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Incorporating grade uncertainty into open-pit long-term production planning using loss and profit functions

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ABSTRACT

Long-term production planning for open-pit mines is recognised as one of the vital decision-making issues in open-pit mining operations. In addition, the ore grade is one of the most significant sources of uncertainty in a mining operation, as it classified run-of-mine material into ore and waste. In the classical approach, the destination of mining blocks is determined by comparing the estimated grade with a pre-determined cut-off grade. However, the uncertainty of material grade dramatically affects production planning. In this paper, a novel model was developed based on the idea of simulating the grade to incorporate the risk of grade uncertainty. In the proposed model, the economic consequences of the assigned destination are calculated using the profit and loss functions and they are integrated with the production scheduling. The proposed production planning was implemented in an iron ore mine, and the results were discussed for classical, loss, and profit models. Results show that the net present value increases by 3.64% by implementing the profit function. In contrast, the loss function method reduces the net present value by 2.23% compared to the classic model. This happens because the amount of ore class is increased by 7.46% using the profit function method and decreased by 2.49% using the loss function method. Additionally, the coefficient of variation, as an index of uncertainty, was investigated. The results show that the loss function approach attempts to extract more reliable blocks in the early years and postpone the high-uncertain blocks to the later years of the production.

Keywords: *Mine planning, Profit and loss functions, Grade uncertainty, Open-pit mines.*

1. Introduction

The declining trend in mineral grades, increasing mining costs, and environmental considerations confirm that the economic life of today's mines depends on careful planning and management. Open-pit mining planning is a decision-making issue related to determining which blocks, enclosed within the ultimate pit, should be extracted each year and sent to the process, waste dump, or ore dump. The problem of open-pit mine production planning is usually solved to maximize the project's net present value under constraints, such as mining and processing capacities and the precedence of block extraction [1].

Operational research techniques have been widely applied to solve the problem of long-term production planning since the 1960s; however, the solution obtained from mathematical techniques may not be practically feasible and for the larger instances of the problem, finding a solution is computationally expensive. In contrast, heuristic methods can provide an acceptable, but not necessarily optimal solution. But, the ease of use and flexibility in considering operational constraints make heuristic methods widely used. Depending on the nature of the input parameters, the proposed methods for planning open-pit mining are divided into deterministic and uncertainty-based solution approaches. In the deterministic approach, all input data is assumed to be definite. In the uncertainty-based methods, instead of an absolute value, the distribution of a value is used depending on the source of uncertainty. Ore grade is the most crucial source of uncertainty in mining operations. Kriging is the primary geostatistical method used to estimate block attributes; however, to include estimation uncertainty, the geostatistical simulations, such as sequential Gaussian simulations are applied. The simulation generates realizations of the block model and investigates the uncertainty in the estimation [2-5].

To solve the deterministic classic long-term production planning problem, three types of solution methods including exact, heuristic, and meta-heuristic approaches have been developed. Johnson [6] proposed the first known mathematical model, which sets the variables to decide the sequence of block extraction in each period and determines their destination. Constraints on the capacity of resources required for extraction and processing are also considered. Tolwinski [7] developed a dynamic programming routine using the depth-first search method. However, despite the possibility of applying large block models, the highest NPV was still infeasible. Sattarvand [8] applied Ant Colony Optimization (ACO) to solve open-pit production planning for a two-dimensional hypothetical block model. Shishvan and Sattarvand [9] used the Ant Colony Optimization (ACO) algorithm to solve a real case study with 350,000 blocks under mining and processing capacity constraints.

Moreno et al. [10] examined various linear and nonlinear integer models for production planning in open-pit mines and developed a nonlinear model for production planning considering stockpiles.

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Rezakhah [11] developed the Moreno model by considering blending and concluded that the mixing plan increased the NPV by about 19%. Lotfian et al. [12] proposed a new method based on the block clustering to reduce model size. In the first step, the blocks are grouped into "mother clusters," using the concept of clustering as a mathematical programming problem and solved using a genetic algorithm (GA). In the second stage, the mother clusters are transformed into mineral clusters for practical reasons. Fathollahzadeh et al. [13] have presented a mathematical model based on mixed integer programming with the grade engineering framework for mining operations. Mariz and Soofastaei [14] focused on improving surface mine planning using advanced analytical approaches, such as machine learning and artificial intelligence. Flores-Fonseca et al. [15] presented models based on mixed integer programming considering the sequencing of power shovels and storage. This model has been developed to maximize the work efficiency of shovels and NPV. The results show an increase in operational NPV by including storage in the model.

Despite the valuable research in the field of deterministic open-pit planning, uncertainties associated with the input parameters can lead to increased differences between the realized and calculated NPV. Therefore, the direct involvement of uncertainties, especially grade and price uncertainties, in the production planning process can lead to planning that maximizes the NPV of the project with a high degree of reliability [16]. Albach [17] first investigated the issue of grade changes in reserves and their impact on mining production planning. The author used the linear programming technique with random constraints to plan a lignite mine where the amount of resources is uncertain. Darwen [18] examined a genetic algorithm's capabilities to plan mines' production in the face of mineral price and grade uncertainty. Godoy and Dimitrakopoulos [19] developed a model that considers the grade of uncertainty by modifying the Tan and Ramani [20] model and combining it with the simulated annealing method. The results show that the use of optimal mining rates and grade uncertainty leads to plans that meet the objectives through significant improvements in the NPV of the project. Ramazan and Dimitrakopoulos [21] used the stochastic integer programming method to perform production planning in a gold mine in Australia and utilized multiple conditional simulations to investigate supply uncertainty. The objective function in this model will eventually lead to the maximization of the NPV. Morales et al [22] combine geo-metallurgical models using co-probability scenarios that evaluate spatial variability and mineral deposit uncertainty. Additionally, using a stochastic integer programming model and considering simultaneous methods for direct block scheduling will determine long-term open-pit mining planning. Rimélé et al. [23] proposed a dynamic stochastic programming approach with geological and commodity price uncertainty for open-pit mining planning. The proposed method first uses a two-stage model to manage geological uncertainty. Then, the stochastic dynamic programming algorithm is developed and applied to determine the appropriate strategy for metal production based on the price evolution of the commodity. Gilani et al. [24] have developed a stochastic integer planning (SIP) model to integrate geological uncertainty and use a PSO-based algorithm to solve the SIP model. Four strategies were proposed according to the population topology and the risk block model. Implementing the proposed approach in a large-scale mine demonstrates its performance in creating a unique application considering geological uncertainties with a maximum NPV and a minimum risk of not achieving production goals. Tolouei et al. [25] have proposed a new approach to finding the optimal solution for long-term mining planning by considering grade uncertainty. The augmented Lagrangian relaxation (ALR) method speeds up the optimisation convergence in large samples. The grey wolf optimiser (GWO) algorithm is then used to update the programmed Lagrangian multipliers. Birch [26] presented an approach to optimize the cut-off grade by considering estimation uncertainty to improve the materials' classification in open-pit mines.

