



The Extension of the Spokes Model to the Streets with Unequal Length Model (Explosion Model)

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Abstract

Locating is one of the key elements in the success and survival of industrial centers and greatly affects cost reduction of the establishment and launching of various economic activities. This study has used the streets with unequal-length model through the classic extension of the Spoke model (Lijesen and Reggiani, 2013) but with an unlimited number of streets with uneven lengths. The results show that the spoke model is a special case of the streets with unequal-length model. According to the results of this study, if the strategy of Enterprises and firms is to select both price and location, there would be no balance in the game. Furthermore, the increased length of streets leads to an increased profit of Enterprises, while the Enterprises choose locations far from the center (the maximum differentiation) with the increased number of streets, consequently decreasing the Enterprises' output. Moreover, the Enterprise production rate will incline toward zero when the number of streets goes to infinity, and a complete competition outcome will be achieved.

Keywords: game theory, locating, Nash equilibrium, streets with equal-length model, streets with unequal-length model.

JEL Classification: C62, C70, D43, L11, R30.

Introduction

Location has long been recognized as one of the most important strategic decisions made by firms and one of the principal determinants of market performance, competitive advantage, and long-term profitability. The geographical position of a firm directly influences production efficiency, transportation costs, accessibility to consumers, market coverage, and ultimately the firm's ability to compete successfully in spatially differentiated markets. Consequently, location decisions affect not only the profitability of individual firms but also the allocation of economic activities, regional development, market

efficiency, and social welfare. Since the pioneering contributions of von Thünen (1826) and Weber (1909), location theory has evolved into one of the fundamental disciplines of regional science, industrial organization, operations research, and spatial economics. Later developments by Isard (1956), Fujita et al. (1999), and Duranton and Puga (2004) further demonstrated that firms' spatial decisions play a central role in determining industrial concentration, regional specialization, and market performance. Consequently, inappropriate location decisions or inefficient industrial location policies may substantially reduce firms' competitiveness and generate considerable economic inefficiencies (Vinay and Chakravorty, 2005).

The increasing integration of markets, globalization, technological progress, and intensified competition have transformed location from a purely geographical concept into a strategic economic asset. Modern firms compete not only through prices, product quality, and innovation but also through their spatial positions relative to consumers and rival firms. Likewise, consumers no longer make purchasing decisions solely on the basis of product characteristics or prices; transportation costs, accessibility, travel time, convenience, and other behavioral considerations increasingly influence purchasing decisions (Shy, 2014). Consequently, firms' competitive advantage depends not only on production efficiency but also on selecting locations that maximize accessibility while minimizing transportation and operating costs. Empirical evidence consistently demonstrates that accessibility, transportation costs, and consumer distribution significantly influence firms' market performance and consumers' purchasing behavior (Gelormino et al., 2015; Zhao et al., 2018; Pereira et al., 2017).

From a managerial perspective, facility location represents one of the few strategic decisions whose consequences persist throughout the entire life cycle of a firm. Unlike pricing or production decisions, location choices are often costly or impossible to reverse once investments have been made. An inappropriate location may permanently reduce demand, increase operating costs, weaken competitive position, and ultimately threaten business survival. Evidence reported by Melaniphy (1999) indicates that a substantial proportion of newly established businesses fail despite satisfactory managerial performance because of inappropriate location choices. These observations have motivated economists, operations researchers, and management scientists to develop increasingly sophisticated analytical models capable of identifying optimal facility locations under different competitive environments (Drezner, 1995; ReVelle and Eiselt, 2005).

Theoretical studies on facility location are generally classified into competitive and non-competitive frameworks. While non-competitive models focus primarily on minimizing transportation or operating costs without explicitly considering firms' strategic interactions, competitive location models recognize that firms simultaneously choose locations while anticipating competitors' responses. Consequently, competitive facility location has become one of the most important applications of game theory in industrial organization and spatial economics. Within this framework, firms strategically determine their locations, pricing policies, market coverage, and other decision variables in order to maximize expected profits. The resulting interactions are commonly analyzed within a Nash equilibrium framework, where each firm's optimal strategy depends explicitly on the equilibrium behavior of competing firms (Hotelling, 1929; d'Aspremont et al., 1979; Anderson and Neven, 1991).

