

GeV-Scale Electron Acceleration in a Magnetized Plasma-Filled Waveguide by Twisted Electromagnetic Waves

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Abstract

In this paper, a test particle model is developed to study the electron acceleration in a magnetized plasma-filled waveguide by a twisted electromagnetic wave with variable amplitude and phase along the longitudinal position. With appropriately assigned initial values, the electron total energy gain is obtained using numerical analysis without calculating the dispersion relation. Numerical results show that as long as the twisted electromagnetic waves significantly affect the electron acceleration, during the passage of an electron through the waveguide, one may employ an optimum value of the external magnetic field to obtain the maximum energy gain.

Keywords: Electron Acceleration; Twisted Electromagnetic Waves; Magnetized Plasma-Filled Waveguide.

Introduction

Electron acceleration mechanics, including microwave acceleration, plasma acceleration, and laser acceleration, have been operationalized (1-13). In this regard, the mechanism of electron acceleration in the interaction of twisted electromagnetic waves with plasmas has attracted significant attention because of its fundamental interest in twisted electromagnetic waves physics as well as its relation to many potential applications such as inertial fusion (1), metal cutting (2), surface modification of materials (3), magnetic fusion (4), transmutation of fission products (5), and electron beam welding (6). In this field, the recent developments in high-power pulse lasers, e.g., twisted laser pulses (7), have opened new research frontiers (8-10).

In the 1990s, for the first time, Allen et al. reported (7) that optical beams with helical phase fronts, like, for instance, Laguerre Gaussian (LG) laser modes, carry a

well-defined orbital angular momentum (OAM) (14), called twisted laser beams. Nowadays, twisted electromagnetic waves are used in many research fields of laser-plasma systems due to their unique feature, which allows us to employ them in a wide range of applications, e.g., in optical microscopy (15, 16) and optical micromanipulation (17, 18).

From the laser electron acceleration point of view, it has been discussed how high-intensity twisted electromagnetic waves influence the electron acceleration mechanism in the vacuum (19). In this regard, it has been shown that using a linearly polarized LG laser pulse can reduce the electron beam beam's divergence angle. This can also change the acceleration of off-axis electrons. The interaction of a twisted laser beam with a plasma mirror has recently been investigated. It has been shown that the twisted laser generates high-quality ultrarelativistic electron bunches (20).

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This paper examines the acceleration of a relativistic electron during the propagation of high-intensity twisted electromagnetic waves in a cold magnetized plasma filling the cylindrical waveguide using an advanced mathematical method combined with numerical calculations. The amplitude profile and phase of the wave depend on the longitudinal Z component. Therefore, the nonlinear ponderomotive force $\propto \nabla |E^2|$ cannot be ignored in this study. In this study, we assume that an electron with finite initial energy injected inside the waveguide interacts with the TM mode excited in the system. The electron gets acceleration along the waveguide length. In the acceleration of electrons by twisted waves, the electric and magnetic fields play an important role. In our study, while the B_z component of the twisted wave is zero due to considering the propagation of the TM mode only in the system, the longitudinal electric fields of TM modes (E_z) play the dominant role in the acceleration of the electron. The external magnetic field, providing electron confinement, enables efficient electron acceleration by the longitudinal electric field.

Considering the effect of variation in the twisted electromagnetic waves' amplitude and phase along the longitudinal position, we find that an electron with an initial energy of 100keV gets 1.6 GeV energy in 2.1 cm of propagation along the waveguide length. Results show that the acceleration gradient of electrons is very sensitive to the initial phase and amplitude of the nonlinear TM mode, and the external static magnetic field.