The most influential effect of grade uncertainty occurs around the cut-off grade. In the classical open-pit production planning model, the cut-off grade is used to classify mining blocks. If the estimated grade is higher than the cut-off, this ore block is classified; otherwise, it is considered waste. Therefore, any mistake in the estimated grade around the cut-off grade can change the destination of the material parcel. Thus, an alternative approach is to simultaneously consider the distribution of the grade and the economic consequences of sending the block using simulation-based methods, such as the loss and profit functions. Dimitrakopoulos [27] proposed the idea of classifying ore/waste based on estimation and conditional simulation. The results indicate that conditional simulation is generally better than the fixed estimation of material grade in material classification. This is especially true when the profit value of a block is a nonlinear function of the block grade. Mousavi et al. [28] have used the profit and loss functions to classify ore/waste material. Studies show that this method performs better than conventional methods and is more adjustable to actual data. Vasylchuk and Deutsch [29] have presented a new approach to enhancing grade control using the loss function. The research aims to show how to improve grade control, i.e., sending less ore to the waste dump or sending tailings to the plant through numerical studies. Verly and Parker [30] have provided a practical review of conditional simulation to improve mineral resource estimation in classification, mining selectivity or dilution, and grade uncertainty.

In this paper, we integrated the idea of loss and profit functions in classifying material and in the long-term production planning of open-pit mines. Thus, the cut-off grade is not fixed and changes based on the probability that the block is ore or waste, and the economic consequences of materials classification. The proposed model considers monetary loss and misclassification and applies actual loss or profit figures in the mine planning by considering the discounting rate and linking them to the mining sequence. Finally, the proposed model was applied to a real case study, and the results were discussed for the classic, loss, and profit function approaches.

2. The Profit and Loss Functions

One of the essential steps in production planning is to use a method to determine the class of mining blocks. In other words, the destination of the extraction blocks (plant, waste dump, and stockpile) must be specified. One of the methods for classifying materials and determining their destination is using the profit and loss functions.

2.1. The Loss Function

The loss function is defined as the amount of money lost due to incorrect classification. Losses associated with the wrong classification can be expressed as a function of actual but unknown grade losses. In other words, the amount of loss for a given block is defined as the potential value of the block minus the recovered value [31].

The article introduces p, r, and Cp as the price, recovery, and processing cost. Cm and z denote mining cost and the grade of block, respectively. In the case of a real ore block initially categorized as waste:

Potential value $prz - C_m - C_p$
Recovered value $-C_m$
In the case of a real waste block initially categorized as ore:
Potential value $-C_m$
Recovered value $prz - C_m - C_p$

Then, the loss function as a result of the incorrect classification of the waste block as ore (Lo) and also the loss function for the ore block that is incorrectly classified as waste (Lw) are given in Equations (1) and (2), respectively [28].

$$L_o = P_0 \times (-(prz^- - C_p)) \tag{1}$$

$$L_w = (1 - P_0) \times (prz^+ - C_p) \tag{2}$$

Where, P0 is the probability of having a grade higher than the cut-off grade, and z+ and z- are the average values of the simulated grade greater than and less than the cut-off grade, respectively. If Lo is less than Lw for a specific block, it is categorized as ore; otherwise, it falls into the waste category. Within a given block and category (ore or waste), the estimated grade (z) represents the mean value of simulated outcomes.

2.2. Profit Function

The profit function specifies the expected profit associated with each classification scenario [31]. The profit function approach calculates the expected profit for each class, and the classification that gives the highest expected profit will be selected [28]. The profit function for the blocks estimated as ore (Pr_o) and also the profit for the blocks classified as waste (Pr_w), respectively, in equations (3) and (4) are given [28].

$$Pr_o = P_0 \times (prz^+ - C_p) + (1 - P_0) \times (prz^- - C_p) \quad (3)$$

$$Pr_w = -(P_0) \times (prz^+ - C_p) \quad (4)$$

If Pr_o exceeds Pr_w, the block is categorized as ore; otherwise, it is designated as waste. For instance, when the average simulated value for a specific block surpasses the cut-off grade, the profit function categorizes it as ore, given that $-(P_0) \times (prz^+ - C_p) < 0$. Likewise, when the average simulated value for a particular block falls below the cut-off grade, the block is directed to a waste dump, provided that $P_0 \times (prz^+ - C_p) + (1 - P_0) \times (prz^- - C_p) < 0$. In both the loss and profit approaches, the assumption is that every block needs to be mined, and the mining costs for ore and waste are considered equal. Consequently, mining cost are not factored into the equations.

