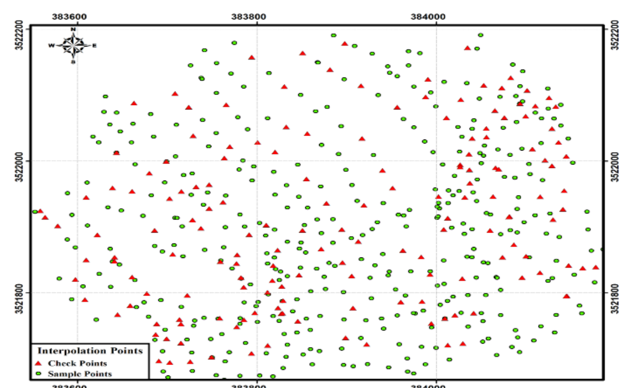

Figure 4 Distribution of sample and check points in the third studied area (The circles and triangles are sample points and check points respectively)

3.2. The Design of Algorithm

Parameters of the network must be determined to solve the interpolation problem by the RBFNN model. In this research, the optimal values of the centers and radii of RBFs are optimized by the PSO algorithm initially, and then, the weights are determined using an analytical method. Thus, firstly particles must be defined and initialized to be used in the PSO algorithm. According to Figure 5, each neural network is assumed to be a particle. The length of each particle is equal to 3nc considering nc for the number of RBFs. According to Figure 5, the horizontal position of the center (x_{C1}, y_{C1}) *and radius* (σ_{C1}) *of the first RBF are placed in the particle and this continues until the placement of all RBFs in the particle.*

x_{C1}	y_{C1}	σ_{C1}	…	x_{Ci}	y_{Ci}	σ_{Ci}	…	x_{Cm}	y_{Cm}	σ_{Cm}

Figure 5 Configuration of particles of PSO algorithm in order to determine RBFNN parameters.

The parameters of the optimization algorithm must be set after the initialization of the particles. One of the most important parameters is the number of suitable particles which is called the initial population. The small initial population reduces the chance of reaching a global solution and a large population causes a significant increase in the computational cost of the algorithm (Bai and Zhang 2002).

The K-means clustering method was used for the Initialization of the centers (Moertini 2002). This algorithm is iterative and does not necessarily produce the same results on the same input data. Therefore, consecutively running this algorithm can produce distinct initial values (centers) for the PSO algorithm.

The radius of RBFs is one of the effective parameters for the accuracy of the RBFNN model. Selecting a very large radius leads to over-smoothing of the surface and a lack of modeling of all the elevation changes. On the other hand, too small a radius leads to over-adapting the surface as well as reducing the coefficient matrix rank (Fasshauer and Zhang 2007). In this study a unique radius is considered for each RBF based on the KNN method (equation 13) (Larose 2005):

$$\sigma_i = \frac{\sum_{j=1}^{k} \sqrt{(x_{ci}-x_{cj})^2 + (y_{ci}-y_{cj})^2}}{k} \tag{13}$$

The mean distance of k RBFs neighboring the i-th function is equal to its radius. One of the major challenges of initializing the radius is the determination of k in the above equation. The k has been chosen randomly in each particle to consider the various ranges of search.

The weights of RBFs are determined by calculating the pseudo-inverse matrix. The objective function is RMSE which is used to evaluate the accuracy of the interpolation method and is computed using equation 14 (Awad 2010).

$$RMSE^2 = \frac{1}{p} \sum_{i=1}^{p} (Z_{ci} - Z_{ci}^{C})^2 \tag{14}$$

where p is the number of checkpoints, Z_{ci}^{C} is the computational elevation of the i-th checkpoint and Z_{ci} denotes the actual elevation of the i-th checkpoint. Eventually, the PSO algorithm will terminate if either the maximum iteration condition is satisfied, or the optimum value does not considerably improve over some iterations.

4. Experimental Results

The proposed methodology is implemented using MATLAB software for the introduced datasets. In this section the results of this implementation is reported.

