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The Complete Model of Wet Air Analysis in Mine Ventilation Design (CMWAA method)

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ABSTRACT

The ventilation design of mines is based on fluid dynamics. Air is the main fluid in mining ventilation. This fluid is analyzed by two models: incompressibility and compressibility. The air-fluid in the compressibility model is examined in two models of dry and wet air along with thermal analysis. This paper argues that a number of presented parameters in the common thermodynamic analysis of mine ventilation should be modified. Accordingly, issues of the Earth's true gravity acceleration, potential energy difference, the specific heat capacity of air at constant pressure and volume, enthalpy difference of air, the average volumetric mass of dry air, and the amount of output moisture are rechecked. Therefore, a new method is presented in this paper for correcting thermodynamic equations in mine ventilation design. The name of this method is the Complete Model of Wet Air Analysis (CMWAA method). The results of this paper show that the CMWAA method can accurately perform thermodynamic analysis of air-fluid in mine ventilation without requiring a specified evaporation rate in mine networks, with minimal iteration of calculations.

Keywords: *Ventilation, Thermodynamic, Compressible, Thermal, CMWAA..*

1. Introduction

Liquids and gases fall within the category of “fluids” as different states of matter. The incompressible flows are mainly dealt with cases of constant density. Also, when the density variation in the flow domain is negligible, the flow can be treated as incompressible. Invariably, it is valid for liquids because liquid density decreases slightly with temperature and moderately with pressure over a broad range of operating conditions. Hence, the liquids are considered incompressible. Conversely, compressible flows are routinely defined as “variable-density flows”. Thus, it applies only to gases, where they may be considered incompressible or compressible, depending on the conditions of operation. During the flow of gases under certain conditions, the density changes are so small that the assumption of constant density can be made with reasonable accuracy in a few other cases, the density changes of the gases are significant. Due to the dual nature of gases, they need special attention, and the broad area of the study of the motion of compressible flows is dealt with separately in the subject of “gas dynamics”. Thus, gas dynamics is the study of fluid flows where compressibility and temperature changes become important. Here, the entire flow field is dominated by Mach waves and shock waves when the flow speed becomes supersonic. Most of the flow properties change across these waves from one state to other. In addition to basic fluid dynamics, the knowledge of thermodynamics and chemical kinetics is also essential to the study of gas dynamics [1].

Gas is considered a collection of particles (molecules, atoms, ions, electrons, etc.) that undergo random motion under certain intermolecular forces. These forces vary with distances and thus influence the microscopic behavior of the gases. However, the thermodynamic aspect mainly deals with the global nature of gases. Over wide ranges of pressures and temperatures in compressible flow fields, it is seen that the average distance between the molecules is more than the molecular diameters (about 10 times). So, all the flow properties may be regarded as macroscopic in nature. A perfect gas follows the relation of pressure, density, and temperature in the form of the fundamental equation [1].

The simple definition of compressible flow is variable-density flows. In general, the density of gases can vary either by changes in pressure or temperature. In fact, all high-speed flows are associated with significant pressure changes [1].

In order to be more precise, the compression process for a gas involves an increase in temperature depending on the amount of heat added or removed from the gas. If the temperature of the gas remains constant, the definition is refined as isothermal compressibility [1].

According to the above topic regarding the thermodynamic analysis of fluids, extensive research has been done by various researchers. Some of this research has been presented in references 2 to 29 for civil and mining projects, such as water supply, gas supply, and mine ventilation. The oldest and most famous researchers in the field of civil and mining projects were Hardy Cross and Wong, respectively, and subsequent researchers improved and completed theories them. Several researchers in mining projects have stated methods for analyzing the incompressibility and compressibility of air-fluid for ventilation design.

A subsurface ventilation system is comprised of a closed cycle of thermodynamic processes. Baden Hinsley (1900-1988) realized that a mine ventilation system is a big heat engine. That engine pushes air

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inside the mine and the air becomes compressed and heated by gravitational energy. This action happens more in down casting shafts or slopes. Also, the strata, machines, and other sources cause increased air heat in a ventilation system. If air expands during its ascent through upcast shafts or slopes, work is done. A section of the produced heat is converted temporarily into mechanical energy and it helps to increase airflow. When the exhaust air re-enters the pressure sink of the surface atmosphere, it cools the original entry conditions, closing the cycle. Figure 1 illustrates the descending flow through downcast shafts 1 and 2, level working 2 to 3, and returning to the surface through upcast shafts 3 to 4 [25].

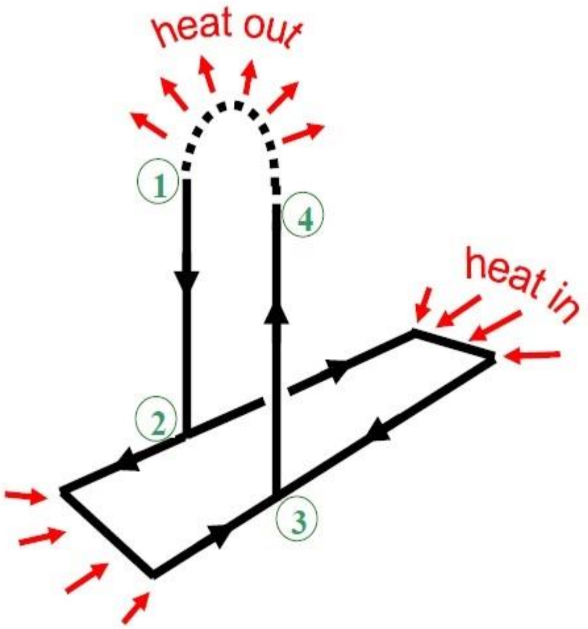

Figure 1. Elements of ventilation system [25].