The purpose of using the loss function is to minimize the loss from sending the block to the wrong destination. Also, the profit function is used to maximize the profit from sending the block to the correct destination. In other words, for each block in these two methods, four cases occur [19]:

- Acceptance: The actual grade of the block is higher than the cut-off grade, and it is sent to the processing plant.
- Correct rejection: The actual block grade is less than the cut-off grade sent to the waste dump.
- Wrong acceptance: The block grade is lower than the cut-off grade and is sent to the processing plant.
- Wrong rejection: The block grade is higher than the cut-off grade sent to the waste dump

In this context, the loss function aims to minimize errors in acceptance and rejection, while the profit function strives to increase acceptance rates and rectify rejections. Simulating the probability of profit for a block is feasible by estimating the likelihood of a block having a grade higher than the cut-off grade. For instance, if the estimated block grade surpasses the cut-off grade, factoring in the potential error in this estimation helps gauge the cost associated with sending it to the processing plant. This approach clarifies the determination of block profitability under various scenarios.

3. Open-pit Production Planning Problem

In classical long-term open-pit mining planning, mineral block destinations are determined by comparing each block's grade with the economic cut-off grade. However, grade uncertainty affects production planning results in practice. This uncertainty makes it impossible to achieve the production intended for the mine in some periods. For this purpose, this paper uses the profit and loss function to determine the destination and sequence of block extraction. In this regard, the classic production planning model is presented, and then revised models based on the loss and profit functions are developed.

3.1. The Classical Approach for Open-Pit Production Planning

In the classical production planning model, the objective function maximizes the NPV. Each block's economic value (BEV) is included in the formula, and only the cut-off grade is used to determine the destination of the blocks. First, the sets, parameters, and decision variables are introduced, and then the mathematical model of the objective function and constraints is expressed.

Sets:

T: The set of time period, t = (0, 1, ..., T)

I: The set of extraction blocks, i = (0,1,..., I)

P_i: The set of blocks that must be extracted before block i.

I_o: The set of blocks that are sent to the processing plant

D: The set of block's destination, d = (0, 1, ..., D), where d=0 and d=1 denote waste and ore, respectively.

P_i denotes the collection of blocks having a precedence connection originating from block i. The grouping of these precedence connections is established based on both slope angle and height, forming what is referred to as the "cone of precedence".

Parameters:

C_m:	The mining cost (dollars per tonne)
C_p:	The processing cost (dollars per tonne)
p:	The price of product (dollars per tonne of product).
r:	Total recovery (%)
Z_c:	Cut-off grade (%)
e:	Discount rate (%)
Bev_i:	The economic value of block i (dollars)
b_i:	The tonnage of block i (tonne)
M^t_{ml}:	The minimum mining capacity in period t (tonne)
M^t_{mu}:	The maximum mining capacity in period t (tonne)
M^t_{pl}:	The minimum capacity of processing plant in period t (tonne)
M^t_{pu}:	The maximum capacity of processing plant in period t (tonne)
Z_i:	Grade of block i (%)
Z_{max}:	Maximum acceptable grade in the processing plant (%)
Z_{min}:	Minimum acceptable grade in the processing plant (%)

Decision variable:

X^t_{id}: If block (i) is extracted in period (t) and sent to the destination (d) is equal to one; otherwise, it will be zero.

The objective function

The objective function of the long-term production planning problem is to maximize the net present value. The objective function, shown in equation (6), is obtained from the total discounted revenue of the blocks minus the total discounted cost of extracting and processing. The Block Economic Values (BEV) are calculated using equation (5).

$$BEV = \begin{cases} prz - C_p - C_m & z > Z_c \\ -C_m & z \le Z_c \end{cases} \quad (5)$$

$$Max \sum_{d=0}^{1} \sum_{i=0}^{I} \sum_{t=0}^{T} \frac{Bev_i X^t_{id}}{(1+\varepsilon)^t} \qquad X^t_{id} = \begin{cases} 1 \\ 0 \end{cases} \quad (6)$$

Constraints

Precedence constraint:

Equation (7) shows the precedence constraint of block extraction. The precedence constraint ensures that all immediate blocks, which have restricted the target block, are extracted before the extraction of the target block.

$$\sum_{d=0}^{1} \left(\sum_{r=1}^{t} X^r_{jd} - X^t_{id}\right) \ge 0 \qquad \forall\, i \in I;\ j \in P_i;\ t \in T;\ d \in D \quad (7)$$

Constraint of the mining capacity:

Equations (8) and (9) show the constraints related to the mining capacity, which are determined according to the amount of mineral reserve, technical and economic constraints, and the processing plant's capacity.

$$\sum_{d=0}^{1} \sum_{i=0}^{I} b_i\, X^t_{id} \ge M^t_{ml} \qquad \forall\, i \in I;\ t \in T;\ d \in D \quad (8)$$

$$\sum_{d=0}^{1} \sum_{i=0}^{I} b_i\, X^t_{id} \le M^t_{mu} \qquad \forall\, i \in I;\ t \in T;\ d \in D \quad (9)$$

Constraint of the processing plant capacity:

The processing capacity is determined according to mineral reserve, extraction capacity, and market conditions. As shown in Equations (10) and (11), the total tonnage of ore sent to the processing plant should not be less than the minimum feed required by the plant in period t; also, this tonnage should not exceed the capacity of the processing plant.

$$\sum_{d=1}\sum_{i=0}^{I_o} b_i\, X_{id}^t \geq M_{pl}^t \qquad t \in T \tag{10}$$

$$\sum_{d=1}\sum_{i=0}^{I_o} b_i\, X_{id}^t \leq M_{pu}^t \qquad t \in T \tag{11}$$

Constraint of grade limits:

The grade limits constraint given in Equations (12) and (13) controls the amount of metal content sent to the processing plant in period t. This constraint ensures that the plant feed is within a steady range.

$$\sum_{d=1}\sum_{i=0}^{I}(Z_i - Z_{max})\, X_{id}^t \leq 0 \qquad \forall\, i \in I;\ t \in T \tag{12}$$

$$\sum_{d=1}\sum_{i=0}^{I}(Z_i - Z_{min})\, X_{id}^t \geq 0 \qquad \forall\, i \in I;\ t \in T \tag{13}$$

Reserve constraint:

The constraint given in Equation (14) ensures that each block is extracted only once during the life of the mine.

$$\sum_{d=0}^{1}\sum_{t=0}^{T} X_{id}^t \leq 1 \qquad \forall\, i \in I \tag{14}$$

3.2. Profit and Loss Approaches for Open-Pit Production Planning

Open-pit mining planning can be formulated as an integer program with the objective function of NPV maximisation with a set of technical and practical constraints. This research has developed a mathematical model considering the grade uncertainty, which calculates the potential profit and loss from sending blocks to the processing plant or waste dump. Ultimately, it chooses the destination that maximises the potential profit of the block or minimises the potential loss. Then, the extraction sequence of the blocks is determined. The structure of the proposed model requires new parameters related to the concept of economic function, which will be described below. Additional detail includes sets, decision variables, and constraints based on the classic model.