The accuracy of the model in the three different areas has been determined by calculating RMSE for checkpoints and comparing it with other interpolation methods. The parameters of the PSO algorithm are experimentally determined for three studied areas according to Table 2.

According to the table, 30 particles have been considered as the initial population of the PSO algorithm in the first study area. Moreover, the radius of each RBF is equal to the mean distance of this function to a random number (from 1 to 20) of its neighboring functions. One of the most important parameters in determining the accuracy of the RBFNN model is the number of RBFs which is experimental and pre-determined. Furthermore, the type of RBFs has a significant impact on the accuracy of the RBFNN model. The proposed algorithm has been implemented for three MQ, IQ, and GA functions in a various number of RBFs, and the RMSE is shown in Figure 6.

*Table **2** Parameters of PSO algorithm for three studied areas*

Parameters of PSO	***First Study Area***	***Second Study Area***	***Third Study Area***
Initial population	***30***	***30***	*20*
Inertia weighting factor (w)	***1***	***1***	*1*
Self-recognition component coefficient (C_1)	***5***	***5***	*5*
Social component coefficient (C_2)	***10***	***10***	*10*
Minimum speed	***0***	***0***	*0*
Maximum speed	***50***	***20***	*50*
Maximum iteration	***150***	***150***	*150*
Maximum neighbor for calculating the radius of basis function (k)	***20***	***20***	*20*

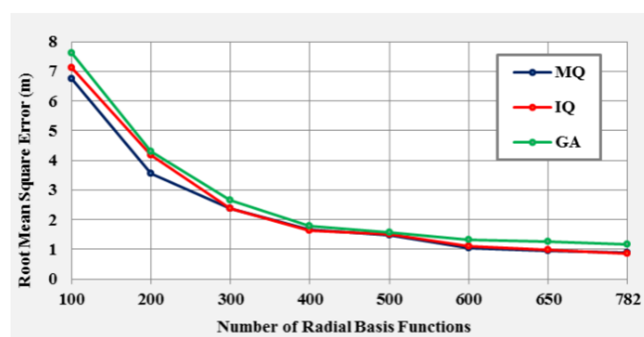

Figure 6 RMSE based on the number of centers for three types of RBFs in the first studied area.

According to Figure 6 MQ function is more accurate compared to IQ and GA functions in the case of 100 to 300 functions. The accuracy of all three types of functions significantly is improved by increasing the number of functions to 500 and it is approximately the same. Ultimately, the number of functions becomes equal to the number of sample points (square coefficients matrix), and the accuracy of IQ and MQ functions becomes nearly the same but still better than the GA function.

The effect of the number of RBFs on RMSE is shown in detail for the MQ function in Figure 7. Moreover, the accuracy of the proposed method is evaluated in the first *studied area and compared to the other interpolation methods.*

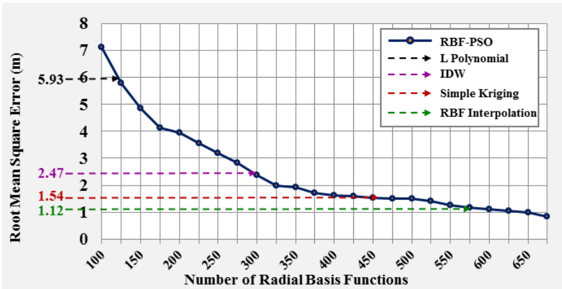

Figure 7 RMSE based on the number of centers for MQ function and in comparison with other interpolation methods in the first study area

Figure 7 depicts that increasing the number of RBFs from 100 to 375 can dramatically enhance the accuracy of the method. In addition, the accuracy of the method is better than IDW, Simple Kriging, and RBF interpolation methods when the number of RBFs is more than 300, 450, and 575 respectively. Despite achieving the most accurate results (with the RMSE = 0.89 meters) in the case of the square coefficients matrix, the accuracy is not improved considerably when the number of functions is more than 650. Thus, the convergence of the PSO algorithm in the number of 650 functions is shown in Figure 8.

