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Heavy Tailed Distribution of Binary Classification Model

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Abstract

The proposed research incorporates the utilization of a heavy-tailed skewed distribution referred to as the inverse Weibull as a link function in the context of a binary classification model. This selection is motivated by the need to address the existence of rare or extreme events in random processes. The study introduces a model that relies on the Inverse Weibull (TYPE II) distribution, and the estimation of model parameters is accomplished through the application of maximum likelihood methods. When the outcomes are compared to those derived from other link functions such as TYPE I (Complementary log) and TYPE III (Weibull) based on extreme value distributions using standard classification data as well as real-life data, it becomes apparent that the Inverse Weibull (TYPE II) model exhibits exceptional performance. This assessment of performance takes into account several criteria, encompassing the Akaike information criterion, Bayesian information criterion, Area under the curve, and Brier scores. In conclusion, the study establishes that the proposed model demonstrates considerable robustness in its performance, rendering it a viable choice for the modeling of binary classification problems.

Keywords: Extreme value Distribution; Inverse-Weibull; Classification Model; Heavy-Tailed Distribution.

Introduction

Classification involves determining the category to which an observation belongs. The application of classification models extends across various aspects of life. In the realm of medical science, the significance of classification models cannot be overstated, given the field's structure and operations. A common scenario is assigning a diagnosis to a patient based on observable patient characteristics, including gender, blood pressure, and the presence or absence of specific symptoms. These individual observations are often transformed into quantifiable properties referred to as features or covariates. Classifiers function by comparing observations to previous ones using a similarity or distance measure.

Unique modeling approaches for classification problems include linear discriminant models, probit models, and logit models, which are commonly employed by frequentist statisticians. Bayesian statisticians, on the other hand, use methods like naïve Bayes and Bayesian networks. Additional methods include dynamic linear models (1), nonlinear models, hidden Markov models, and more (2).

In the general linear model, numerous link functions have been developed by researchers for modeling classification data, which include the inverse Gaussian, logistic distribution, and a class of two-parameter link

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functions that also generalize the logit model. Some studies have explored skewed distributions, such as the Weibull (3, 4). However, the Weibull distribution, being a member of the Extreme Value distribution family, exhibits a lighter tail compared to the Gumbel distribution. This characteristic may limit its capability to capture extreme or rare events resulting from random processes.

In this proposed study, we adopt a heavy-tailed skewed distribution known as the inverse Weibull as a link function within a binary classification model. This choice enables us to account for extreme or rare events that may occur in random processes.

Materials and Methods

1. Extreme Value Distribution

Extreme Value Theory (EVT) is a branch of statistics that deals with the stochastic behavior of extreme events found in the tails of probability distributions. A stochastic model represents a situation where uncertainty is present, essentially a model for a process that exhibits some degree of randomness. EVT's primary goal is to predict the probabilities of rare events that are greater (or smaller) than previously recorded events. An extreme value distribution serves as a limiting model for the maximums and minimums within a dataset (5). A limiting distribution simply models how large (or small) your data is likely to become. Let Y be a random variable, represented as Y = (y1, y2, y3, ...). Extreme value distributions are categorized into three groups: Type I, Type II, and Type III. These three types are defined as follows, with parameters μ, α, and β, corresponding to the location, scale, and shape parameters, respectively.

Type I

$$f_I(y)=\frac{1}{\beta}\exp\left\{-\frac{y-\mu}{\beta}-\exp\left(\frac{y-\mu}{\beta}\right)\right\}\ y\varepsilon\Re \qquad (1).$$

$$F_I(y)=\exp\left\{-\exp\left(\frac{y-\mu}{\beta}\right)\right\} \qquad (2)$$

Type II

$$f_{II}(y)=\left(\frac{\alpha}{\beta}\right)\left(\frac{y-\mu}{\beta}\right)^{-\alpha-1} e^{\left\{-\left(\frac{y-\mu}{\beta}\right)^{-\alpha}\right\}}$$

$$y\geq\beta,\alpha>0 \qquad (3).$$

$$F_{II}(y)=\exp\left\{-\left(\frac{y-\mu}{\beta}\right)^{-\alpha}\right\} \qquad (4).$$