The new parameters in the mathematical formulation of long-term planning using the profit and loss function are as follows:

Pr_{id}:	Potential profit of block i sent to destination d (dollars)
L_{id}:	Potential loss of block i sent to destination d (dollars)
P_0:	The probability that the block is an ore (%)
z^+:	The average values of the simulated grade are higher than the cut-off grade
z^-:	The average values of the simulated grade which are less than the cut-off grade

As mentioned earlier, the sequence of block extraction has been determined using the profit and loss functions. Objective functions are introduced below; however, the constraints are the same as those of the classical method.

The objective function in the profit function approach is to maximize potential profits from correctly classifying blocks; by observing physical and technical constraints, including precedence, reserve, mining capacity, processing capacity, and grade limit.

$$Max \sum_{d=0}^{1}\sum_{i=0}^{I}\sum_{t=0}^{T}\frac{Pr_{id}X_{id}^t}{(1+\varepsilon)^t} \qquad X_{id}^t = \begin{cases}1\\0\end{cases} \tag{15}$$

The Pr value for the blocks that are sent to the processing or dump will be calculated using Equations (16) and (17), respectively. According to this function, a block is sent to the processing plant, if it has a grade higher than the cut-off grade and its potential profit (Pr_o) is equal to zero (Equations (16)). Also, if a block has grades less than the cut-off grade, its potential profit will be calculated with the probability of having a grade higher than the cut-off grade. A block is sent to the dump (Pr_w), if it has a grade lower than the cut-off grade and the potential gain is zero. However, if the grade is higher than the cut-off grade, the potential profit is calculated according to the probability of the block having a grade higher than the cut-off grade [28].

$$Pr_o = \begin{cases}0 & z > Z_c\\ P_0 \times (prz^+ - C_p) + (1-P_0)\times(prz^- - C_p) & z \leq Z_c\end{cases} \tag{16}$$

$$Pr_w = \begin{cases}-(P_0)\times(prz^+ - C_p) & z > Z_c\\ 0 & z \leq Z_c\end{cases} \tag{17}$$

In this loss function approach, the objective function leads to the minimization of potential losses, resulting in incorrect classification of blocks; following physical and technical constraints, such as precedence, reserve, mining capacity, and grade limit.

$$min \sum_{d=0}^{1}\sum_{i=0}^{I}\sum_{t=0}^{T}\frac{L_{id}X_{id}^t}{(1+\varepsilon)^t} \qquad X_{id}^t = \begin{cases}1\\0\end{cases} \tag{18}$$

In this case, using the functions given in Equations (19) and (20), the amount of possible losses of a given block as a result of sending it to the processing plant (L_o) and waste dump (L_w) is calculated [28].

$$L_o = \begin{cases}0 & z > Z_c\\ P_0 \times \left(-(prz^- - C_p)\right) & z \leq Z_c\end{cases} \tag{19}$$

$$L_w = \begin{cases}(1-P_0)\times(prz^+ - C_p) & z > Z_c\\ 0 & z \leq Z_c\end{cases} \tag{20}$$

4. Case study

The Chah-Gaz iron ore mine was selected as a case study to evaluate and validate the mathematical models developed for long-term production planning of open-pit mines using the profit and loss functions. The Chah-Gaz mine, shown in Figure (1) is located at longitude 29 degrees and 55 minutes east and latitude 7 degrees and 32 minutes north. The average altitude of the deposit is about 1700 meters above sea level and it is considered a mountainous area. The first studies of the Chah-Gaz deposit were carried out in 1976, in which most of the profiles were perpendicular to the trend of the eastern and western massifs, of which Techno Export drilled 59 wells. The Chah-Gaz ore deposit is concentrated in two almost parallel masses, east and west, and most of the deposit is located at depth [32].

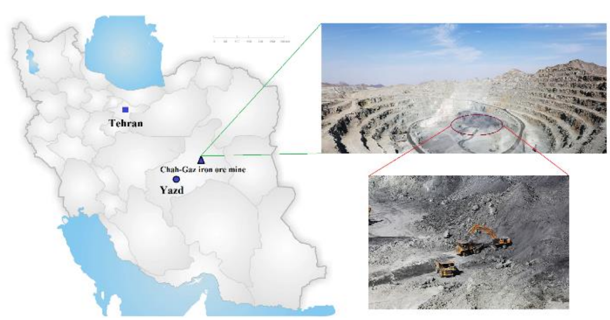

Figure 1. The geographical location of the Chah-Gaz mine. a) The position of the mine b) Chah-Gaz iron ore mine.

4.1. Geostatistical Studies

Variography output determines three critical parameters, namely: the sill, the nugget effect, and the range. Each reserve has a unique variogram model, the parameters of which are not precisely determinable but can be approximated to some extent. Anisotropy can be determined by drawing a variogram in different directions, and its parameters can be obtained. Variography in the study area was outlined in different steps using Datamine software to identify the spatial structure of the deposit. Table (1) shows the variogram parameters and maximum and minimum anisotropy variability. Figure (2) also shows the variogram of the maximum and minimum anisotropy variability, respectively.