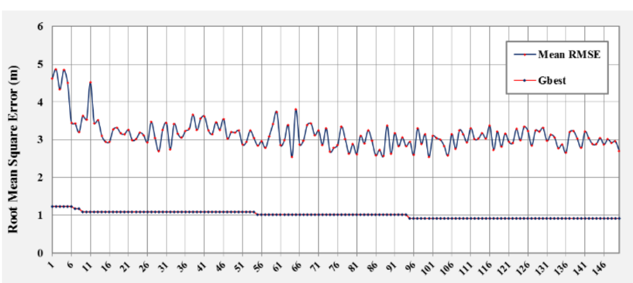

Figure 8 Convergence diagram of PSO algorithm for the first study area

The accuracy of the proposed method is improved from 1.22 to 0.93 meters after 150 iterations. The topography of the first studied area (generated by the proposed method) is shown in Figure 9.

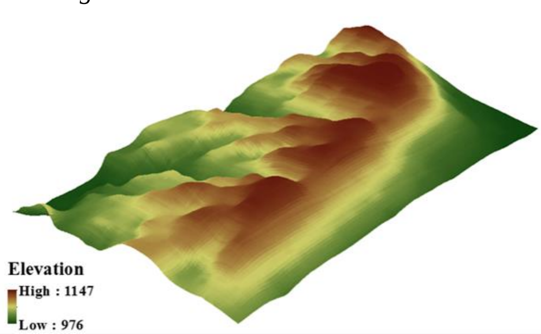

Figure 9 3D view of the topography of the first studied area calculated by proposed method.

A total of 30 particles are considered as the initial population of the PSO algorithm in the second study area. In addition, the radius of each RBF is equal to the mean distance of this function to one up to 20 of its neighboring functions. It should be noted that the number of neighbors is selected randomly. The proposed algorithm is implemented for three MQ, IQ, and GA functions (Figure 10). A different number of RBFs is considered to find out how the accuracy improves by increasing the number of RBFs.

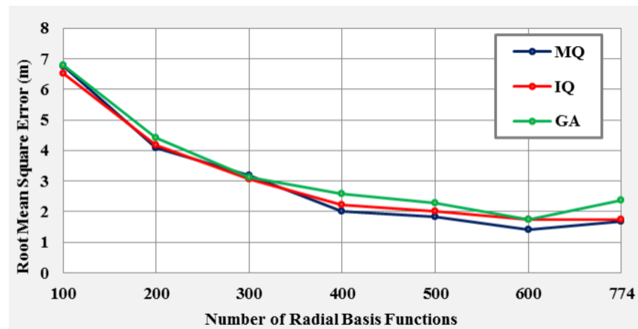

Figure 10 RMSE for the number of centers for all three types of radial basis functions in the second study area

The accuracy of all three types of functions is approximately the same in the number of 100 to 300 functions according to Figure 10. By increasing the number of RBFs to more than 300, the MQ function is more accurate than the IQ and GA functions. The MQ function is the most accurate function when the number of functions is approximately equal to 600 functions (550 RBFs). Moreover, in the square coefficients matrix condition, the accuracy of MQ and GA functions reduces, while the accuracies of IQ functions improve.

As shown in Figure 11, the accuracy of the proposed method (in the number of 550 MQ functions) is compared to other interpolation methods. This figure depicts how the RMSE changes by increasing the number of MQ functions.

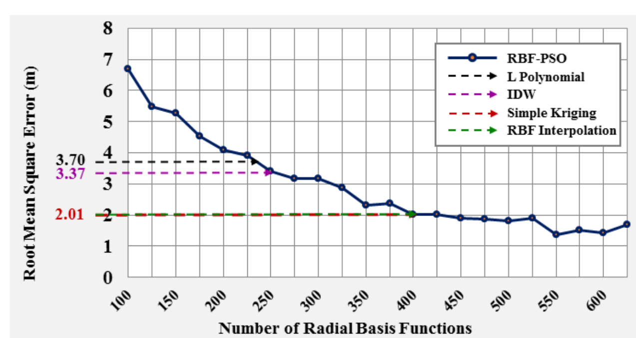

Figure 11 RMSE based on the number of centers for MQ function and in comparison, with other interpolation methods in the second study area.