Physical model and theoretical analyses

Consider a circular cylindrical waveguide filled with a uniform plasma of density n_e , immersed in a uniform static axial magnetic field $\vec{B}_{ext} = B_0 \hat{z}$. For the plasma system under study (21), using linear perturbation theory, assuming small amplitude perturbations, the fluid equations enclosed with Maxwell equations result in the following dielectric tensor:

$$\vec{\epsilon} = \begin{bmatrix} \epsilon_{11} & i\epsilon_{12} & 0 \\ -i\epsilon_{12} & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \quad (1)$$

In which the dielectric tensor elements are as follows:

$$\begin{aligned} \epsilon_{11} = \epsilon_{22} &= 1 + \frac{\omega_{pe}^2}{(\omega_{ce}^2 - \omega^2)}, & \epsilon_{12} &= \frac{(\omega_{ce})(\omega_{pe}^2)}{[\omega(\omega_{ce}^2 - \omega^2)]}, \\ \epsilon_{33} &= 1 - \frac{\omega_{pe}^2}{\omega^2} \end{aligned} \quad (2)$$

$\omega_p = \sqrt{4\pi n_e e^2 / m_e}$ is the plasma frequency, $\omega_c = e B_0 / m_e c$ is the cyclotron frequency, c is the velocity of light in the vacuum, and m_e is the mass of the electron, respectively. The zero elements in the dielectric tensor could be attributed to neglecting the calculations'

pressure and temperature gradient terms.

TM mode components

To illustrate the electron acceleration mechanism by the twisted electromagnetic waves, the dynamics of a single electron in the plasma-filled circular cylindrical waveguide interacting with the twisted electromagnetic waves are considered, as shown in Figure 1.

Let us consider the electric field component evolution of the TM mode in the system under study:

$$\nabla^2 E_z + \frac{\omega^2}{c^2} [\vec{\epsilon} \cdot \vec{E}]_z - [\nabla(\nabla \cdot \vec{E})]_z = 0 \quad (3)$$

Using the obtained dielectric tensor, the spatio-temporal evolution of the electric field of the TM mode propagating in the cylindrical waveguide filled by the magnetized plasma could be found as follows:

$$\begin{aligned} \frac{1}{r} \frac{\partial E_z}{\partial r} + \frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 E_z}{\partial \theta^2} + \frac{\omega^2}{c^2} \epsilon_{33} E_z + \frac{i \epsilon_{12}}{r \epsilon_{11}} \frac{\partial E_\theta}{\partial z} \\ + \frac{i \epsilon_{12}}{\epsilon_{11}} \frac{\partial}{\partial r} \frac{\partial E_\theta}{\partial z} - \frac{i \epsilon_{12}}{r \epsilon_{11}} \frac{\partial}{\partial z} \frac{\partial E_r}{\partial \theta} \\ + \frac{\epsilon_{33}}{\epsilon_{11}} \frac{\partial^2 E_z}{\partial z^2} = 0 \end{aligned} \quad (4)$$

Now, to examine the interaction of the twisted electromagnetic waves with the cold magnetized plasma-filled waveguide, consider the twisted form of the TM mode $E_z \sim A(z) J_n(\mu r) e^{i z_1}$, where $z_1 = -\omega t + n\theta + k_z z + \varphi(z)$, in which the phase of the TM mode $\varphi(z)$, depends on the Z component and k_z ($k_r = \mu$), is the propagation constant in the Z direction. Using equation (4) for the defined nonlinear twisted form of the TM mode, the electric field components in cylindrical coordinates are obtained as follows:

$$E_r(r, \theta, z) = \Sigma_n \mu f'(z) J'_n(\mu r) e^{i(-\omega t + n\theta)} \quad (5)$$

$$E_\theta(r, \theta, z) = \Sigma_n \frac{f'(z)}{r} J_n(\mu r) e^{i(-\omega t + n\theta)} \quad (6)$$

$$E_z(r, \theta, z) = \Sigma_n \mu^2 f(z) J_n(\mu r) e^{i(-\omega t + n\theta)} \quad (7)$$