To better realize the thermodynamic analysis of Figure 1, it is necessary to go through five stages. These stages are as follows:

a. Downcast shaft

The main equation for thermodynamic analysis in this stage between stations 1 to 2 is according to equation 1 [2].

$$\frac{V_1^2-V_2^2}{2}+(Z_1-Z_2)g=\int_1^2 VdP+F_{12}=h_{12}-q_{12} \tag{1}$$

b. Working level

The main equation for thermodynamic analysis in this stage between stations 2 to 3 is according to equation 2 [2].

$$\frac{V_2^2-V_3^2}{2}+(Z_2-Z_3)g=\int_2^3 VdP+F_{23}=h_{23}-q_{23} \tag{2}$$

c. Upcast Shaft

The main equation for thermodynamic analysis in this stage between stations 3 to 4 is according to equation 3 [2].

$$\frac{V_3^2-V_4^2}{2}+(Z_3-Z_4)g=\int_3^4 VdP+F_{34}=h_{34}-q_{34} \tag{3}$$

d. Naturally Ventilation Mine

The main equation for thermodynamic analysis in this stage between stations 4 to 1 is according to equation 4 [2].

$$\frac{V_4^2-V_1^2}{2}+(Z_4-Z_1)g=\int_4^1 VdP+F_{41}=h_{41}-q_{41} \tag{4}$$

e. Combined fan and natural ventilation

The main equation of for thermodynamic analysis in this stage between stations 4 to 5 and 5 to 1 are according to equations 5 and 6 [2].

$$\frac{V_4^2-V_5^2}{2}+(Z_4-Z_5)g+W_{45}=\int_4^5 VdP+F_{45}=h_{45}-q_{45} \tag{5}$$

$$\frac{V_5^2-V_1^2}{2}+(Z_5-Z_1)g=\int_5^1 VdP+F_{51}=h_{51}-q_{51} \tag{6}$$

In general, the air mixture is a combination of different gases and water vapor. The presence or lack or absence of water vapor in the air mixture causes or results in the creation of two combinations of dry air and humid air in nature [25-29]. The specifications of each of the two air combinations are as follows:

1.1. The properties of the dry air

The compositions and characteristics of dry air have been reported according to Table 1 by some researchers. According to Table 1, the molecular mass, gas constant, and volumetric mass are three important characteristics of dry air that are calculated as follows:

Table 1. The combination of the dry air properties [26].

gas	molecular mass	volumetric percentage
N_2	28.015	78.03
O_2	32	20.99
Co_2	44.003	0.03
H_2	2.016	0.01
Ar	39.943	0.94
Total	-	100

a. The molecular mass of the dry air

Using Table 1, the molecular mass of dry air (Ma) is calculated as follows:

$$M_a=M_d=\sum m_i p_i=28.015\times 0.7803+32\times 0.2099+$$
$$44.003\times 0.03+2.016\times 0.01+39.943\times 0.94$$
$$=28.966gr$$

b. The gas constant of the dry air

Using the global gas constant ( ) to calculate the gas constant of the dry air (Ra) is as follows:

$$R_a=R_d=\frac{\bar{R}}{M_a}=\frac{8.31436}{0.28966}=287.04\frac{J}{kg^oK}=29.27\frac{m}{oK}$$

c. The volumetric mass of the dry air

Under standard conditions, atmospheric pressure is assumed to be equal to 1 atm or 101325 Pa and its temperature 15oc. Therefore, the volumetric mass of the dry air (ρa) is calculated as follows:

$$\rho_a=\rho_d=\frac{P}{R_dT}=\frac{101325}{287.04\times(273.15+15)}=1.225055\frac{kg}{m^3}$$

1.2. The properties of the wet air

The number of parameters related to humid air is numerous. In this section, some important parameters have been reported in equations 7 to 18 [25-29].

$$P=P_d+P_v \tag{7}$$

$$P_d=\rho_d R_d T=287.04\ \rho_d T \tag{8}$$

$$P_v=\rho_v R_v T=w\rho_d R_v T=461.5w\rho_d T \tag{9}$$

$$\rho_d=\frac{P-P_v}{287.04T} \tag{10}$$

$$\rho_m=\frac{P-0.378P_v}{287.04T} \tag{11}$$

$$L = (2502.5 - 2.386t_w) \times 10^3 \tag{12}$$

$$P_{vs} = 610.6Exp(\frac{17.27t_w}{237.3 + t_w}) \tag{13}$$

$$w_s = 0.622\frac{P_{vs}}{P - P_{vs}} \tag{14}$$

$$w = \frac{Lw_s - c_p(t_d - t_{w)}}{L + c_v(t_d - t_{w)}} \tag{15}$$

$$P_v = \frac{Pw}{0.622 + w} \tag{16}$$

$$P_{vsd} = 610.6Exp(\frac{17.27t_d}{237.3 + t_d}) \tag{17}$$

$$R_h = \frac{P_v}{P_{vsd}} \tag{18}$$

P: Air pressure (Pa)
P_d: Dry air pressure (Pa)
P_v: Water vapor pressure (Pa)
ρ_d: Volumetric mass of dry air (kg/m^3)
ρ_m: Volumetric mass of moisture air (kg/m^3)
L: Latent heat of water evaporation (J/kg)
P_{vs}: Pressure of saturation water vapor (Pa)
w_s: Humidity of saturation air (kg/kg)
w: Humidity of air (kg/kg)
P_{vsd}: Water vapor pressure of dry air (Pa)
R_h: Relative Humidity (%)
c_p: Special heat capacity of air at constant pressure (1005 J/kg^{o}k)
c_v: Special heat capacity of air at constant volume (1884 J/kg^{o}k)

2. The common methods of the wet air analysis

The fluid of the mine ventilation is an air mixture. This fluid exists in nature in two models of dry or wet but any fluid can behave in two models of incompressible or compressible. If the amount of moisture in the air is not assumed to be zero, then this air is named wet. Researchers believe that if the average interval between molecules is more than 10 times the molecular diameter in any fluid, then this fluid is assumed a compressible model. Also, researchers in the ventilation of underground mining believe that if the mining depth is more than 500 meters, the air-fluid in mine ventilation design is assumed a compressible model. Therefore, the wet air analysis in mine ventilation is better done in the compressible model. Accordingly, there are different methods such as general, delta, and K methods [25-29]. The analysis for each of these theories is done according to its own rules, which are expressed as follows:

2.1. The general method of wet air analysis

In this method, wet air analysis is often done/conducted by the thermodynamic analysis method of the compressible model, and both heat and moisture issues are considered in mine networks. Thermodynamic equations for this theory have been presented in equations 19 to 30 [2-5]. In this method, the equations are divided into two groups. The first group calculates the dry temperature of the outlet air but this value is done based on equations 19 to 26. The second group calculates the dry volumetric mass of the outlet air but this value is done based on equations 27 to 30. The main conditions for solving equations in the two groups are to use approximate mathematical methods and a certain vaporization rate [25-29].