Table 1. The variogram parameters along with the maximum and minimum anisotropy variability.

	Type	Extension	Slope	tolerance	Nugget effect	Range	Sill
Maximum	Spherical	150	55	10	0.2	128	0.89
Minimum	Spherical	70	80	10	0.2	47	0.87

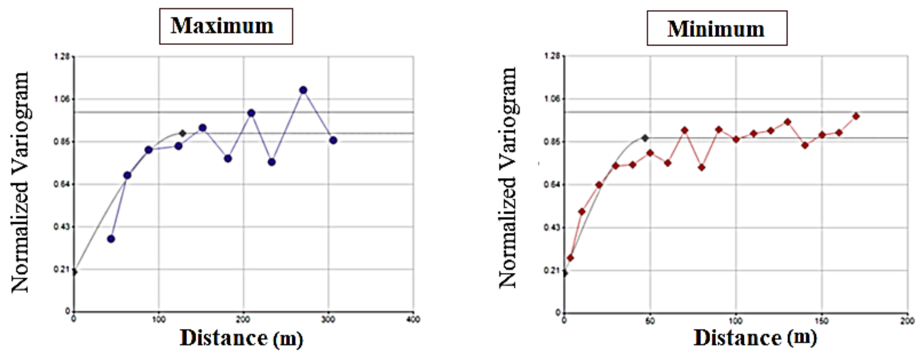

Figure 2. Variogram along with the maximum and minimum anisotropy variability.

4.2. Geostatistical Simulation

In this study, the sequential Gaussian simulation (SGS) method was applied for simulation. The SGS is an efficient and popular geostatistical simulation method for continuous variables [28]. The statistical and geostatistical parameters of each realization that the SGS generates are identical to those of the raw data, and the actual known data remain constant throughout all realizations. Following the generation of realizations, the material is divided into the ore and waste categories using the profit and loss functions as the two primary simulation-based methods.

Utilizing the available data, we performed simulations for 50 realizations of the block model. When selecting 50 simulations for the SGS in our study, several critical factors were considered. First and foremost, we aimed to strike a balance between achieving statistically significant results and managing computational resources effectively. Given the complexity of our geological model and the computational demands of the SGS, 50 simulations were deemed sufficient to provide reasonably stable estimates of the spatial distribution of the variable under investigation. This choice also aligned with established practices in our field, ensuring the comparability and credibility of our research within the geological community. However, we acknowledge that the choice of the number of simulations can influence precision and computational time.

The statistical parameters obtained from the results of five realisations of the simulation and the original data are given in Table (2). Figure (3) also depicts the non-directional variogram of five realisations (R) and the original data. The results shown in Figure (3) and the variograms of all 50 realisations are matched by the histograms and variograms of the original data and the simulations have high validity. Finally, Figure (4) shows the ore block of the case study along with the grade obtained from geostatistical studies.

5. Result and Discussion

Production planning at the Chah-Gaz iron ore mine based on the grade uncertainty was performed with three mentioned methods to evaluate the developed models, and each technique produced results according to its nature. All approaches were coded in Python, and CPLEX was used to solve the models and compare the profit and loss function results with the classical model. All models were conducted on a Windows 10 workstation equipped with an Intel Core i7-8557U CPU and 12GB of RAM. The computational time for the classical, profit, and loss approaches amounted to 692 seconds, 17,060 seconds, and 10,412 seconds, respectively.

5.1. Block Classification

The classification of blocks will have a significant impact on the long-term planning of production because if an ore block is sent to the waste dump, a significant amount of money will be wasted. Furthermore, if a waste block is processed in the plant, the recovery of the plant will be reduced. In particular, processing plants susceptible to incoming feed can experience severe problems due to wrongly sending blocks to destinations. Therefore, using the most appropriate method to determine destinations can lead to better planning and reduce the risk associated with mining operations.

This paper discusses ore/waste classification using economic classification functions, which are indicators of the economic impacts of different classification decisions. Along with the classification process, two approaches were presented. One involved minimizing loss and the other involved maximizing profit. Both incorporate conditional simulations of the ore body, as well as the calculation of economic classification functions for the ore body blocks. In addition, these functions allow for the inclusion of asymmetric relationships between monetary loss and misclassification.

Table 2. The statistical parameters of five realizations and raw data.

	Original data	R 5	R 15	R 25	R 35	R 45
Average	46.31	44.37	43.78	44.72	44.13	46.39
Standard deviation	21.67	22.85	23.04	22.48	23.07	22.17
Middle	57.22	56.46	55.76	56.46	56.20	57.50
Skewness	-0.92	-0.99	-1.06	-0.92	-1.03	-0.68

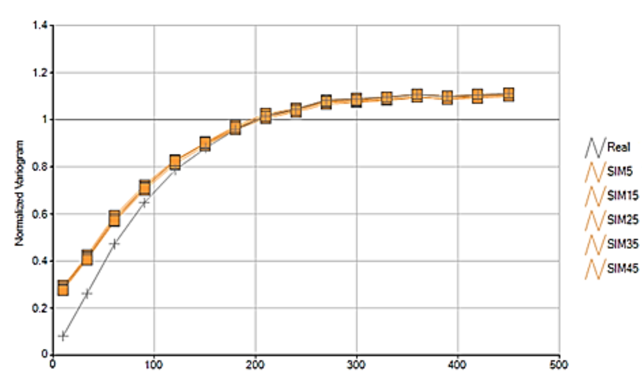

Figure 3. The non-directional variograms related to five realisations and the original data.