Where $J_n(\mu r)$ is the nth-order of the Bessel function and $2J'_n(\mu r) = J_{n-1}(\mu r) - J_{n+1}(\mu r)$, $2nJ_n(\mu r)/(\mu r) = J_{n-1}(\mu r) + J_{n+1}(\mu r)$, $f(z) = A(z) e^{ik_z z + i\varphi(z)}$, and $\partial J_n / \partial (\mu r) = J'_n(\mu r)$. In these equations, A (z) and the

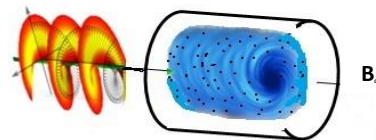


Figure 1. Schematic of twisted electromagnetic waves interacting with a plasma-filled circular cylindrical waveguide.

$\varphi(z)$ are the amplitude profile and phase of the TM mode wave, respectively. Using the obtained electric field components, Maxwell's equation results in the following relations for the magnetic field components:

$$B_z = 0 \quad (8)$$

$$B_\theta(r, \theta, z) = \Sigma_n \frac{ic\mu J'_n(x)}{\omega} A(z) \left(\mu^2 + k_z^2 + \frac{b}{c} \right) e^{iz_1} \quad (9)$$

$$B_r(r, \theta, z) = \Sigma_n \frac{ncJ_n(x)}{r\omega} A(z) \left(\mu^2 + k_z^2 + \frac{b}{c} \right) e^{iz_1} \quad (10)$$

Where $x = \mu r$, $k_r = \mu$ and $J'_n(x) = dJ_n(x)/dx$. In the last step, insert the obtained electric and magnetic field equations (5- 10) into equation (4), separating the real and imaginary parts of the resulting equation. Then, give equations for the amplitude and phase of the twisted wave.

$$A'' = \frac{d^2 A}{dz^2} = [-b_1 A(z)/c_1 + 2k_z A\varphi' + A\varphi'^2] \quad (11)$$

and

$$\varphi'' = \frac{-[2k_z A' + 2A'\varphi']}{A(z)} \quad (12)$$

in which, $b_1 = -\mu^4 - \left[\frac{k_z^2 \mu^2 \epsilon_{33}}{\epsilon_{11}} \right] + [(\omega^2 \epsilon_{33} \mu^2)/c^2]$ and $c_1 = (\mu^2 \epsilon_{33})/\epsilon_{11}$ do not change with z , $A' = dA/dz$, and $\varphi' = d\varphi/dz$. It must be mentioned that while in the linear study of the wave propagation in the system finding the analytical dispersion relation is possible, in most nonlinear cases with boundary conditions, such as those we investigated in the present paper, finding the analytical dispersion relation is impossible. To analyze the system, the dispersion relation has not been neglected, It is considered by the numerical solution of the amplitude $A(z)$ and the phase $\varphi(z)$ with appropriate boundary conditions.

Power transmission through the waveguide

The initial amplitude of TM mode $A(z = 0)$ could be related to the average wave power transmitted by the Poynting vector. The average wave power in cylindrical coordinates is obtained as follows:

$$\bar{p} = \left[\frac{c}{8\pi} \int_0^{2\pi} d\theta \int_0^R (E_r B_\theta^* - E_\theta B_r^*) r dr \right] \quad (13)$$

Inserting equations (5)-(10) into equation (13) gives:

$$\bar{p} = \frac{c^2 A^2}{8\omega} \left[\left(k_z^2 + \mu^2 + \frac{b}{c} \right) (k_z + \varphi') R^2 \mu^2 J_n'^2(\mu R) \right] \quad (14)$$

Using the differential equation (12), one can obtain:

$$(k_z + \varphi') A^2 = (k_z + \varphi'_0) A_0^2 \quad (15)$$

Where $A_0 = A(z = 0)$ and $\varphi'_0 = [d\varphi/dz]_{z=0}$.