Recent decades have witnessed substantial progress in competitive location theory. Successive generations of models have progressively incorporated richer representations of transportation costs, market structures, consumer behavior, and strategic interactions. At the same time, advances in behavioral economics have demonstrated that consumers frequently exhibit bounded rationality, heterogeneous preferences, imperfect information, and reference-dependent decision-making rather than the perfectly rational behavior assumed in many classical spatial models (Kahneman and Tversky, 1979). Consequently, firms' location decisions increasingly depend on consumers' behavioral responses as well as geographical considerations. Integrating behavioral insights with game-theoretic location models therefore represents an important step toward improving the explanatory power and practical relevance of competitive spatial analysis.

Despite these advances, existing competitive location models continue to rely on several simplifying assumptions introduced primarily for analytical convenience. Although such assumptions facilitate the derivation of equilibrium solutions, they frequently reduce the realism of theoretical predictions. Previous studies have repeatedly shown that competitive location models are highly sensitive to relatively small modifications in transportation costs, consumer distribution, market geometry, and firms' strategic interactions (Murata, 2003; Duranton and Puga, 2004; Behrens et al., 2017). Consequently, improving the realism of these assumptions has become one of the principal directions of contemporary research in spatial competition.

One of the most restrictive assumptions maintained in almost all existing game-theoretic location models concerns the geometry of transportation networks. In particular, the widely used spokes framework assumes that every street connected to the city center possesses an identical length. While this assumption considerably simplifies mathematical analysis, it rarely reflects the spatial organization of actual cities, where transportation networks exhibit substantial heterogeneity in street lengths, accessibility, travel distances, and network structure. Such heterogeneity directly affects consumers' travel costs, firms' effective market areas, equilibrium prices, and competitive strategies. Consequently, maintaining identical street lengths may significantly limit the explanatory power and practical applicability of existing competitive location models.

Motivated by this limitation, the present study develops a generalized game-theoretic facility location model in which streets connected to the city center are allowed to have heterogeneous lengths. By relaxing one of the most restrictive assumptions of the classical spokes framework, the proposed model provides a more realistic representation of urban transportation networks while preserving analytical tractability. More importantly, the proposed framework demonstrates that several well-known competitive location models—including the Hotelling model, the circular city model, and the classical spokes model—can all be interpreted as special cases under particular parameter configurations.

This study contributes to the literature in several important respects. First, it generalizes one of the most influential game-theoretic location models by introducing heterogeneous street lengths into firms' strategic location decisions. Second, it establishes a unified analytical framework capable of encompassing several existing spatial competition models within a single theoretical structure. Third, it provides new insights into how transportation network geometry influences firms' equilibrium locations, pricing strategies, market shares, and profitability under Nash equilibrium. Finally, by integrating concepts from game theory, spatial competition, and behavioral economics, the proposed model offers a more realistic analytical foundation for studying facility location decisions and provides useful implications for industrial organization, regional science, operations research, and spatial economic policy.

This paper is structured in six sections. The second section presents a literature review, and the third section presents the model. The fourth section focuses on location in the streets with unequal-length model in five subsections.

The fifth section presents the selection of price and location, and finally, summary, conclusion, and recommendation are presented in the sixth section.

Literature Review

The development of competitive facility location theory has been characterized by a gradual relaxation of the restrictive assumptions underlying earlier spatial competition models. Although the origins of location theory can be traced to the pioneering contributions of Saffle (1878), Launhardt (1882), and Weber (1909), modern analytical research on competitive location began with Hotelling's (1929) seminal linear city model. By formulating firms' location choices as a strategic game, Hotelling established the theoretical foundations of spatial competition and demonstrated that firms tend to agglomerate toward the market center under linear transportation costs. His model initiated an extensive body of research devoted to understanding how firms' strategic location decisions influence market outcomes and how equilibrium changes under alternative assumptions regarding transportation costs, consumer behavior, and market structure.