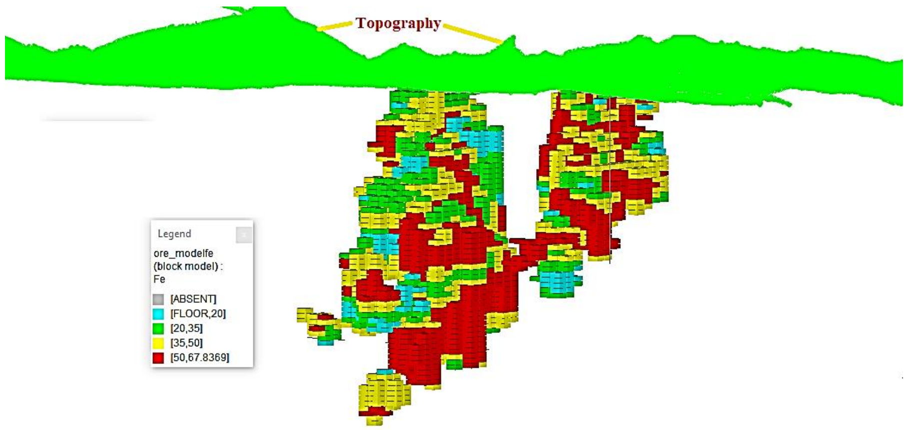

Figure 4. The ore block of the case study along with the grade obtained from geostatistical studies.

First, the possible profit and loss for each block were calculated to classify the blocks using the profit and loss functions. Then, blocks were classified using the traditional method and economic functions. Figure (5) illustrates the result of classifying blocks using three methods. After determining each function's parameters, the destination of each block was specified by implementing the production planning model. The extraction and processing tonnage diagrams of the plant used to analyse and compare the performance of each model are shown in Figure (6).

In all three methods, profit, loss, and classical, the extraction capacity is approximately the same. But, the results of ore production in Figures (5) and (6) differ for each approach over a period of five years. Some blocks with a lower grade than the cut-off grade (with a high probability) in the profit function are classified as ore and sent to the plant. Hence, the tonnage of plant feed in the profit model is greater than in the other two models. In the loss function model, not sending blocks with a grade higher than the cut-off grade to the processing plant, due to its harmfulness, has reduced the annual tonnage of the processing plant in this model.

The profit function generally directs several blocks with a grade lower than the cut-off grade to the plant (due to profitability). This classification change caused a 7.46% increase in the total tonnage sent to the plant. The loss function method is conservative, which is why blocks with a higher grade than the cut-off grade was not delivered to the plant due to losses, and blocks provided the minimum tonnage of the plant with lower losses in all years. As a result, this conservative choice reduced the total tonnage delivered to the plant by 2.49%.

5.2. Block Sequencing

Each of the developed models uses its theory to classify blocks. This change in the block classification approach will affect the mine production planning and block sequencing. As mentioned in the previous section, the first effect of applying economic functions is the change in the classification of blocks. This classification will change the annual extraction tonnage in planning. Table (3) and Figure (7) show the details of the annual production (Ore and Waste) for all three approaches.

Table (3) and Figure (7) show that the total extraction tonnage is approximately equal; however, the loss function has extracted less ore, proportionate to its economic nature (2.4% lower). Also, the concept of the loss function in classifying the blocks has caused it to extract 1.2% more waste than the other two methods. This classification contrasts with the profit function and has caused more tonnes of ore to be sent to the plant.

A change in the classification of blocks and the annual extraction amount will also affect the sequencing of blocks. Therefore, Figure (8) shows the block extraction sequencing in three developed models in the east-west view. Each color represents the planning period (year). According to the description and Figure (8), it is clear that the classification approach affects the sequencing of blocks. Also, the change in tonnage sent to the plant and block sequencing leads to a change in the economic parameters of mine planning.

According to Figure 8, the sequencing of blocks in two classical methods and the profit function have differences, but they use the same extraction pattern. However, this extraction pattern is different in the loss function method. The reason is the concept of this method, which tries to extract blocks that are more likely to be ore in the early planning periods. Giving priority to blocks with the highest probability of having ore causes a change in their sequencing. This concept can be better understood with the coefficient of variation.

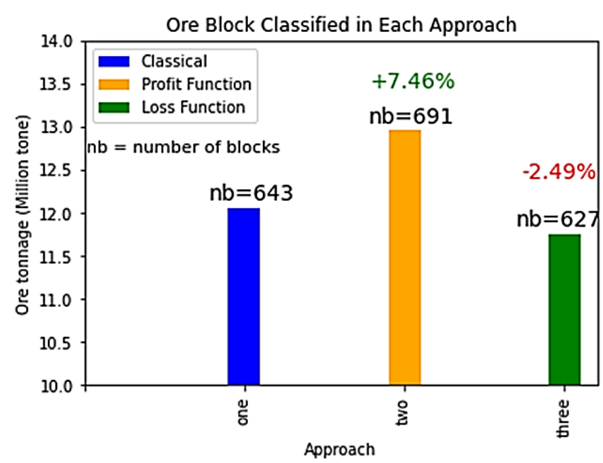

Figure 5. The ore blocks classified by classical, profit, and loss approaches.

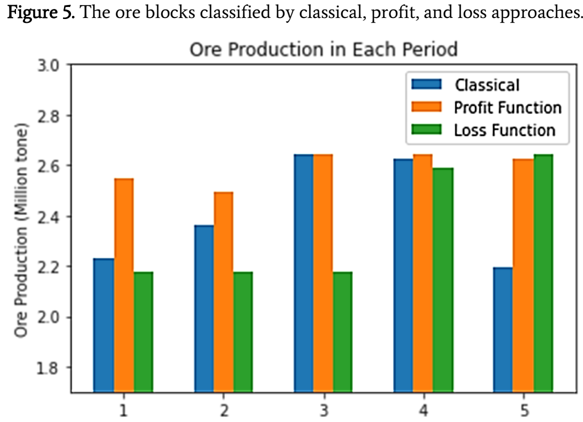

Figure 6. The annual ore production by classical, profit, and loss approaches

Table 3. The details of annual production (Ore and Waste).