$$-\Delta^*_{ep} - \Delta^*_{ek} + W_{12} = h^*_{12} - q_{12} \tag{19}$$

$$\Delta^*_{ep} = g(Z_2 - Z_1)(1 + \frac{w_2 + w_1}{2}) \tag{20}$$

$$\Delta^*_{ek} = \frac{1}{2}(V_2^2 - V_1^2)(1 + \frac{w_2 + w_1}{2}) \tag{21}$$

$$w_2 = w_1 + \Delta w \tag{22}$$

$$h^*_{12} = c_p(t_{d2} - t_{d1}) + w_2h_{v2} - w_1h_{v1} - (w_2 - w_1)\overline{h_w} \tag{23}$$

$$h_{V1} = (1.81t_{d1} + 2501) \times 1000 \tag{24}$$

$$h_{V2} = (1.81t_{d2} + 2501) \times 1000 \tag{25}$$

$$\overline{h_w} = 2100 \times (t_{d2} + t_{d1}) \tag{26}$$

$$-\Delta^*_{ep} - \Delta^*_{ek} + W_{12} = \int VdP + F_{12} \tag{27}$$

$$\int VdP = (\frac{P_2}{\rho_{d2}} - \frac{P_1}{\rho_{d1}})\frac{k}{k-1} \tag{28}$$

$$k = \frac{Ln({}^{P_2}\!/\!{}_{P_1})}{Ln({}^{\rho_{d2}}\!/\!{}_{\rho_{d1}})} \tag{29}$$

$$F_{12} = \frac{f}{2}\frac{LP}{S}V^2 \tag{30}$$

Z_1: Height of station 1(m)
Z_2: Height of station 2 (m)
V_1: Fluid velocity at station 1 (m/s)
V_2: Fluid velocity at station 2 (m/s)
Δ_{ep}: Difference of potential energy (J/kg)
Δ_{ek}: Difference of kinetic energy (J/kg)
w_1: Humidity at station 1 (kg/kg)
Δw: Evaporation rate between two stations (kg/kg)
w_2: Humidity at station 2 (kg/kg)
W_{12}: Fan power (J/kg)
F_{12}: Frictional resistance of mine work (J/kg)
q_{12}: The amount of heat between two stations (J/kg)
R_d: Constant coefficient of dry air (J/kg^{o}k)
t_{d1}: The temperature of dry air at station 1 (^{oc})
t_{d2}: The temperature of dry air at station 2 (^{oc})
ρ_{d1}: Volumetric mass of fluid at station 1 (kg/m^3)
ρ_{d2}: Volumetric mass of fluid at station 2 (kg/m^3)
P_1: Total pressure at station 1 (Pa)
P_2: Total pressure at the station 2 (Pa)
f: Friction coefficient without dimension
L: Excavation length (m)
P: Excavation perimeter (m)
S: Excavation area (cross sectional) (m^2)
V: Average velocity of air between two stations (m/s)

2.2. The delta method of wet air analysis

This method was presented by Elahi Zeyni et al. in 2022 [29]. Also, this method is similar to the general method for wet air analysis. However, the difference is that instead of using two groups of equations and performing approximate mathematical methods twice, the final answer can be calculated with one group of equations and performing approximate mathematical methods once. The thermodynamic equations for this theory are presented inequations [2-5]. In this method, the equations are only one group. The main condition for solving these equations is a certain vaporization rate.

$$V_2 = 1.1V_1 \tag{31}$$

$$t_{d2} = 1.1t_{d1} \tag{32}$$

$$\rho_{d2} = 1.1\rho_{d1} \tag{33}$$

$$\Delta^*_{ep} = g(Z_2 - Z_1)(1 + \frac{w_2 + w_1}{2})$$

$$\Delta^*_{ek} = \frac{1}{2}(V_2^2 - V_1^2)(1 + \frac{w_2+w_1}{2})$$

$$w_2 = w_1 + \Delta w$$

$$c = w_2h_{v2} - w_1h_{v1} - (w_2 - w_1)\overline{h_w} \tag{34}$$

$$t_{d2} = t_{d1} + \frac{q_{12} - c - \Delta^*_{ep} - \Delta^*_{ek} + W_{12}}{c_p} \qquad (35)$$

$$F_{12} = \frac{f}{8}\frac{LP}{S}(V_2 + V_1)^2 \qquad (36)$$

$$P_2 = (R_d + R_v w_2)T_{d2}\rho_{d2} \qquad (37)$$

$$k = \frac{Ln(^{P_2}/_{P_1})}{Ln(^{\rho_{d2}}/_{\rho_{d1}})}$$

$$\Delta = (\frac{P_2}{\rho_{d2}} - \frac{P_1}{\rho_{d1}})\frac{k}{k-1} + F_{12} + \Delta^*_{ep} + \Delta^*_{ek} - W_{12} \qquad (38)$$

$$\rho^{'}_{d2} = \rho_{d2} - \Delta \times 10^{-5} \qquad (39)$$

$$V^{'}_2 = \frac{V_1 S_1 \rho_{d1}}{S_2 \rho^{'}_{d2}} \qquad (40)$$

P_1: Total pressure at station 1 (Pa)
P_2: Total pressure at the station 2 (Pa)
f: Friction coefficient without dimension
L: Excavation length (m)
P: Excavation perimeter (m)
S: Excavation area (cross sectional) (m^2)
V: Average velocity of air between two stations (m/s)

2.3. The delta method of wet air analysis

This method was presented by Elahi Zeyni et al. in 2022 [29]. Also, this method is similar to the general method for wet air analysis. However, the difference is that instead of using two groups of equations and performing approximate mathematical methods twice, the final answer can be calculated with one group of equations and performing approximate mathematical methods once. The thermodynamic equations for this theory are presented inequations [2-5]. In this method, the equations are only one group. The main condition for solving these equations is a certain vaporization rate.