Type III

$$f_{III}(y)=\left(\frac{\alpha}{\beta}\right)\left(\frac{\mu-y}{\beta}\right)^{\alpha-1} e^{\left\{-\left(\frac{\mu-y}{\beta}\right)^{\alpha}\right\}}$$

$$y<\mu,\alpha>0 \qquad (5).$$

$$F_{III}(y)=\exp\left\{-\left(\frac{\mu-y}{\beta}\right)^{\alpha}\right\}\ y<\mu \qquad (6).$$

The basic idea is that three types of extreme value distributions (EVD Types I, II, and II) can model the extremes from any set of data, as long as the distribution is "well-behaved" (5). Figure 1, depicts the tail of the three extreme value distributions the figure shows that TYPE II has a heavier tail than TYPE I and TYPE III.

2. Weibull Distribution and Inverse Weibull Distribution

The Extreme value distribution of type III was named after a Swedish engineer and scientist called Waloddi Weibull, well-known for his work on the

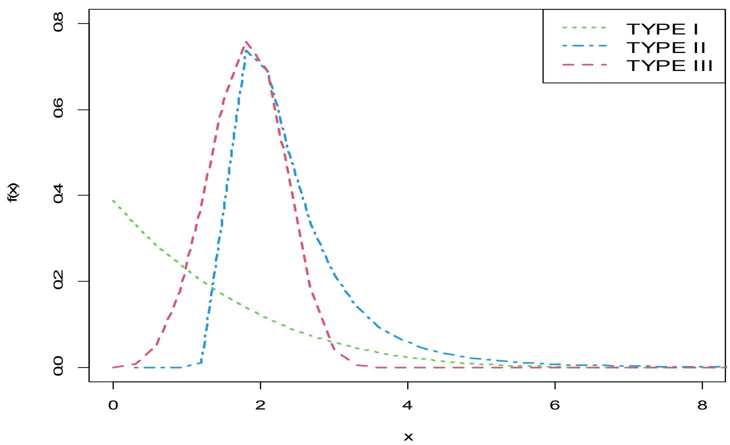

Figure 1. Plot of Extreme Value Distribution

strength of materials and fatigue analysis (6). The Weibull distribution was first created to analyze the distribution of material lifetimes or failure times in the realm of material science. It proved highly valuable in comprehending material behavior and predicting potential failures Let X represent a random variable that denotes the lifetime or time to occurrence of an event. The Weibull distribution is defined as

$$f(x) = \left(\frac{\alpha}{\beta}\right)\left(\frac{x-\mu}{\beta}\right)^{\alpha-1} e^{\left\{-\left(\frac{x-\mu}{\beta}\right)^{\alpha}\right\}}$$

$x \geq \mu, \beta \neq 0, \alpha \varepsilon \Re$ (7).

In equation 7 (above), if α = 1, the Weibull distribution function reduces to the Exponential model, whereas for α = 2, it mimics the Rayleigh distribution which is mainly used in the telecommunications field (6). Furthermore, it resembles the Normal distribution when α = 3.5.

Given that X follows the Weibull distribution of $X \sim w\,(\mu, \beta, \alpha)$, let $y = \frac{\beta^2}{X-\mu}$'s call it reciprocal transformation to a Weibull distribution X therefore the newly generated distribution is inverse Weibull and is stated as follows:

$$f(y) = \left(\frac{\alpha}{\beta}\right)\left(\frac{y-\mu}{\beta}\right)^{-\alpha-1} e^{\left\{-\left(\frac{y-\mu}{\beta}\right)^{-\alpha}\right\}}$$

$y \geq 0 \;\; \beta \neq 0, \alpha \varepsilon \Re$ (8).