Subsequent theoretical studies demonstrated that the conclusions of the Hotelling model are highly sensitive to its underlying assumptions. Salop (1979) addressed several limitations of the linear city framework by proposing the circular city model, which eliminated boundary effects and provided a symmetric representation of competition among multiple firms. Later contributions extended spatial competition models by incorporating endogenous pricing, quantity competition, product differentiation, and multidimensional strategic interactions (Shaked and Sutton, 1982; Kreps and Scheinkman, 1983; Anderson and Neven, 1991; Anderson et al., 1992). Collectively, these studies established that firms' equilibrium locations cannot be analyzed independently of pricing behavior or market structure and confirmed Nash equilibrium as the dominant analytical framework for competitive location problems.

An important milestone in the literature was achieved by Chen and Riordan (2007), who introduced the spokes model as a generalized representation of urban spatial competition. Unlike the linear and circular city models, the spokes framework considers a city composed of multiple streets connected through a common center, thereby allowing firms to compete within a network rather than along a single dimension. Assuming identical street lengths, uniformly distributed consumers, and a two-stage game in which firms first choose locations and subsequently compete in prices, they demonstrated that market entry influences equilibrium prices, consumer surplus, market coverage, and

social welfare. Their analysis further showed that free entry does not necessarily generate socially efficient outcomes because equilibrium prices may remain above marginal costs even in relatively competitive markets. Owing to its analytical flexibility, the spokes model has become one of the principal theoretical frameworks in the competitive facility location literature.

Building upon the spokes framework, Lijesen and Reggiani (2013; 2016) investigated firms' specialization strategies and the effects of consumer confusion on equilibrium outcomes. Their results indicate that firms' optimal locations depend critically on consumers' ability to distinguish among competing products. When consumers experience confusion, firms have stronger incentives to differentiate themselves in order to increase market power. Conversely, when competition is primarily driven by market-share considerations, firms tend to locate closer together. These findings demonstrate that relatively small behavioral changes on the demand side may substantially alter firms' equilibrium location strategies, reinforcing the argument that competitive location models are inherently sensitive to their behavioral assumptions.

Another important extension was proposed by Shahbazi and Salimian (2017), who relaxed one of the principal assumptions of previous models by replacing the conventional uniform distribution of consumers with a triangular distribution. Motivated by the observation that population density is generally higher near city centers than in suburban areas, they reformulated the spokes model as a two-stage location-price game under a non-uniform distribution of consumers. Their analytical results showed that increases in transportation costs and the number of streets reduce the intensity of competition and increase equilibrium prices. Moreover, adopting a triangular distribution produced lower Nash equilibrium prices than those obtained under the traditional uniform distribution while enlarging firms' effective market areas. These findings highlighted the critical role of consumer distribution in determining equilibrium outcomes.

More recently, Salimian et al. (2023) expanded the scope of competitive location theory by incorporating sales agents into firms' strategic decisions. Rather than focusing exclusively on firms' physical locations, they simultaneously determined the optimal number and optimal locations of sales agents within the spokes framework. Their analysis demonstrated that firms' distribution strategies depend jointly on transportation costs, market size, the number of consumers, consumers' willingness to pay, the number of streets, expected revenues generated by each sales agent, and the fixed costs associated

with establishing additional facilities. Their results also showed that city centers are not always the optimal locations for sales agents, emphasizing the trade-off between accessibility and establishment costs.