The proposed method results in better accuracy in comparison with IDW and local polynomial methods when the number of functions is raised to more than 250. Moreover, the accuracy of the proposed method is better *than the accuracy of the Simple Kriging and RBF interpolation methods by increasing the number of functions to more than 400 functions. Increasing the number of functions from 400 to 550 reduces the RMSE to 1.36 m gradually. Moreover, according to Figure 11, the accuracy of the algorithm decreases in the case of the square coefficients matrix. Thus, since the minimum RMSE occurs in the number of 550 functions, the convergence diagram of the PSO algorithm has been drawn in Figure 12 for this number of functions. As shown in the figure, the RMSE of the proposed method is reduced from 1.59 to 1.36 meters after 150 iterations applying the PSO algorithm. The three-dimensional topography of the second studied area is drawn in Figure 13.*

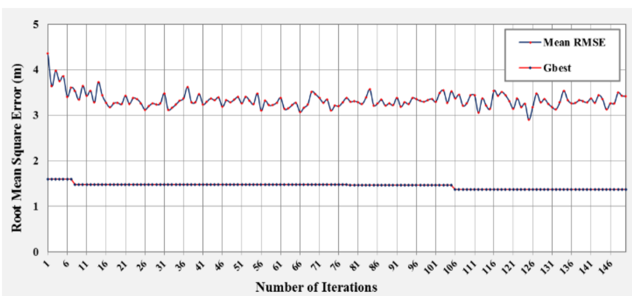

Figure 12 Convergence diagram of PSO algorithm for the second studied area

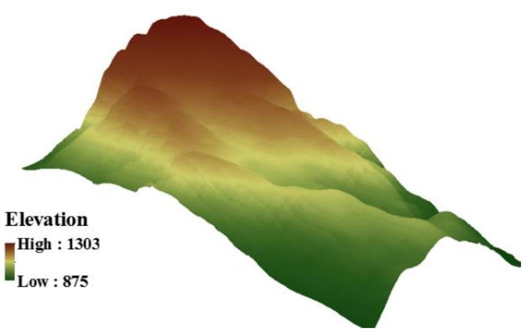

Figure 13 3D view of the topography of the second studied area calculated by proposed method.

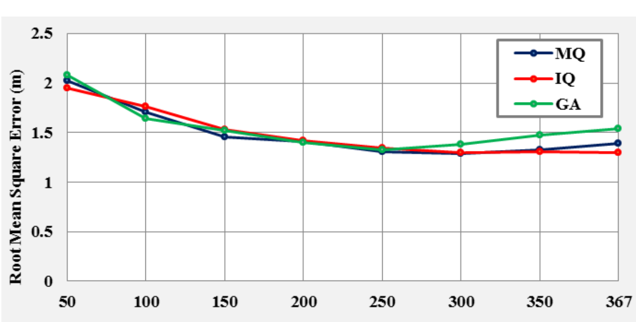

Figure 14 RMSE based on the number of centers for all three types of radial basis functions in the third study area.

The number of 20 initial population has been considered in the PSO algorithm in the third studied area due to the low elevation changes of the area. Likewise, the radius of each RBF is equal to the mean distance of this function to a random number (between 1 and 20) of its neighboring functions. Figure 14 shows how RMSE depends on the number of RBFs for all three types of functions.

Figure 14 illustrates that all three types of functions have approximately equal accuracy when up to 250 RBFs are used. Unlike the GA function, the accuracy of MQ and IQ functions becomes better by increasing the number of RBFs to more than 250. Eventually, in the square coefficients matrix condition, the accuracy of the IQ function remains constant, whereas the accuracy of MQ and GA functions reduces. Thus, the effect of the number of functions on the accuracy of the proposed method has been evaluated for IQ function in Figure 15. The accuracy of this method has been compared with other interpolation methods in this figure.