The following represent essential statistical properties of the Inverse Weibull distribution.

$$E(y) = \int_0^{\infty} y\left(\frac{\alpha}{\beta}\right)\left(\frac{y-\mu}{\beta}\right)^{-\alpha-1} e\left\{-\left(\frac{y-\mu}{\beta}\right)^{-\alpha}\right\}$$

$E(y) = \mu + \beta\Gamma\left(1-\frac{1}{\alpha}\right)$ where Γ is the gamma function

$$0.5 = \int_0^{m} \left(\frac{\alpha}{\beta}\right)\left(\frac{y-\mu}{\beta}\right)^{-\alpha-1} e\left\{-\left(\frac{y-\mu}{\beta}\right)^{-\alpha}\right\}$$

Median= $\mu + \frac{\beta}{\alpha\sqrt{\log_e 2}}$

$$E(y-\mu)^2 = \int_0^{\infty} (y-\mu)^2\left(\frac{\alpha}{\beta}\right)\left(\frac{y-\mu}{\beta}\right)^{-\alpha-1} e\left\{-\left(\frac{y-\mu}{\beta}\right)^{-\alpha}\right\}$$

Variance= $\beta^2\left(\Gamma\left(1-\frac{2}{\alpha}\right)-\left(\Gamma\left(1-\frac{1}{\alpha}\right)^2\right)\right)$

Skewness=

$$\frac{\Gamma\left(1-\frac{3}{\alpha}\right)-3\Gamma\left(1-\frac{2}{\alpha}\right)\Gamma\left(1-\frac{1}{\alpha}\right)+2\Gamma^3\left(1-\frac{1}{\alpha}\right)}{\sqrt{\left(\Gamma\left(1-\frac{2}{\alpha}\right)-\Gamma^2\left(1-\frac{1}{\alpha}\right)\right)^3}}$$

Figure 2 depicts the relationship between extreme value distributions and the inverse Weibull distribution.

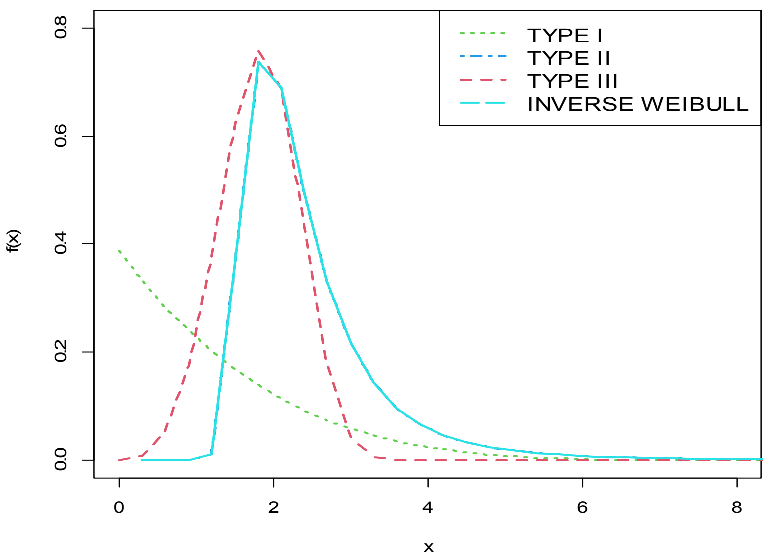

Figure 2. Plot of Extreme Value Distribution and Inverse Weibull distribution

The figure illustrates that the inverse Weibull and TYPE II extreme value distributions are equivalent and exhibit heavier tails compared to other distributions.

3. Inverse Weibull link Function

The study introduces a novel link function that relies on the highly asymmetric (inverse Weibull) distribution. This new link function is expressed as:

$$F(y) = e\left(-\left(\frac{y-\mu}{\beta}\right)^{-a}\right)$$

$$Y = \lambda_0 + \lambda X_i^{'}$$

$$\Pr\left(y_i = 1/w_i = \lambda^0{}_0 + \lambda^0 X_i\right) = F_y\left(\lambda^0{}_0 + \lambda^0 X_i^{'}\right) =$$

$$F\,F_y\left(\lambda^0{}_0 + \lambda^0 X_i^{'}\right)$$

$$e\left\{-\left(\frac{\lambda^0{}_0 + \lambda^0 X_i^{'} - \mu}{\beta}\right)^{-a}\right\}$$

$$= e\left\{-\left(\frac{\lambda_0^0 + \lambda_i^0 X_{ii} + \cdots + \lambda_k^0 X_{ik} - \mu}{\beta}\right)^{-a}\right\}$$