2.4. The K method of wet air analysis

This method was presented/introduced by Elahi Zeyni et al. in 2022 [29]. Also, this method is similar to the delta method for wet air analysis. However, the difference is that the speed of convergence in this method is higher than the delta method. Thermodynamic equations for this theory are presented in equations [2-5]. In this method, the equations are only one group. The main condition for solving these equations is a certain vaporization rate.

$$\begin{aligned}
&V_2 = 1.1V_1\\
&t_{d2} = 1.1t_{d1}\\
&\rho_{d2} = 1.1\rho_{d1}\\
&P_2 = 1.1P_1 \qquad (41)\\
&\Delta^*_{ep} = g(Z_2 - Z_1)(1 + \frac{w_2 + w_1}{2})\\
&\Delta^*_{ek} = \frac{1}{2}(V_2^2 - V_1^2)(1 + \frac{w_2 + w_1}{2})\\
&w_2 = w_1 + \Delta w
\end{aligned}$$

$$\begin{aligned}
&c = w_2 h_{v2} - w_1 h_{v1} - (w_2 - w_1)\overline{h_w}\\
&t_{d2} = t_{d1} + \frac{q_{12} - c - \Delta^*_{ep} - \Delta^*_{ek} + W_{12}}{c_p}\\
&V_2 = \frac{V_1 S_1 \rho_{d1}}{S_2 \rho_{d2}} \qquad (42)\\
&F_{12} = \frac{f}{8}\frac{LP}{S}(V_2 + V_1)^2
\end{aligned}$$

$$K = \frac{F_{12} + \Delta^*_{ep} + \Delta^*_{ek} - W_{12}}{F_{12} + \Delta^*_{ep} + \Delta^*_{ek} - W_{12} + R_d(t_{d2} - t_{d1}) + R_v(w_2 T_{d2} - w_1 T_{d1})} \qquad (43)$$

$$\rho'_{d2} = \rho_{d1}\left[\frac{(R_d + R_v\, w_2)T_{d2}}{(R_d + R_v\, w_1)T_{d1}}\right]^{\frac{1}{K-1}} \qquad (44)$$

$$P_2{}' = P_1(\frac{\rho'_{d2}}{\rho_{d1}})^K \qquad (45)$$

$$V'_2 = \frac{V_1 S_1 \rho_{d1}}{S_2 \rho'_{d2}}$$

$$\Delta = \left|P_2{}' - P_2\right| \qquad (46)$$

3. The complete model of wet air analysis

The general, delta, and K methods for wet air analysis depend on a certain vaporization rate. Evaporation rate measurement requires a special instrument. If this special instrument is not available in mine, it is not possible to solve the equations by the above three methods. Therefore, to solve this problem, it is necessary to present a new method. This new method should not depend on the value of the evaporation rate. Accordingly, this paper presents the Complete Model of Wet Air Analysis (CMWAA method). Before presenting this new method, it is necessary to review some parameters. A number of parameters, such as the Earth's gravitational acceleration (g), the specific heat capacity of air at constant pressure (cp), and the constant volume (cv) have been considered to have constant values. However, these constant values are related to specific environmental conditions. Therefore, the true values of the above parameters are related to environmental conditions. Accordingly, in this paper, the true values of the above parameters are improved and their effects on thermodynamic equations are checked.

3.1. Correction of thermodynamic equations

3.1.1. The acceleration of the Earth's true gravity

The acceleration of the Earth's actual gravity is a function of the height difference of the sea level. Therefore, according to Equation 47, the acceleration of the Earth's true gravity has been corrected by researchers [20].

$$g_o = 9.80665\frac{m}{s^2} \rightarrow g = g_o(\frac{6357000}{6357000+H})^2 \qquad (47)$$

g: The acceleration of the Earth's actual gravity (m/s^2)
g_0: The acceleration of the Earth's constant gravity (m/s^2)
h: The height difference of the sea level (m)

3.1.2. Correction of potential energy difference

The acceleration of the Earth's gravity is varied. Also, potential energy is a function of it. Therefore, it is necessary to correct potential energy. These corrections have been done or made in equations 48 and 49.

$$\Delta_{ep} = g_2 Z_2 - g_1 Z_1 \qquad (48)$$

$$\Delta^*_{ep} = (g_2 Z_2 - g_1 Z_1)(1 + \frac{w_2 + w_1}{2}) \qquad (49)$$

3.1.3. Correction of specific heat capacity

The specific heat capacity of air at constant pressure is a function of environmental temperature. So, this variable is not constant. The specific heat capacity of air at constant pressure at different temperatures is reported in Table 2. Accordingly, this variable was modified and its result is presented in Equation 50.

$$c_p = 1000 + 1.87 \times 10^{-7}T^3 \qquad (50)$$

Moreover, the specific heat capacity of air at constant volume is a function of environmental temperature. So, this variable is not constant. The specific heat capacity of air at constant volume at different temperatures is reported in Table 3. Accordingly, this variable was modified and its result is presented in Equation 51.

Table 2. The specific heat capacity of air at constant pressure and different temperatures [21]

Temperature	*oK*	250	300	350
C_p	$\frac{J}{kg^oK}$	1003	1005	1008

Table 3. The specific heat capacity of air at constant volume and different temperatures [21].

Temperature	*oK*	250	300	350
C_v	$\frac{J}{kg^oK}$	716	718	721

$$c_v = 713 + 1.87 \times 10^{-7}T^3 \tag{51}$$

3.1.4. Correction of enthalpy difference of air

Air enthalpy is a function of the specific heat capacity of air. Besides, the specific heat capacity of air is not constant. Accordingly, the enthalpy difference of air was modified and its result is presented in equations 52 and 53.

$$h_{12} = c_{p2}t_{d2} - c_{p1}t_{d1} \tag{52}$$

$$h^*{}_{12} = c_{p2}t_{d2} - c_{p1}t_{d1} + w_2h_{v2} - w_1h_{v1} - (w_2 - w_1)\overline{h_w} \tag{53}$$

3.1.5. Correction of average volumetric mass of dry air

In thermodynamics science, the flow intensity of air mass is always constant during excavation. Accordingly, the average volumetric mass of dry air is corrected according to equation 54.

$$M_1 = M_2 = M_3$$

$$\rho_{d1}V_1S_1 = \rho_{d2}V_2S_2 = \rho_dVS$$

$$V = 0.5 \times (V_2 + V_1)$$

$$\frac{M_3}{\rho_dS} = 0.5 \times \left(\frac{M_2}{\rho_{d2}S_2} + \frac{M_1}{\rho_{d1}S_1}\right)$$

$$\frac{1}{\rho_d} = 0.5 \times (\frac{1}{\rho_{d2}} + \frac{1}{\rho_{d1}})$$

$$\rho_d = \frac{2\rho_{d1}\rho_{d2}}{\rho_{d1}+\rho_{d2}} \tag{54}$$

3.2. The CMWAA Method

According to the upper corrected equations and evaporation rate measurement problems, it is necessary to present a new method under the title of the most Complete Model of Wet Air Analysis (CMWAA method) in this section. The steps for solving the CMWAA method are as follows:

a. primary assumptions

Solve the CMWAA method requires a number of hypotheses. These hypotheses have been presented in the equations below.