Now, substituting equation (15) into equation (14) yields:

$$\bar{p} = \frac{c^2 A_0^2}{8\omega} \left[\left(k_z^2 + \mu^2 + \frac{b}{c} \right) (k_z + \varphi'_0) R^2 \mu^2 J_n'^2(\mu R) \right] \quad (16)$$

Or, equivalently,

$$A_0^2 = \frac{(8\omega\bar{p})}{[c^2(k_z^2 + \mu^2 + \frac{b}{c})(k_z + \varphi'_0) R^2 \mu^2 J_n'^2(\mu R)]} \quad (17)$$

Boundary conditions

The boundary conditions at the perfectly conducting waveguide wall requires that the component of the electric field vanish at the wall, i.e.

$$E_z(r = R) = 0 \quad (18)$$

Thus,

$$J_n(\mu R) = 0, \mu R = p_{nm} \quad (19)$$

Where p_{nm} is the m th root of the n th-order Bessel function of the first kind.

Acceleration gradient and energy gain of electrons

The acceleration gradient and energy gain of electrons in the system under study are obtained by using the following momentum and energy equations:

$$\frac{d}{dt} (m_e \gamma_e \vec{v}_e) = -e [\vec{E} + \vec{v}_e \times (\vec{B} + \vec{B}_{ext})/c] \quad (20)$$

$$\frac{dU}{dt} = -e \vec{v}_e \cdot \vec{E} \quad (21)$$

Where $(m_e \gamma_e \vec{v}_e)$ is the relativistic electron momentum, $\vec{B}_{ext} = B_0 \hat{z}$ is the external static magnetic field along the waveguide axis, $U = (\gamma_e - 1)m_e c^2$ is the energy, \vec{v}_e is the velocity of the test electron in the system and, \vec{E} and \vec{B} are the electric and magnetic fields of the TM twisted mode interacting with the test particle. From equation (20), the electron acceleration inside the plasma-filled waveguide could be found as follows:

$$\frac{dv_r}{dt} = \frac{v_\theta^2}{r} - \frac{v_r}{\gamma_e} \frac{d\gamma_e}{dt} - \frac{eE_r}{\gamma_e m_e} - \frac{eE_r}{c\gamma_e m_e} [v_\theta(B_z + B_0) - v_z B_\theta] \quad (22)$$

$$\frac{dv_\theta}{dt} = -\frac{v_\theta v_r}{r} - \frac{v_\theta}{\gamma_e} \frac{d\gamma_e}{dt} - \frac{eE_\theta}{\gamma_e m_e} + \frac{eE_\theta}{c\gamma_e m_e} [v_r(B_z + B_0) - v_z B_r] \quad (23)$$

$$\frac{dv_z}{dt} = -\frac{v_z}{\gamma_e} \frac{d\gamma_e}{dt} - \frac{eE_z}{\gamma_e m_e} - \frac{e}{c\gamma_e m_e} [v_r B_\theta - v_\theta B_r] \quad (24)$$

Where $v_r = dr/dt = \dot{r}$, $v_\theta = r\dot{\theta}$, $v_z = \dot{z}$. In the next section, using the obtained equations (5- 12), and (17) as well as the appropriate boundary condition (19), the numerical calculation will be employed to find the energy gain and acceleration of the test electron in the interaction with the twisted electromagnetic waves in the plasma filled waveguide, based on the fourth-order Runge-Kutta method.

Results

In the interaction of the twisted TM mode with the magnetized plasma-filled waveguide, let us consider the numerical calculations in investigating the electron total energy gain under various initial conditions based on the physical model presented in Sec. III. For this purpose, a code is programmed in Fortran language using the Runge-Kutta method of order four.