Pr

$$\left(y_i = 1/X_i = x_i\right) = e\left\{-\left(\frac{\lambda_0^0 - \mu}{\beta} + \frac{\lambda_i^0 X_{ik}}{\beta} + \cdots + \frac{\lambda_k^0 X_{ik}}{\beta}\right)^{-a}\right\}$$

let $\lambda_0 = \frac{\lambda_0^0 - \mu}{\beta}$ and $\lambda = \frac{\lambda_i^0}{\beta} \cdots \frac{\lambda_k^0}{\beta}$

While

$$\lambda_0 + \lambda x_i^{'} > 0, \alpha > 0, \lambda_0 = \frac{\lambda^0{}_0 - \mu}{\beta}\ and\ \lambda = \lambda^0$$

4. Estimation Method

Let Y be a random variable from the Bernoulli distribution then, its Probability Mass function (PMF) is defined as

$$\pi^{y_i}\left(1-\pi\right)^{1-y_i} \qquad y=0,1$$

The likelihood function of Bernoulli distribution and Inverse Weibull link is given by

$$L(\pi) = \prod_{i=1}^{n} \pi_i^{\,y_i}\left(1-\pi_i\right)^{1-y_i}$$

Where $\pi_i = e\left\{-\left(\psi\right)^{-a}\right\}$

The likelihood function for the new link function can be written as

$$= \prod_{i=1}^{n}\left\{e^{-}\left(\left(\psi\right)^{-\alpha}\right)\right\}^{y_i}\left\{1 - e^{-}\left[\left(\psi\right)^{-\alpha}\right]\right\}^{1-y_i}$$

Let $\lambda_0 + \lambda X_i^{'}$ be ψ

Where $\lambda_0 + \lambda X = \Lambda X^{'}$

$$\Lambda = \begin{pmatrix} \lambda_0 \\ \lambda_1 \\ \lambda_2 \\ . \\ . \\ . \\ \lambda_k \end{pmatrix}$$

Consider a sample of size n from the binary response Y, with $\Pr[Y_i = y_i = 1] = \pi_i$ for y_i=1,…,n. we denote the observed observation as $U = \{n, Y = y, X = x\}$, where $y = (y_{1,\cdots,}y_n)$ is the observed vector of $Y\ and\ X = (1, x_{1,\cdots,}x_r)^{'}$, is the observed design matrix of $X = (1, x_{1,\cdots,}x_r)^{'}$. Also, denote the Log Likelihood as LL.

$$LL = \sum_{i=1}^{n} y_i \log(\pi_i) + (1-y_i)\log(1-\pi_i)$$

$$= \sum y_i \log\left(e^{-}(\psi)^{-\alpha}\right) + (1-y_i)\log\left(1 - e^{-}(\psi)^{-\alpha}\right)$$

$$= \sum -y_i(\psi)^{-\alpha} + (1-y_i)\log\left(1 - e^{-}(\psi)^{-\alpha}\right)$$

Where Λ is a vector? A numerical method will be adopted to obtain the MLE for (Λ, α).

5. Model Efficiency and Selection

In selecting the most competitive models, this study adopted the Log-likelihood(LL) value, Akaike Information Criterion(AIC), and Bayesian Information Criterion (BIC). Also, for goodness of fit Kolmogorov-Smirnov (KS) statistic was used, for classification performance, model accuracy and Brier Score were used.