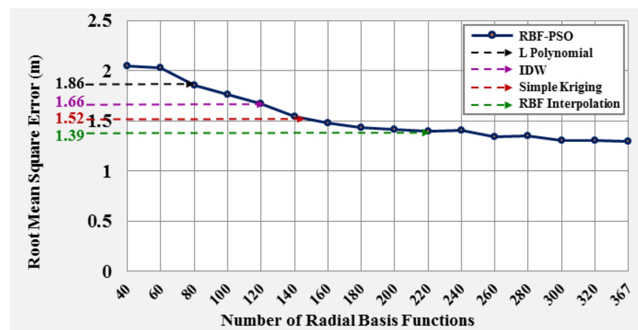

Figure 15 RMSE based on the number of centers for IQ function and in comparison, with other interpolation methods in the third study area.

As can be seen in Figure 15, the implementation of the method with more than 120 functions leads to a more accurate method than the IDW interpolation method. Moreover, the proposed method becomes more accurate than Simple Kriging and RBF interpolation methods when the number of RBFs is more than 140 and 220 functions respectively. The method reaches the maximum accuracy (the RMSE of 1.30 meters) in 300 functions with an approximately constant gradient. Moreover, the accuracy of the proposed method remains approximately constant by increasing the number of functions to more than 300 functions. The convergence diagram of the PSO algorithm in the number of 300 RBFs is drawn in Figure 16. Applying the proposed method turns out to be effective in terms of accuracy since the RMSE is reduced from 1.73 to 1.30 meters while the algorithm ran 150 times. Moreover, the Three-dimensional topography of the third studied area has been drawn in Figure 17.

In this study, pseudo-inverse matrix calculation is preferred to the least squares method to determine the weights because it can compute the best answer in the case of a rank-deficient coefficients matrix. The RMSE in three *studied areas has been compared in Table 3 in the case of using a pseudo-inverse matrix and the least squares method to determine the weights of RBFs.*

*Table **3** RMSE of pseudo-inverse matrix calculation in comparison with the least square method in all three studied areas*

The first studied area			***The second studied area***			***The third studied area***		
n	***Pseudo Inverse***	***Least Squares***	***n***	***Pseudo Inverse***	***Least Squares***	***n***	***Pseudo Inverse***	***Least Squares***
100	***7.11***	***6.64***	***100***	***6.69***	***6.50***	***50***	***1.95***	***2.09***
250	***3.19***	***3.21***	***250***	***3.40***	***3.59***	***150***	***1.53***	***1.54***
400	***1.61***	***1.70***	***400***	***2.02***	***2.19***	***250***	***1.30***	***1.37***
550	***1.26***	***1.26***	***500***	***1.82***	***1.68***	***300***	***1.30***	***1.63***
650	***0.98***	***1.07***	***600***	***1.41***	***1.49***	***350***	***1.28***	***2.21***
782	***0.85***	***50.74***	***774***	***1.68***	***59.36***	***367***	***1.29***	***2.17***

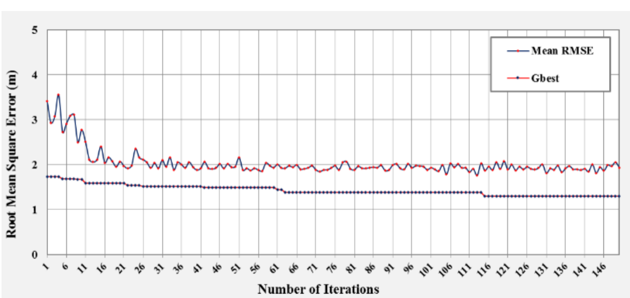

Figure 16 Convergence diagram of PSO algorithm for the third studied area.