In the following simulations, the plasma density is $n_0 = 1.08 \times 10^{11} \text{cm}^{-3}$, the plasma frequency is $\omega_p = 18.53 \text{ GHz}$, the wave power is $\bar{p} = 14 \times 10^{13} \text{ (erg/sec)}$, the cylindrical waveguide radius is $R = 2.1 \text{ (cm)}$, and the initial electron energy is 100 (keV) . An electron is injected into the waveguide at $r_0 = R/2 = 1.05 \text{ cm}$, $\theta_0 = \pi/6$, $z_0 = 0$, $v_{0r} = 0$, $v_{0\theta} = 0$, and $v_{0z} = 0.55 c$ interacts with the twisted TM mode with the initial phase $\varphi_0 = 0$ (22). To show the effect of twisted electromagnetic waves on electron acceleration, we have considered the initial conditions of electron entry into the waveguide similar to reference (22) (in this reference, waves do not have twisted properties).

For simplicity, numerical calculations have been done by defining the dimensionless parameters as $r = c r' / \omega_p$, $\theta = \theta'$, $z = c z' / \omega_p$, $\omega = \omega_p \omega'$, $t = t' / \omega_p$, $v = c v'$, $E = m_e c \omega_p E' / e$, $B = m_e c \omega_p B' / e$, $A = c^3 m_e A' / e \omega_p$, $k_z = \omega_p k'_z / c$, $k_r = \mu = \omega_p k'_r / c$, $\omega_c = \omega_p \omega'_c$ and $\bar{p} = c^5 m_e^2 \bar{p}' / e^2$.

Figure 2 illustrates the variation of electron total energy gain vs. waveguide length in the interaction with various twisted electromagnetic waves modes $n = 3$, $n = 4$, and $n = 5$, where the electromagnetic waves frequency corresponds to the case where $\omega = 41.44 \text{ GHz}$. Each curve corresponds to a fixed mode value. This figure shows that increasing the mode number increases the gradient electron's total energy gain. In this regard, for the case of $n = 5$, an electron with initial an energy of 100 keV gets 1.6 GeV energy in 2.1 (cm) of traveling along the waveguide.

In Figure 3, the effect of the external magnetic field on the electron total energy gain in the system understudy has been shown for the mode number $n = 1$. The other parameters are the same as in Figure 2. Results indicate that while increasing the magnetic field magnitude from the zero value decreases the electron's total energy for weak magnetized systems, in the case of a magnetic field around $B_0 = 2834 \text{ (G)}$ the electron's total energy suddenly increases. This behavior could be attributed to the resonance at this magnetic field value, which will be considered in detail in Figure 5.

The electron total energy versus waveguide length in interaction with non-twisted ($n = 0$) and twisted ($n = 1$) electromagnetic waves has been depicted in Figures. 4(a)

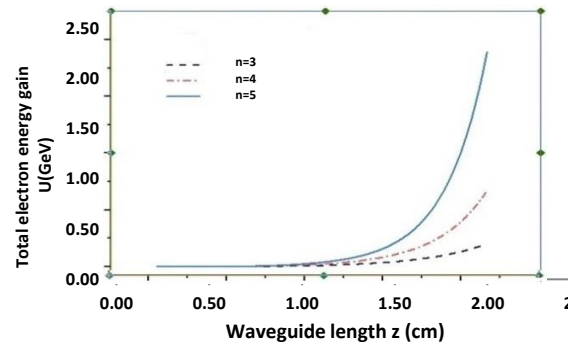


Figure 2. Variation in the electron energy gain vs. waveguide length. The choices for system parameters are $\omega = 41.44 \text{ GHz}$, $n_0 = 1.08 \times 10^{11} \text{cm}^{-3}$, $\varphi_0 = 0$, $\bar{p} = 14 \times 10^{13} \text{ (erg/sec)}$, and $R = 2.1 \text{ cm}$.