Planning model	Classical (Mt)		Profit function (Mt)		Loss function (Mt)	
	Ore	Waste	Ore	Waste	Ore	Waste
Year one	2.231	6.094	2.55	5.775	2.175	6.113
Year two	2.363	5.963	2.494	5.831	2.175	6.019
Year three	2.644	4.969	2.644	4.931	2.175	5.531
Year four	2.625	4.2	2.644	4.181	2.587	4.313
Year five	2.194	4.631	2.624	4.238	2.644	4.18

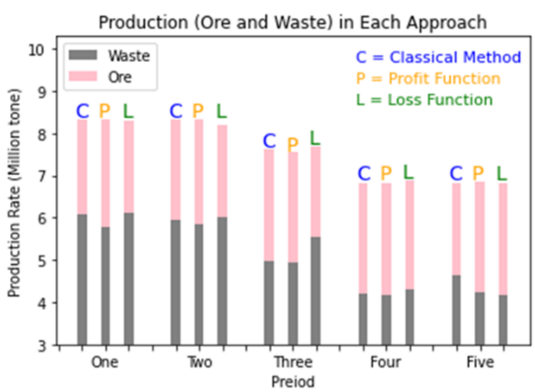

Figure 7. The detail of annual production (Ore and Waste) by classical, profit, and loss approaches.

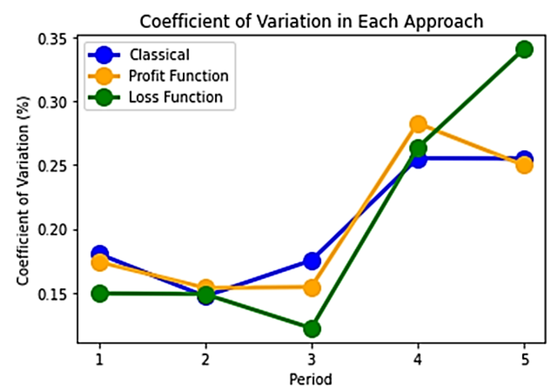

Figure 9. The coefficient of variation for blocks of ore in classical, profit, and loss approaches.

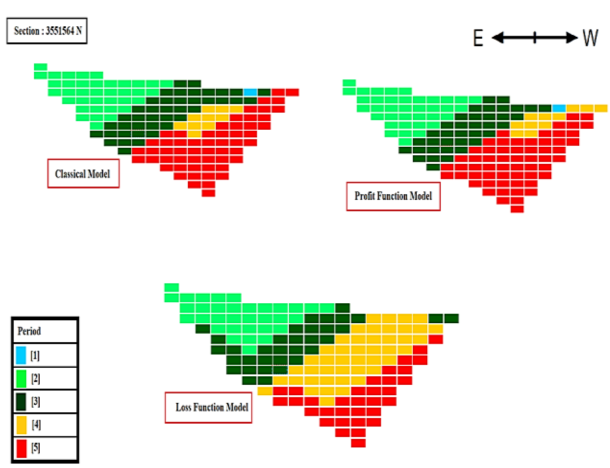

Figure 8. The block extraction sequencing by classical, profit, and loss approaches in the east-west view.

5.3. Grade Variability

As mentioned, the loss and profit functions can include grade uncertainty in production planning. To explore this ability, the coefficient of variation has been calculated and summarized for the yearly schedule. The coefficient of variation measures dispersion which is a measure of data variability used to gauge the extent of data variability. Hence, the coefficient of variation measures how far data are from the average or mean value [33]. Therefore, the coefficient of variation is a useful way to express the level of uncertainty, especially for grades. Points with higher coefficients of variation indicate lower accuracy and higher risk. Figure 9 shows the coefficient of variation for blocks of ore in all three methods.

As it is known, using the loss function causes the selection of blocks with a lower coefficient of variation than ore. The reason for this is that in the calculation of profit and loss functions, the role of the coefficient of variation is entered as a probability distribution function for each block.

In this study, the coefficient of variation was used as an index of uncertainty in mine planning. The results of the loss function approach show that blocks with a high probability of being ore are mined in the first years. In contrast, blocks with more uncertainty are mined in the last years when exploration is complete. In this regard, Figure 10 illustrates the annual probability of being ore in each approach. In Figure 10, the concept of extracting blocks with higher probability (more reliability) in the first years and extracting blocks with lower probability (more uncertainty) in the last years is displayed.

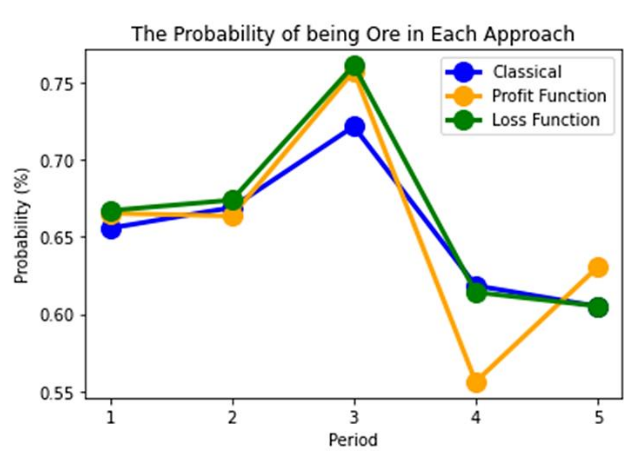

Figure 10. The annual probability of being ore in classical, profit, and loss approaches.

5.4. Grade limits

The purpose of this section is to compare classical grade limit practices in an iron mine with the mine's comparison and reconciliation of the economic classification functions (the loss and profit functions) using geostatistical simulations. The next step is to determine whether grade limits based on the economic classification functions and simulations will further improve performance.

Figure 11 illustrates the comparison between the mine's grade control ordinary (blue), the classification from the minimum loss approach (green), and the classification from the maximum profit approach (orange) to compare differences in the metal content between three approaches. As a result, the application of the minimum loss economic classification function and indicator sequential simulation shows improvement in the mine's grade control performance.

Also, because the profit function labeled a larger number of blocks as ore, it caused the average grade in periods two and four to be lower than the traditional method.