$$V_2 = 1.1V_1$$

$$t_{d2} = 1.1t_{d1}$$

$$c_{p2} = 1.1c_{p1} \tag{55}$$

$$w_2 = 1.1w_1 \tag{56}$$

b. main calculations

At this stage, 14 equations are used. These equations are as follows:

$$\Delta_{ep} = g_2Z_2 - g_1Z_1 \tag{57}$$

$$\Delta_{ek} = \frac{1}{2}(V_2^2 - V_1^2)$$

$$\Delta_{ep}^* = (g_2Z_2 - g_1Z_1)(1 + \frac{w_2 + w_1}{2}) \tag{58}$$

$$\Delta_{ek}^* = \frac{1}{2}(V_2^2 - V_1^2)(1 + \frac{w_2 + w_1}{2})$$

$$c = w_2h_{v2} - w_1h_{v1} - (w_2 - w_1)\overline{h_w}$$

$$t_{d2} = \frac{q_{12} - c + c_{p1}t_{d1} - \Delta_{ep}^* - \Delta_{ek}^* + W_{12}}{c_{p2}} \tag{59}$$

$$F_{12} = \frac{f}{2}\frac{LP}{S}(\frac{V_2 + V_1}{2})^2 \tag{60}$$

$$K = \frac{F_{12} + \Delta_{ep}^* + \Delta_{ek}^* - W_{12}}{F_{12} + \Delta_{ep}^* + \Delta_{ek}^* - W_{12} + R_d(t_{d2} - t_{d1}) + R_v(w_2T_{d2} - w_1T_{d1})}$$

$$\rho_{d2} = \rho_{d1}\ \left[\frac{(R_d + R_vw_2)T_{d2}}{(R_d + R_vw_1)T_{d1}}\right]^{\frac{1}{K-1}} \tag{61}$$

$$P_2 = P_1(\frac{\rho_{d2}}{\rho_{d1}})^K \tag{62}$$

$$c_{p2} = 1000 + 1.87 \times 10^{-7}{T_{d2}}^3 \tag{63}$$

$$w_2 = \frac{c_{p2}t_{d2} - c_{p1}t_{d1} - w_1h_{v1} + w_1\overline{h_w} - q_{12} + (\Delta_{ep} + \Delta_{ek})(1 + \frac{w_1}{2}) - W_{12}}{-h_{v2} + \bar{h}_w - 0.5(\Delta_{ep} + \Delta_{ek})} \tag{64}$$

$$V_2 = \frac{V_1S_1\rho_{d1}}{S_2\rho_{d2}}$$

$$\Delta = \left|{P_2}' - P_2\right|$$

c- The number of calculations iteration

If the amount of delta in the final equation becomes equal to or smaller than the accuracy of calculations, solving the above 14 equations is stopped.

4. The validation of the CMWAA method

Two hypothetical examples were used to validate this new method. these examples are as follows:

4.1. The first hypothetical example

In this model, a hypothetical vertical shaft has been dug in a square type with the dimension of 3 meters, the height of the portal shaft is 5000 meters above sea level, and the shaft length is 4500 meters. Therefore, the height of the shaft end is 500 meters above sea level. The inlet air properties of the shaft portal are assumed with a velocity of 5 (m/s), a temperature of 10 (oc), relative humidity of %15, friction factor of 0.025, barometric pressure of 50 KPa, and generated heat during the shaft is assumed to be 50 KW. Accordingly, or Consequently, the specifications of the outlet air at the end of the shaft are calculated as follows:

4.1.1. The calculations of the first model

a. primary calculations

$$V_2 = 1.1V_1 = 1.1 \times 5 = 5.5\ {}^{m}/_{s}$$

$$t_{d2} = 1.1t_{d1} = 1.1 \times 10 = 11^{oc}$$

$$T_1 = 273.15 + t_{d1} = 283.15^{oK}$$

$$c_p = 1000 + 1.87 \times 10^{-7}T^3 \rightarrow c_{p1} = 1004.245133\ \frac{J}{kg^oK}$$

$$c_{p2} = 1.1c_{p1} = 1104.669646\ \frac{J}{kg^oK}$$

$$t_{d1}=10^{oc}\rightarrow P_{vsd1}=610.6Exp(\frac{17.27t_{d1}}{237.3+t_{d1}})=1227.560536\ Pa$$

$$R_{h1}=\%15\rightarrow P_{v1}=R_{h1}\times P_{vsd1}=184.134080\ Pa$$

$$P_1=50000\ Pa\rightarrow w_1=\frac{0.622P_{v1}}{P_1-P_{v1}}=0.002299\ {}^{\text{kg}}\!/_{\text{kg}}$$

$$w_2=1.1w_1=0.002529\ {}^{\text{kg}}\!/_{\text{kg}}$$

$$\rho_{d1}=\frac{P_1-P_{v1}}{287.04T_{d1}}=0.612927\frac{kg}{m^3}$$

$$g=g_o(\frac{6357000}{6357000+H})^2\rightarrow\begin{cases}H_1=5000\ m\rightarrow g_1=9.791242\ \frac{m}{s^2}\\ H_2=500\ m\rightarrow g_2=9.805108\ \frac{m}{s^2}\end{cases}$$

$$h_{V1}=(1.81t_{d1}+2501)\times 1000=2519100\ {}^{J}\!/_{\text{kg}}$$

$$h_{V2}=(1.81t_{d2}+2501)\times 1000=2520910\ {}^{J}\!/_{kg}$$

$$\overline{h_w}=2100(t_{d1}+t_{d2})=44100\ {}^{J}\!/_{kg}$$

$$R_d=287.04\ {}^{J}\!/_{\text{kg}\ {}^oK}$$

$$R_v=461.5\ {}^{J}\!/_{\text{kg}\ {}^oK}$$

$$W_{12}=0$$

b. the first iteration of calculations

$$\Delta_{ek}=\frac{1}{2}\times(5.5^2-5^2)=2.625\ {}^{J}\!/_{kg}$$

$$\Delta^*_{ek}=\frac{1}{2}(V_2^2-V_1^2)(1+\frac{w_2+w_1}{2})=2.631337{}^{J}\!/_{\text{kg}}$$