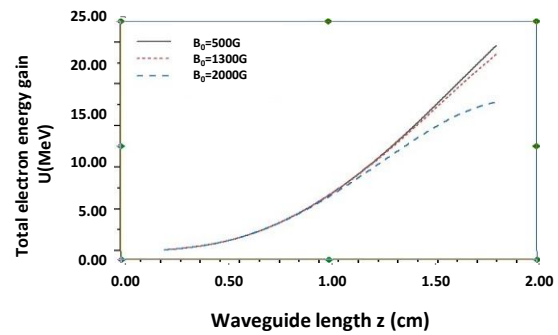


Figure 3. Variation of electron energy gain vs. waveguide length for the case where twisted electromagnetic waves frequency corresponds to $\omega = 40.82 \text{ GHz}$, the mode number $n = 1$, and different external magnetic field values, e.g., $B_0 = 500 \text{ (G)}$, $B_0 = 1300 \text{ (G)}$ and $B_0 = 2000 \text{ (G)}$. The system parameters are otherwise identical to Fig. 2.

and 4(b) for three values of the electromagnetic wave frequencies. The other parameters are the same as in Figure 2. As seen in these figures, increasing the wave frequency decreases the electron energy gain in both cases. Comparing Figures 4(a) and 4(b) shows that for the case of a non-twisted pulse, one can see the optimum value for the energy gain at the traveling of electrons in the waveguide. In contrast, the electron can get higher energy from the twisted electromagnetic waves by traveling in the waveguide at a more significant distance. Moreover, the electron gets more energy in the interaction with twisted electromagnetic waves than non-twisted ones for the same traveling length in the waveguide, particularly at large distances.

The maximum total electron energy gain versus the external magnetic field magnitude has been illustrated in

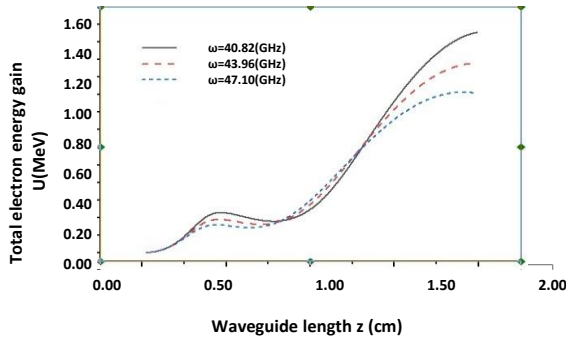


Figure 4(a). Variation of electron energy gain vs. waveguide length for the non-twisted electromagnetic waves ($n = 0$) with different frequencies, e.g., $\omega = 40.82$ (GHz), $\omega = 43.96$ (GHz) and $\omega = 47.10$ (GHz). Here, $B_0 = 1500$ (G) and the system parameters are otherwise identical to Fig.2.

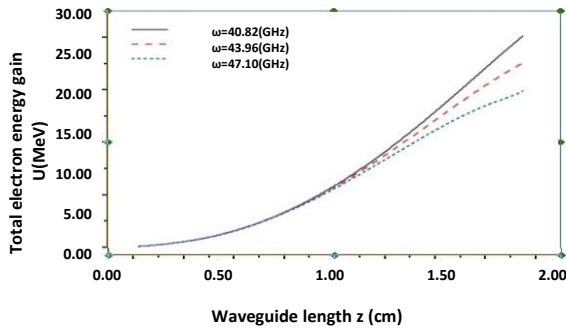


Figure 4(b). Variation of electron energy gain vs. waveguide length for twisted electromagnetic waves ($n = 1$) with different frequencies, e.g., $\omega = 40.82$ (GHz), $\omega = 43.96$ (GHz) and $\omega = 47.10$ (GHz). Here, $B_0 = 1500$ (G) and the system parameters are otherwise identical to Fig. 2.

Figure 5 for three twisted electromagnetic wave frequencies. As a significant result, the resonance absorption of twisted waves could occur in the system with the specific value of the external magnetic field, $B_0 = 2834$ (G), for the parameters used in the figure.