Results

1. Roland Fisher Irish Data (Standard Data)

Roland Fisher Iris dataset which contains four features sepal length, sepal width, petal length, and iris Virginica (7). This dataset will be used as standard data to test the proposed model using Virginica as a species of interest while Setosa and Versicolor will be used as

Table 1. Comparison of the link functions under maximum likelihood Estimate for Standard Data

MODEL	LL	AIC	BIC	KS	Brier Score	Accuracy
TYPE II(INVERSE WEIBULL)	98.287	-184.58	-183.517	0.1333	0.1832	82.3%
LOGISTIC	-5.949	21.899	36.953	0.01431	0.0133	98.7%
TYPE I (COM LOG LOG)	-5.689	21.378	36.431	0.0141	0.0133	96.2%
TYPE III (WEIBULL)	9.90E+290	-1.98E+291	-1.98E+291	0.3333	0.3541	66.7%

LL=Loglikelihood, AIC=*Akaike* Information Criterion, BIC= Bayesian Information Criterion, KS= Kolmogorov-Smirnov

Table 2. Area under the curve for Irish data

	Obs	ROC Area	Std. Err.	[95% Conf. Interval]	
Inweibullirish	150	0.8100	0.0261	0.75882	0.86118
Logisticiris	150	0.9986	0.0012	0.99621	1.00000
Comloglogirish	150	0.9990	0.0010	0.99709	1.00000
Weibullirish	150	0.540	0.0194	0.502	0.5779

Std. Err.

references.

The best model based on log-likelihood, AIC, and BIC is TYPE III (Weibull) followed by TYPE II (Inverse Weibull) while the logistic model performed better based on KS and Brier's score followed by TYPE I (Complementary log log). The coefficients of TYPE III (Weibull) distribution result in a very small standard error which makes all the coefficients highly significant. However, it has the lowest accuracy compared to other models (Tables 1& 2). The Area under the Curve (AUC) in Table 2, revealed that both logistic and TYPE I (Complementary log log) were classified better than TYPE II (Inverse Weibull) and TYPE I (Weibull). However, TYPE II (inverse Weibull) has a higher AUC than TYPE I (Weibull). Also, in Table 3, the TYPE III model consistently reveals strong associations for all factors (constant, sepal length, sepal width, petal length, and petal width) with notably low p-values. The TYPE II and TYPE I models similarly display substantial associations for certain variables. In contrast, the Logistic model generally suggests less robust associations with elevated p-values and a lack of uniformity in identifying significant predictors.

2. Stillbirth Data

Data used in validating the new proposed model were collected from six primary Health Centres across the six Area Councils of Federal Capital Territory of Nigeria. Data consists of patients who delivered babies in Federal Capital territory clinics in the year 2019. The information elicited from the patient's record were maternal age, antenatal status, birth size, gestational age, and birth outcome (alive or stillbirth).

Maximum likelihood estimates were obtained for the proposed model (Inverse Weibull), Logistic, Complementary log log, and Weibull. The most competitive model was selected based on the model efficiency method which includes Log-likelihood, AIC,