$$\Delta_{ep}=g_2Z_2-g_1Z_1=\text{-}44053.656\ {}^{J}\!/_{kg}$$

$$\Delta^*{}_{ep}=(g_2Z_2-g_1Z_1)(1+\frac{w_2+w_1}{2})=\text{-}44160.002157\ {}^{J}\!/_{kg}$$

$$q_{12}=\frac{N}{\rho_{d1}V_1S_1}=\frac{50\times 1000}{0.612927\times 5\times 9}=1812.795415\ {}^{J}\!/_{kg}$$

$$c=w_2h_{v2}-w_1h_{v1}-(w_2-w_1)\overline{h_w}=573.603459\ {}^{J}\!/_{kg}$$

$$t_{d2}=\frac{q_{12}-c+c_{p1}t_{d1}-\Delta^*_{ep}-\Delta^*_{ek}+W_{12}}{c_{p2}}$$

$$t_{d2}=50.186057^{oc}$$

$$F_{12}=\frac{f}{2}\frac{LP}{S}(\frac{V_2+V_1}{2})^2=2067.1875\ {}^{J}\!/_{kg}$$

$$K=\frac{F_{12}+\Delta^*_{ep}+\Delta^*_{ek}-W_{12}}{F_{12}+\Delta^*_{ep}+\Delta^*_{ek}-W_{12}+R_d(t_{d2}-t_{d1})+R_v(w_2T_{d2}-w_1T_{d1})}$$

$$K=1.380992$$

$$\rho_{d2}=\rho_{d1}\ \left[\frac{(R_d+R_vw_2)T_{d2}}{(R_d+R_vw_1)T_{d1}}\right]^{\frac{1}{K-1}}=0.869183\ \frac{kg}{m^3}$$

$$P_2=P_1(\frac{\rho_{d2}}{\rho_{d1}})^K=80997.247263\text{Pa}$$

$$c_{p2}=1000+1.87\times 10^{-7}{T_{d2}}^3=1006.321265\frac{J}{kg^oK}$$

$$w_2=\frac{c_{p2}t_{d2}-c_{p1}t_{d1}-w_1h_{v1}+w_1\overline{h_w}-q_{12}+(\Delta_{ep}+\Delta_{ek})\left(1+\frac{w_1}{2}\right)-W_{12}}{-h_{v2}+\bar{h}_w-0.5(\Delta_{ep}+\Delta_{ek})}$$

$$w_2=0.004483\frac{\text{kg}}{\text{kg}}$$

$$V_2=\frac{V_1S_1\rho_{d1}}{S_2\rho_{d2}}=3.525878\frac{m}{s}$$

c. the other iterations of calculations

The other iterations of the CMWAA method have been presented or provided in Table 4. If the accuracy of the calculations is assumed to be 0.001, according to the information in Table 4, the repeat number of calculations for the output air pressure becomes equal to 4.

Table 4. The results of the CMWAA method for example 1.

Description	Iteration 1	Iteration 2	Iteration 3	Iteration 4	Iteration 5
$\Delta^*{}_{ek}$	2.631337	-6.305404	-6.364058	-6.364814	-6.364824
$\Delta^*{}_{ep}$	-44160.002	-44203.050	-44203.050	-44203.050	-44203.050
t_{d2}	50.186057	50.194940	50.194999	50.194999	50.194999
F_{12}	2067.1875	1362.948669	1357.640414	1357.572128	1357.57117
k	1.380992	1.384811	1.384745	1.384744	1.384744
ρ_{d2}	0.869183	0.873300	0.873353	0.873354	0.873354
P_2	80997.247	81637.731	81642.741	81642.806	81642.806
c_{p2}	1006.321	1006.322	1006.322	1006.322	1006.322
w_2	0.004483	0.004483	0.004483	0.004483	0.004483
V_2	3.525878	3.509259	3.509045	3.509042	3.509042

4.1.2. The first validation of the CMWAA method

To validate this new method, the general, delta, and K methods can be used. Basically, the general method is based on two equations and two unknowns, and the steps of their solution are as follows:

$$\begin{cases}-\Delta^*_{ep}-\Delta^*_{ek}+W_{12}=h^*_{12}-q_{12}\\ -\Delta^*_{ep}-\Delta^*_{ek}+W_{12}=\int VdP+F_{12}\end{cases}\rightarrow$$

$$w_1=0.002299\ {}^{\text{kg}}\!/_{\text{kg}}$$

$$w_2=0.004483\ {}^{kg}\!/_{kg}\rightarrow\Delta w=0.002184\ {}^{kg}\!/_{kg}$$

$$\Delta^*_{ep}=g(Z_2-Z_1)\left(1+\frac{w_2+w_1}{2}\right)\rightarrow$$

$$\Delta^*_{ep}=9.80665\times(500-5000)\times(1+\frac{0.004483+0.002299}{2})=\text{-}44279.570\ \frac{J}{\text{kg}}$$

$$V_2=\frac{\rho_{d1}V_1}{\rho_{d2}}=\frac{0.612927\times 5}{\rho_{d2}}\frac{m}{s}$$

$$\Delta^*_{ek}=\frac{1}{2}(V_2^2-V_1^2)\left(1+\frac{w_2+w_1}{2}\right)\rightarrow$$

$$\Delta^*_{ek}=\frac{1}{2}\times((\frac{0.612927\times 5}{\rho_{d2}})^2-5^2)\times(1+\frac{0.004483+0.002299}{2})$$

$$h_{V1}=(1.81t_{d1}+2501)\times 1000=2519100\ {}^{kg}\!/_{kg}$$

$$h_{V2}=(1.81t_{d2}+2501)\times 1000$$

$$\overline{h_w}=2100(t_{d1}+t_{d2})=2100(10+t_{d2})$$

$$h^*_{12}=c_p(t_{d2}-t_{d1})+w_2h_{v2}-w_1h_{v1}-(w_2-w_1)\overline{h_w}\rightarrow$$

$$h^*_{12}=1005\times(t_{d2}-10)+4.483\times(1.81t_{d2}+2501)-5791.411-10.143$$

$$q_{12}=\frac{N}{\rho_{d1}V_1S_1}=\frac{50000}{0.612927\times 5\times 9}=1812.795415{}^{J}\!/_{kg}$$