In electron acceleration by linear waves, the amplitude of the electric field 10^7 (V/m) is required for the acceleration of electrons to 1.6 GeV in a 2.1 (cm) distance, but in our study, due to the presence of nonlinear effects, such as higher mode resonance phenomena, this acceleration rate could be achieved even with an electric amplitude lower than 10^7 V/m. This claim could be confirmed by investigating the Figures (3-5). These figures show that in the absence of nonlinear phenomena (for the primary mode $n = 1$), the maximum electron acceleration by the employed electric field amplitude will be ~ 20 MeV. In contrast, in the presence of nonlinear effects, $n = 3, 4$, and 5 electron

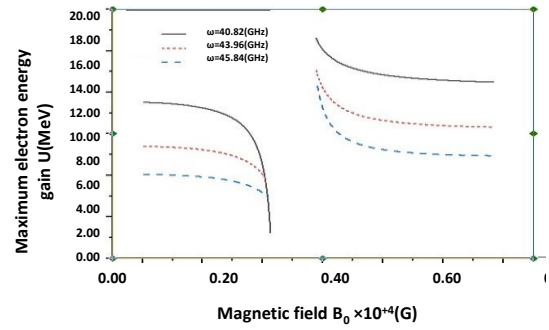


Figure 5. Variation in the maximum electron energy gain vs. magnetic field B_0 for twisted electromagnetic waves ($n = 1$) with different frequencies, e.g., $\omega = 40.82$ (GHz), $\omega = 43.96$ (GHz) and $\omega = 45.84$ (GHz). The system parameters are otherwise identical to Fig. 2.

accelerations can be obtained up to about ~ 2 GeV (Figure 2).

In Figure 6, a three-dimensional trajectory of the electron inside the plasma-filled conducting waveguide has been shown for $\omega = 40.82$ (GHz), $n = 1$, and $B_0 = 1850$ G. The electron is assumed to be initially at $r_0 = R/2 = 1.05$ cm, $\theta_0 = \pi/6$, and $z_0 = 0$, with an initial velocity of $v_{0r} = 0$, $v_{0\theta} = 0$, and $v_{0z} = 0.55$ c. The electron accelerates along the waveguide length due to the twisted TM mode's longitudinal and transverse field components. The axial magnetic field component confines the electron's motion near the Z axis for maximum energy gain, as expected.

Figure 7 demonstrates the variations of perpendicular velocity v_p , total velocity v_{total} , and parallel velocity v_z with waveguide length Z for twisted electromagnetic waves ($n = 1$). The electron experiences a transverse electric force and acquires a transverse velocity component. The interaction between the transverse magnetic field of the twisted wave induces a force $\vec{v}_p \times \vec{B}$ in the axial direction. The resonance between the electrons and the electric field of the electromagnetic wave gives rise to the acceleration of the electrons to high energies.

Figure 8 shows the effect of the axial magnetic field and frequency of the twisted electromagnetic wave on the electron velocity components. This figure illustrates that the axial velocity decreases while the perpendicular velocity v_p increases with increasing twisted wave

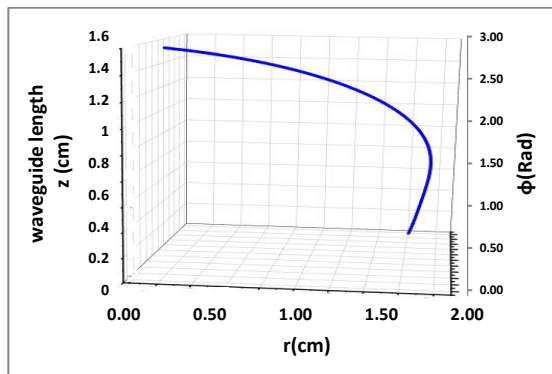


Figure 6. Electron trajectory in the three-dimensional plane for $\omega = 40.82$ (GHz), $n = 1$, $B_0 = 1850$ G.