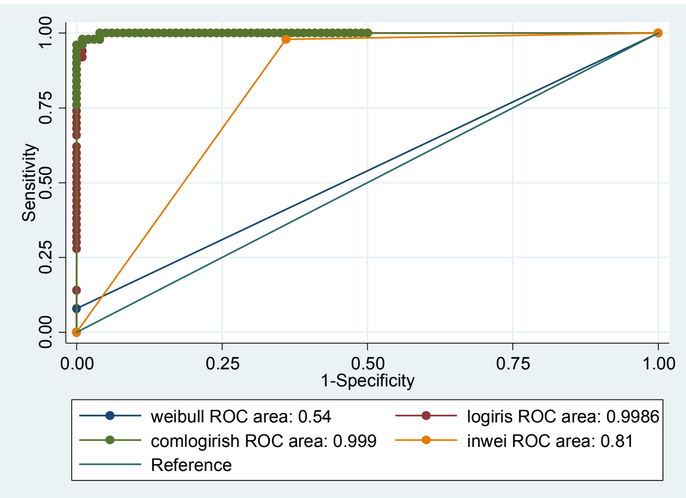

Figure 3. Area under curve for Irish

Table 3. Maximum Likelihood Estimate for Irish Data

	TYPE II (INVERSE WEIBULL)		LOGISTIC		TYPE I (COMPLEMENTARY LOG LOG)		TYPE III (WEIBULL)	
	Coeff(std.error)	p<|z|	Coeff(std.error)	p<|z|	Coeff(std.error)	p<|z|	Coeff(std.error)	p<|z|
Con	-2.28(1.005)	0.02	-42.638(25.708)	0.097	-31.937(17.511)	0.068	28.05(4.3E-145)	<0.0001
sepal.length	0.318(0.3265)	0.3301	-2.46(2.39)	0.303	-1.4629(1.387)	0.292	161(5.205E-145)	<0.0001
sepal.width	0.4267(0.2575)	0.097	-6.681(4.479)	0.136	-5.224(3.113)	0.093	84.14(9.44E-145)	<0.0001
petal.length	-0.529(0.266)	0.047	9.429(4.737)	0.047	6.1047(2.732)	0.025	99.89(9.44E-145)	<0.0001
sepal.width	2.23(0.431)	<0.0001	18.286(9.743)	0.061	15.1167(7.7519)	0.051	30.89(8.68E-145)	<0.0001
α	4.03(0.391)						80.84(5.72E-146)	

Con=Constant, Coeff=Coefficient, std.error=Standard error

BIC, KS, and Brier score. The inverse Weibull has the highest Log-likelihood, lowest AIC, and BIC, and second minimum both in KS and Brier scores. Also, it has a better sensitivity value than other models and it has almost the same accuracy with logistic (see table 4&5). The coefficient of the model shows that an increase in maternal age reduces the risk of stillbirth, booking for antenatal reduces the risk of stillbirth, increase in birth weight will the risk of stillbirth (see Table 7).

In both data examples, the Weibull model predicted value tend towards zero which was aligned with the assumption on which it was being built which affects its sensitivity and accuracy of prediction(5). Inverse Weibull unlike Weibull has both good sensitivity and accuracy of prediction. Also, the result in Table 6 revealed that both logistic and TYPE I (Complementary log log) have 75% value of AUC, and TYPE II (Inverse Weibull) has 76% ability to differentiate between stillbirth and livebirth While TYPE I (Weibull) has 50%.

Discussion

The findings of this study show that Type II and logistic link functions are good in binary classification based on the results of model efficiencies such as loglikelihood, AIC, and Area under the curve. This result aligns with Tahir *et al.* (2016), who in their paper illustrated the application of at-site frequency analysis using relatively nontraditional probability distributions, adopting four methods of parameter estimation using annual maximum rainfall series from 1980 to 2015 from three sites: Muzaffarabad, Garhi Dupatta, and Kotli in

Table 4. Comparison of the link functions under maximum likelihood Estimate for Stillbirth data

MODEL	LL	AIC	BIC	KS	Brier Score
TYPE II(INVERSE WEIBULL)	-67.670	147.34	152.541	0.053	0.2126
LOGISTIC	-336.604	683.207	706.213	0.027	0.2147
TYPE I (COM LOG LOG)	-336.718	683.437	698.4899	0.358	0.3379
TYPE III (WEIBULL)	-1.90E+305	5.163E+302	3.8E+305	0.2377	0.2401

LL=Loglikelihood, AIC=Akaike Information Criterion, BIC= Bayesian Information Criterion, KS= Kolmogorov-Smirnov

Table 5. Comparison of Classification performance for different link functions

MODEL	Sensitivity	Specificity	Accuracy
TYPE II(INVERSE WEIBULL)	65.7%	82.7%	78.4%
LOGISTIC	49.1%	87.7%	78.5%
TYPE I (COM LOG LOG)	23.4%	79.5%	66.2%
TYPE III (WEIBULL)	1.14%	76.2%	76.2%

Table 6. Area under curve for stillbirth data

	Obs	ROC Area	Std. Err.	[95% Conf. Interval]	
Inweibull	736	0.7650	0.0236	0.69879	0.81131
Logistic	736	0.7538	0.0221	0.71038	0.79720
Comploglog	736	0.7527	0.0224	0.70883	0.79649
Weibull	736	0.5048	0.0041	0.4967	0.5129