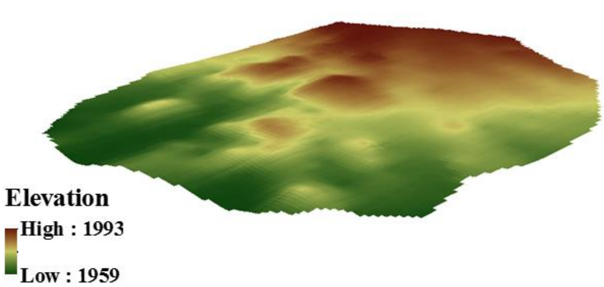

Figure 17 3D view of the topography of the third studied area calculated by proposed method.

According to the table, increasing the number of RBFs raises the probability of the existence of the rank-deficient coefficients Matrix. Rank deficiency may occur when at least two columns (or rows) of the coefficients matrix are parallel. Thus, the RMSE increases significantly in the first and second studied areas due to the inability of the least squares method to determine the weights. In the third studied area, the RMSE of the square coefficients matrix in the least squares method is greater compared to the pseudo inverse calculation method. The initialization of the population might lead to the creation of a particle with at least two approximately the same RBFs. Thus, the RMSE of particles like this particle would be unacceptable due to the inability of the least squares method in determining optimal weights of RBFs. In other words, these particles (with high RMSE) would be removed from the population during the optimization process and the decrease in the initial population reduces the probability of finding a globally optimal solution.

Regarding the results in the areas with small and average elevation changes, the MQ and IQ functions are more accurate compared to the GA function due to the ability of these functions in forming a smoother surface. In other words, the range of elevation reduces by increasing the radius of MQ and IQ functions which leads to fitting a smoother interpolation surface, while the range of elevation in the GA function is always constant between 0 and 1. MQ function is more accurate than IQ and GA in areas with large elevation changes, due to the larger range of elevation of the MQ function in small radii (the radii equal to or greater than the maximum distance to the center of the function). Therefore, this function can model large elevation changes appropriately such as surfaces with Breaks in Slop, drain junction, and hilltop positions.

In the case of the square coefficients matrix, it's not possible to achieve the minimum RMSE of the results. Thus, interpolation by the RBFNN model using a reduced number of RBFs and optimizing network parameters by the PSO algorithm results in better accuracy compared to the RBF interpolation method. Moreover, the proposed method is more accurate than local and global polynomial, IDW, and Simple Kriging interpolation methods. Although the RBFNN interpolation method is more complicated than other algorithms, it has greater efficiency in applications requiring high accuracy. In other words, accuracy could be prioritized over the efficiency of the algorithm based on the existing needs.

*5. **Conclusions***

RBFNN model has been used as an interpolation method in this research to deal with problems in the RBF interpolation method such as the surface fluctuations between sample points and high complexity. Therefore, the centers, radii, and weights of the RBFs were optimized using the PSO algorithm. The centers and radii of RBFs have been initialized using K-means clustering and the KNN method respectively. Moreover, the weights of RBFs have been calculated using the pseudo-inverse method. Eventually, for evaluation of the accuracy of the proposed method, the RMSE of the three studied areas has been accessed.

Regarding the results, the calculation of a pseudo-inverse matrix for coefficients is more efficient than the least square method in the determination of weights for RBFs. The probability of the existence of at least two parallel columns in the coefficient matrix would be raised by increasing the number of basic functions and consequently leads to a rank deficiency. Unlike the least squares method, the pseudo inverse method determines the best answer in the case of a rank-deficient matrix of coefficients.

Moreover, the results show that MQ and IQ functions provide better accuracy than the GA function in flat areas with low elevation changes as well as areas with average elevation changes. This is caused by the ability of these two functions to create a smoother surface in large radii. Furthermore, the MQ function could be more accurate than IQ and GA functions in areas with large elevation changes because of a broader elevation range in small radii. Thus, the MQ function could be regarded as a suitable method to fit surfaces in areas with large elevation changes such as surfaces with breaks in slop.

In a conclusion, it could be conceived from experimental results that the RBFNN model provides higher accuracy than RBF, IDW, and Simple Kriging interpolation methods. Therefore, despite the high complexity due to the iterative process of the PSO algorithm, the proposed method could be used in highly accurate applications.

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