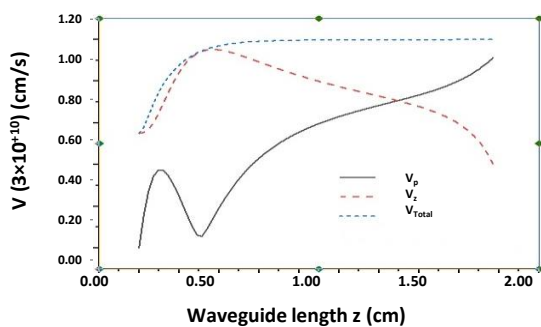


Figure 7. Variation in perpendicular velocity v_p , total velocity v_{total} , and parallel velocity v_z vs. waveguide length Z . The choices for system parameters are $n = 1$, $\omega = 40.82$ (GHz), and $B_0 = 1500$ G.

frequency and axial magnetic field. Physically, this is a result of the fact that wave-particle resonance occurs.

Conclusions

In this paper, the acceleration of electrons up to 1.6 GeV by a twisted electromagnetic wave in a magnetized plasma-filled waveguide is studied. The amplitude profile and phase of the wave depend on the longitudinal Z component (without Fourier Transform). Therefore, in this study, nonlinear terms such as ponderomotive force and others play significant role in the output. The effects of twisted electromagnetic waves and the external magnetic field have been investigated using numerical simulation. It is found that the electron energy grows significantly during the passage of an electron through the waveguide. This is due to its interaction with the twisted electromagnetic waves. This effect is significant when the twisted electromagnetic wave modes increases and the frequency decreases. Further numerical results showed that there is an optimum value for an external magnetic field that changes the resonance conditions, and the electron energy gain increases noticeably. The proposed structure

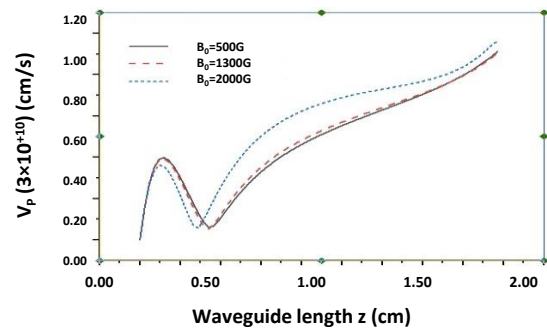


Figure 8(a). Plots of the electron velocity components as a function of the waveguide length for different values of the external magnetic field and twisted wave frequencies. Here $n = 1$, and $\omega = 40.82$ (GHz).

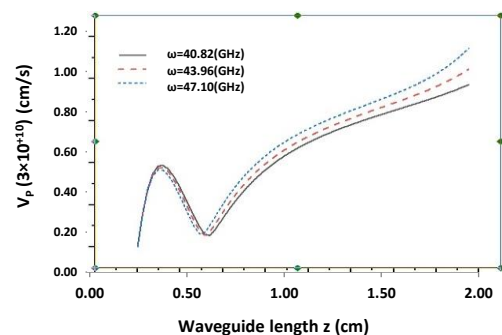


Figure 8(b). Plots of the electron velocity components as a function of the waveguide length for different values of the external magnetic field and twisted wave frequencies. Here, $n = 1$, and $B_0 = 1500$ G.

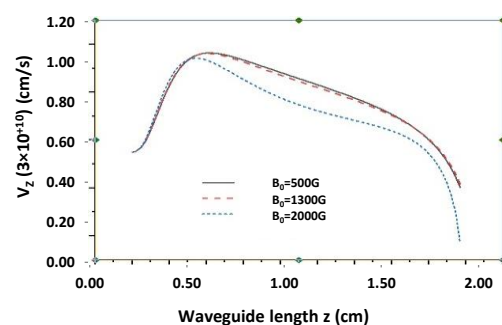


Figure 8(c). Plots of the electron velocity components as a function of the waveguide length for different values of the external magnetic field and twisted wave frequencies. Here, $n = 1$ and $\omega = 40.82$ (GHz).

improves the efficiency of the accelerator devices opening up a whole new area of investigation.

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