Std.Err. standard error

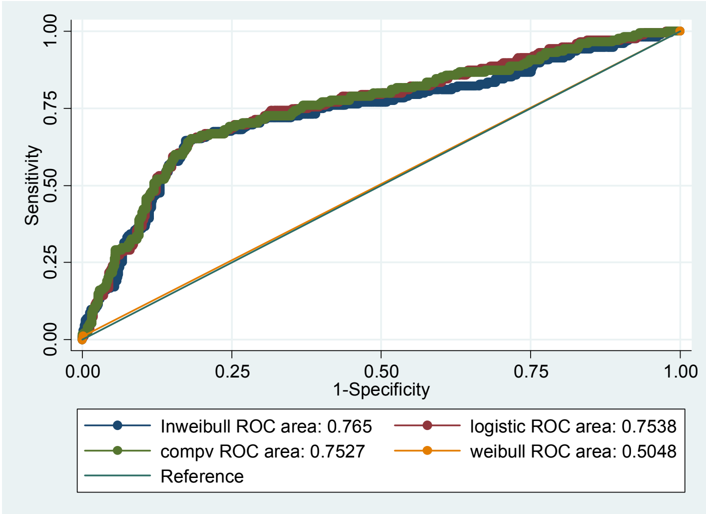

Figure 4. Area under curve for stillbirth Data

Table 7. Maximum Likelihood Estimate for Stillbirth Data

	TYPE II (INVERSE WEIBULL)		**LOGISTIC**	
Variables	Coeff(std.error)	p<|z|	Coeff(std.error)	p<|z|
Con	0.4437(0.2123)	0.03	-1.097(0.778)	0.158
Age	-0.0008(0.0023)	0.725	-0.008(0.009)	0.389
Booked	-2.333(0.3921)	<0.0001	-2.115(0.9937)	<0.0001
birth size	-0.0013(0.000357)	0.017	-0.231(0.151)	0.126
birth month	0.0228(0.0159)	0.154	0.0073(0.0668)	0.913
α	2.25(0.497)		-	-

Con=Constant, Coeff=Coefficient, std.error=Standard error

Azad Jammu and Kashmir, Pakistan (9). Two probability distributions, Fréchet (Type II) and Log-logistic were used as candidate distributions to model the annual maximum rainfall series at given sites. The result of the findings on stillbirth, which indicated that booking antenatal care significantly reduces the likelihood of stillbirth, is in tandem with the findings of Berhe *et al*. (2023) who concluded that "having a good quality of antenatal care significantly reduces antepartum stillbirth (10). Strategies need to be developed on the problems identified to improve the quality of ANC and reduce antepartum stillbirth significantly". This finding also aligns with the position of another study from Lagos State, Nigeria, by Orisakwe *et al*., (2017), who opined that "The prevalence of stillbirth was high in the hospital during the study period (11). The majority of these deaths occurred during the antenatal period and were common in those women who did not receive ANC in the hospital. There is an urgent need to improve the quality of our obstetric healthcare services and encourage early referral of complicated pregnancies and labor to prevent unnecessary fetal deaths due to preventable or manageable obstetric conditions". Another study from Ghana (Afulani, 2016) maintained a similar position with this finding; the author concluded that "Good quality ANC can improve birth outcomes in two ways: directly through preventative measures and indirectly through promoting deliveries in health facilities where complications can be better managed (12).

Conclusion

This study has presented a new link function for binary classification problems. The new model is very flexible and capable of handling different types of data either symmetric or skew. The results of comparison with other extreme value distribution link functions in the previous section indicate that Inverse Weibull (TYPE II), Performed better than TYPE I (Complementary log log) and TYPE III(Weibull). In addition, the numerical procedure of the proposed link function is very easy to implement compared to other Extreme value distributions. Therefore, the performance of this proposed model is good and it can be used for modeling binary classification problems.

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