$$V=0.5\times(V_2+V_1)=\frac{1.532318}{\rho_{d2}}+2.5$$

$$F_{12}=\frac{f}{2}\frac{LP}{S}V^2=75\times(\frac{1.532318}{\rho_{d2}}+2.5)^2$$

$$k=\frac{Ln({}^{P_2}\!/_{P_1})}{Ln({}^{\rho_{d2}}\!/_{\rho_{d1}})}=\frac{Ln({}^{P_2}\!/_{50000})}{Ln({}^{\rho_{d2}}\!/_{0.612927})}$$

$$P_2=(287.04+461.5\ w_2)\rho_{d2}\ T_{d2}$$

$$\int VdP=\left(\frac{P_2}{\rho_{d2}}-\frac{P_1}{\rho_{d1}}\right)\frac{k}{k-1}=$$

$$(\frac{P_2}{\rho_{d2}}-\frac{50000}{0.612927})\times\frac{Ln({}^{P_2}\!/_{50000})}{Ln({}^{P_2}\!/_{50000})-Ln({}^{\rho_{d2}}\!/_{0.612927})}$$

$$\rightarrow \begin{cases} t_{d2} = 50.344691\ ^{\text{oC}} \\ \rho_{d2} = 0.873617\ \dfrac{\text{kg}}{m^3} \\ P_2 = 81705.153\ \text{Pa} \end{cases} \rightarrow \begin{cases} |\Delta t_d| = 0.15\ ^{\text{oC}} \\ |\Delta \rho_d| = 0.00\ \dfrac{\text{kg}}{m^3} \\ |\Delta P_2| = 62.35\ \text{Pa} \end{cases}$$

The performed validation shows that the number of calculation errors is insignificant. Because, in the general method, the value of Earth's gravitational acceleration is assumed to be equal to 9.80665 m/s2, and the specific heat capacity of air at constant pressure equals 1005 J/kg°k. But, in the CMWAA method, these values are not constant.

4.2. The second hypothetical example

In this model, a hypothetical horizontal tunnel has been dug or excavated in a square shape with the dimension of 3 meters. The height of the tunnel portal is zero meters above sea level and the length of the tunnel is 3000 meters. The inlet air properties at the tunnel portal are assumed to be as follows: velocity 7 (m/s), temperature 15 (oc), relative humidity %60, friction factor 0.03, barometric pressure 101.325 KPa, and heat generated during the tunnel is assumed to be 25 KW. Accordingly, the specifications of the outlet air at the end of the tunnel are calculated as follows:

4.2.1. The calculations of the second model

a. Primary Calculations

$$V_2 = 1.1V_1 = 1.1 \times 7 = 7.7\ {}^{m}\!/_{s}$$

$$t_{d2} = 1.1t_{d1} = 1.1 \times 15 = 16.5^{oc}$$

$$T_1 = 273.15 + t_{d1} = 289.65^{oK}$$

$$c_p = 1000 + 1.87 \times 10^{-7}T^3 \rightarrow c_{p1} = 1004.474015\ \frac{J}{kg^oK}$$

$$c_{p2} = 1.1c_{p1} = 1104.921417\ \frac{J}{kg^oK}$$

$$t_{d1} = 15^{oc} \rightarrow P_{vsd1} = 610.6Exp(\frac{17.27t_{d1}}{237.3 + t_{d1}}) = 1704.787835\ Pa$$

$$R_{h1} = \%60 \rightarrow P_{v1} = R_{h1} \times P_{vsd1} = 1022.872701\ Pa$$

$$P_1 = 101325\ Pa \rightarrow w_1 = \frac{0.622P_{v1}}{P_1 - P_{v1}} = 0.006343\ {}^{\text{kg}}\!/_{\text{kg}}$$

$$w_2 = 1.1w_1 = 0.006977\ {}^{\text{kg}}\!/_{\text{kg}}$$

$$\rho_{d1} = \frac{P_1 - P_{v1}}{287.04T_{d1}} = 0.612927\frac{kg}{m^3}$$

$$g = 9.80665\ \frac{m}{S^2}$$

$$h_{V1} = (1.81t_{d1} + 2501) \times 1000 = 2528150\ {}^{J}\!/_{\text{kg}}$$

$$h_{V2} = (1.81t_{d2} + 2501) \times 1000 = 2530865\ {}^{J}\!/_{kg}$$

$$\overline{h_w} = 2100(t_{d1} + t_{d2}) = 66150\ {}^{J}\!/_{kg}$$

$$R_d = 287.04\ {}^{J}\!/_{\text{kg}\ ^oK}$$

$$R_v = 461.5\ {}^{J}\!/_{\text{kg}\ ^oK}$$

$$W_{12} = 0$$

b. The First Iteration of Calculations

$$\Delta_{ek} = \frac{1}{2} \times (7.7^2 - 7^2) = 5.145\ {}^{J}\!/_{kg}$$

$$\Delta_{ek}^* = \frac{1}{2}(V_2^2 - V_1^2)(1 + \frac{w_2 + w_1}{2}) = 5.179267{}^{J}\!/_{\text{kg}}$$

$$\Delta_{ep} = g_2Z_2 - g_1Z_1 = 0.00\ {}^{J}\!/_{kg}$$

$$\Delta^*{}_{ep} = (g_2Z_2 - g_1Z_1)(1 + \frac{w_2 + w_1}{2}) = 0.00\ {}^{J}\!/_{kg}$$

$$q_{12} = \frac{N}{\rho_{d1}V_1S_1} = \frac{25 \times 1000}{1.212688 \times 7 \times 9} = 327.227927\ {}^{J}\!/_{kg}$$

$$c = w_2h_{v2} - w_1h_{v1} - (w_2 - w_1)\overline{h_w} = 1580.615872\ {}^{J}\!/_{kg}$$

$$t_{d2} = \frac{q_{12} - c + c_{p1}t_{d1} - \Delta_{ep}^* - \Delta_{ek}^* + W_{12}}{c_{p2}}$$

$$\rightarrow t_{d2} = 12.497308^{oc}$$

$$F_{12} = \frac{f}{2}\frac{LP}{S}(\frac{V_2 + V_1}{2})^2 = 4051.6875\ {}^{J}\!/_{kg}$$