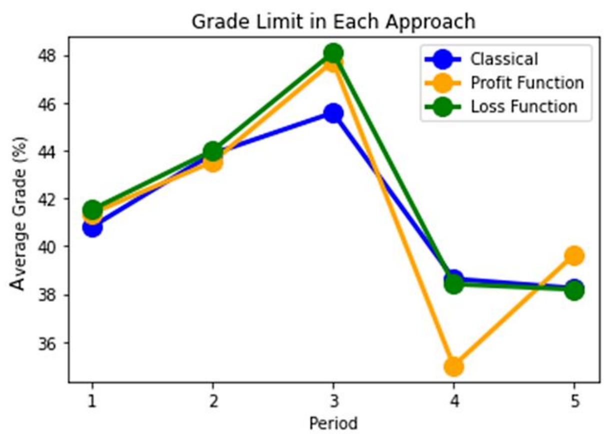

Figure 11. The average grade (%) of ore sent to the plant by classical, profit, and loss approaches.

Figure 12 illustrates the grade distribution of blocks in each period for the selected methods, providing valuable insights into the variability and characteristics of block grades over time. The box in a box plot represents the interquartile range (IQR), which is the range between the first quartile (Q1) and the third quartile (Q3). The distance between Q1 and Q3 (IQR) is a measure of the spread or variability of the data. Based on Figure 12, it is evident that the loss function exhibits a smaller Interquartile Range (IQR) in block grades for mining operations. This suggests a higher degree of consistency and predictability in the ore source, resulting in reduced operational risk.

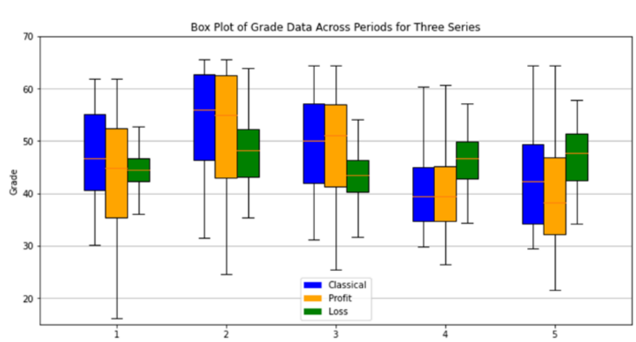

Figure 12. The annual grade distribution analysis by classical, profit, and loss approaches.

5.5. Economical Result

Prior to evaluating, investing, designing, and planning a mining project, it is essential to discriminate between ores and waste. Mining operations can face serious problems as a result of the misclassification of ore and waste. Applying the appropriate ore/waste discrimination technique can lead to better planning and reduce mining risk. In the first stage, the goal is to take into account the asymmetric relationship between monetary loss and misclassification. Then, include actual loss or profit figures in mine planning by considering the discounting rate and linking them to a mining sequence. In this regard, the cumulative cash flows achieved by each approach are shown in Figure (13). Also, Table (4) shows the net present value for all three models.

Table 4. The net present value by each approach

Planning model	Classical	Profit function	Loss function
Net present value (million dollars)	83.11	86.24	81.26
Difference ore classify (%)	-	+7.24	-2.49
Difference NPV (%)	-	+3.64%	-2.23%

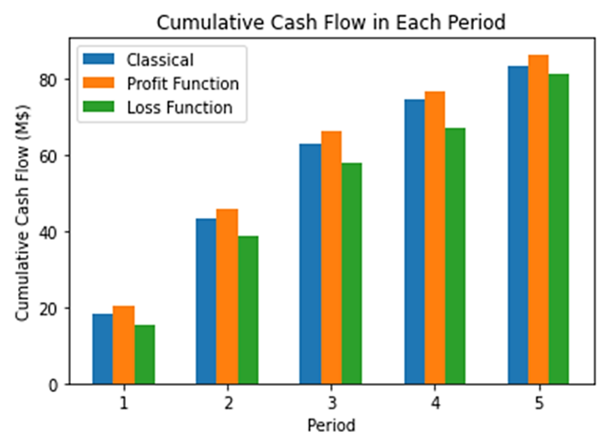

Figure 13. The cumulative cash flow by classical, profit, and loss approaches.

It is evident that, with the nature of the loss and profit functions, we do not simply allow the user to compare the scenarios with the base case. Instead, our approach involves employing these functions to determine block sequencing and subsequently calculating the NPV based on this sequencing. By integrating these two fundamental methods into our analysis, we gain the capability to conduct a thorough and insightful comparison with the base case.

As mentioned, the nature of the profit function causes more tonnage of materials to be sent to the processing plant. As a result, sending these materials will increase the NPV, while the nature of the loss function will reduce it. In the profit function model, an increase of 7.24% in processing plant tonnage caused a rise of 3.64% in NPV. In the loss function model, a decrease of 2.49% in plant tonnage led to a decline of 2.23% in the NPV.

6. Conclusions

The profit and loss functions are methods to classify ore and waste rocks by considering the economic consequences of assigning blocks to different destinations. In this paper, we incorporate the ideas of the profit and loss functions into mine planning models. The proposed model has several advantages, including taking into account the asymmetric relationship between monetary loss and misclassification, and and incorporating the actual loss or profit figures in mine planning by considering the discounting rate and linking them to a mining sequence. The comparative analysis was presented to elucidate the potential performance of the proposed models. Comparisons of the scenarios show that the profit classification function is the most effective approach in terms of returning the highest NPV. Based on a real case study, implementing the classic, loss, and profit functions resulted in an increase of 3.64% in the NPV using the profit function approach. In contrast, the loss function method reduces the NPV by 2.23% compared to the classic model. This happens because the amount of ore class is increased by 7.46% by the profit function method and decreased by 2.49% using the loss function method. Moreover, the quality of ore production can be further improved using the loss function approach, especially in the first periods, because the grade uncertainty is reduced by sending more reliable blocks.

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