$$K = \frac{F_{12} + \Delta_{ep}^* + \Delta_{ek}^* - W_{12}}{F_{12} + \Delta_{ep}^* + \Delta_{ek}^* - W_{12} + R_d(t_{d2} - t_{d1}) + R_v(w_2T_{d2} - w_1T_{d1})}$$

$$\rightarrow K = 1.246532$$

$$\rho_{d2} = \rho_{d1}\ \left[\frac{(R_d + R_vw_2)T_{d2}}{(R_d + R_vw_1)T_{d1}}\right]^{\frac{1}{K-1}} = 1.175329\ \frac{kg}{m^3}$$

$$P_2 = P_1(\frac{\rho_{d2}}{\rho_{d1}})^K = 97448.856298\text{Pa}$$

$$c_{p2} = 1000 + 1.87 \times 10^{-7}{T_{d2}}^3 = 1004.358449\frac{J}{kg^oK}$$

$$w_2 =$$

$$\frac{c_{p2}t_{d2} - c_{p1}t_{d1} - w_1h_{v1} + w_1\overline{h_w} - q_{12} + \left(\Delta_{ep} + \Delta_{ek}\right)\left(1 + \frac{w_1}{2}\right) - W_{12}}{-h_{v2} + \bar{h}_w - 0.5\left(\Delta_{ep} + \Delta_{ek}\right)}$$

$$\rightarrow w_2 = 0.007505\ \frac{\text{kg}}{\text{kg}}$$

$$V_2 = \frac{V_1S_1\rho_{d1}}{S_2\rho_{d2}} = 7.222501\frac{m}{s}$$

c. The Other Iterations of Calculations

The other iterations of the CMWAA method have been presented in Table 5. If the accuracy of the calculations is assumed to be 0.001, according to the information in Table 5, the repeat number of calculations for the output air pressure becomes equal to 6.

4.2.2. The second validation of the CMWAA method

To validate this new method, different approaches can be used , such as the K method. The K method has been presented in section 2.3 and the results of this method have been shown in Table 6.

The performed validation shows that the number of calculation errors is very insignificant. These results are as follows:

$$|\Delta t_d| = 0.0006\ ^{\text{oC}}$$

$$|\Delta \rho_d| = 0.00\ \frac{\text{kg}}{m^3}$$

$$|\Delta P_2| = 0.004\ \text{Pa}$$

5. Conclusions

The design of mine ventilation relies on the principles of fluid mechanics. Air plays a crucial role in ventilation science. As such, it is examined using two models: compressible and incompressible. The compressibility model examines the air-fluid in two different analyses, one for dry air and the other for wet air. The analysis of the moist air provides an accurate description of the ventilation conditions within the mine. So far, various methods have been presented for the analysis of wet air in mine ventilation. These methods include the general, delta, and K methods. The general method has been described in section 2.1, but its purpose is to analyze wet air using two equations and two unknowns. The equations for the general method have been presented in 19 and 27 equations. Also, the delta method is presented in section 2.2. But its purpose is to simplify the analysis of wet air compared to the general method. The delta method has been designed based on 14 equations. Furthermore, the K method is presented in section 2.3, but its purpose is to increase the speed of analysis of wet air compared to the general method.

Table 5. The results of the CMWAA method for example 2.

Description	Iteration 1	Iteration 2	Iteration 3	Iteration 4	Iteration 5	Iteration 6
Δ^*_{ek}	5.179267	1.593219	1.504295	1.500973	1.500849	1.500844
Δ^*_{ep}	0	0	0	0	0	0
t_{d2}	12.497308	12.500880	12.500968	12.500971	12.500972	12.500972
F_{12}	3241.3499	3034.1931	3028.9738	3028.7787	3028.7714	3028.7711
k	1.231882	1.232371	1.232389	1.232390	1.232390	1.232390
ρ_{d2}	1.175329	1.177324	1.177399	1.177402	1.177402	1.177402
P_2	97448.856	97697.422	97703.651	97703.884	97703.892	97703.893
c_{p2}	1004.358	1004.359	1004.359	1004.359	1004.359	1004.359
w_2	0.007505	0.007505	0.007505	0.007505	0.007505	0.007505
V_2	7.222501	7.210264	7.209806	7.209789	7.209788	7.209788

Table 6. The results of the K method for example 2.

Description	Iteration 1	Iteration 2	Iteration 3	Iteration 4	Iteration 5	Iteration 6
Δ^*_{ek}	5.180625	1.628548	1.505790	1.501109	1.500935	1.500929
Δ^*_{ep}	0	0	0	0	0	0
t_{d2}	12.453559	12.501963	12.501548	2.501558	12.501558	12.501558
F_{12}	3241.3499	3036.266	3029.062	3028.787	3028.776	3028.776
k	1.219902	1.231547	1.232297	1.232323	1.232323	1.232324
ρ_{d2}	1.174539	1.177290	1.177396	1.177400	1.177400	1.177400
P_2	97450.155	97694.955	97703.549	97703.877	97703.889	97703.889
V_2	7.227358	7.210469	7.209825	7.209801	7.209800	7.209800

The K method has been designed based on 16 equations. The main condition of the above three methods is the measurement of the evaporation rate. Therefore, general, delta, and K methods all depend on a certain or specific vaporization rate.

Measuring the evaporation rate in a mine requires a special instrument. If this special instrument is not available in the mine, it is not possible to solve the equations by the general, delta, and K methods. Therefore, to solve this problem, it is necessary to present a new method. This new method should not depend on the value of the evaporation rate. Accordingly, this paper presents the Complete Model of Wet Air Analysis (CMWAA method). Also, before presenting this new method, it was checked some issues, such as the acceleration of Earth's true gravity, potential energy difference, the specific heat capacity of air at constant pressure and volume, enthalpy difference, the average volumetric mass of dry air, and the amount of output humidity. Subsequently, these issues were corrected. Moreover, the validation of the CMWAA method with other methods shows or indicates that the number of calculation errors is insignificant. This is because in the general, delta, and K methods, the value of Earth's gravitational acceleration is assumed to be equal to 9.80665 m/s2, and the specific heat capacity of air at constant pressure is 1005 J/kgok. By contrast, the above values in the CMWAA method are not constant and they depend on environmental conditions. Therefore, it can be claimed that the most comprehensive model for wet air analysis in mine ventilation is related to the CMWAA method.

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