Alongside these theoretical developments, numerous empirical studies have investigated the determinants of firms' location decisions across various industries and geographical contexts. Anderson and Neven (1986), Vandenbosch and Weinberg (1995), Yang (1997), Michael et al. (1998), Burdurla and Ejder (2003), Choo and Mazzarol (2003), Hosun et al. (2003), Bautista and Pereira (2007), Redondo et al. (2008), Reggiani (2009), Qin et al. (2010), Choo et al. (2010), Sumetee (2011), Zi et al. (2012), Miquier et al. (2014), Tartavulea (2015), Hu and Wang (2017), Dewita et al. (2020), and Harrison and Campbell (2021) examined firms' location choices in different institutional and industrial settings. Although these studies employed diverse empirical methodologies, they consistently concluded that transportation costs, accessibility, logistics efficiency, infrastructure quality, market potential, production costs, and competitive intensity are among the most influential determinants of firms' location decisions. Collectively, this body of evidence confirms that facility location is a multidimensional strategic decision requiring simultaneous consideration of spatial, economic, and competitive factors.

Recent empirical studies have also emphasized the importance of accessibility and transportation networks in shaping spatial economic outcomes. Baraklianos et al. (2020) demonstrated that alternative accessibility measures significantly affect residential location models, highlighting accessibility as a key explanatory variable in spatial decision-making. Ahrens and Lyons (2021) showed that increasing housing costs in employment centers induce households to accept longer commuting distances, thereby modifying the spatial distribution of demand faced by firms. Similarly, Marval and Silva (2023) used an agent-based model to illustrate how transportation costs and accessibility jointly influence urban spatial structure by shaping residential location choices under alternative policy scenarios. Although these studies primarily analyze household behavior, their findings indirectly reinforce the importance of accessibility and transportation networks for firms' competitive location strategies.

Despite the considerable progress achieved over the past decades, a common limitation persists throughout nearly all game-theoretic facility location models. Whether considering the linear city, circular city, or spokes framework, existing models generally assume transportation networks with highly regular spatial structures. In particular, the spokes model assumes that every street

connected to the city center has an identical length. While this assumption substantially simplifies analytical derivations, it rarely reflects the heterogeneous geometry of real transportation networks, where streets differ considerably in length, accessibility, travel time, and traffic conditions. Such heterogeneity directly influences consumers' travel costs, firms' effective market areas, and the nature of strategic interactions among competing firms.

Accordingly, there remains an important gap in the competitive location literature regarding the incorporation of heterogeneous transportation networks into game-theoretic facility location models. Existing studies have successfully relaxed assumptions related to consumer distribution, pricing behavior, specialization, and market structure, yet the geometry of the transportation network itself has received comparatively little attention. Addressing this limitation provides an opportunity to improve both the realism and the explanatory power of spatial competition models. The present study contributes to this line of research by extending the classical spokes framework to a transportation network composed of streets with unequal lengths, thereby offering a more general representation of spatial competition and providing new theoretical insights into firms' equilibrium location and pricing decisions.

The present study addresses this gap by developing a generalized spatial competition model in which streets connected to the city center are allowed to have heterogeneous lengths. Unlike previous studies that modify consumer distribution or firms' pricing behavior while maintaining identical transportation networks, the proposed framework explicitly incorporates heterogeneity in urban spatial geometry. This extension not only enhances the realism of competitive location modeling but also establishes a unified framework in which several well-known models - including the Hotelling model, the circular city model, and the classical spokes model - can be interpreted as special cases under specific parameter configurations. By relaxing one of the most restrictive assumptions of the existing literature, the proposed model contributes to a deeper understanding of firms' strategic location decisions under more realistic transportation networks and provides new theoretical insights into the relationship between market geometry, consumer accessibility, and spatial competition.

On the other hand, the main problem of these models has been the simplification of assumptions. One of these models, presented by Chen and Riordan (2007) and extended by Lijesen and Reggiani (2013; 2016), was the spokes model, which was the generalized form of the circular city model, considering N streets of equal lengths of $1/2$. All these streets were connected to

the city center, and if the Enterprise was not located on the same street as the consumer or the consumer was on an empty street, the consumer had to pass through the city center to purchase from that Enterprise.

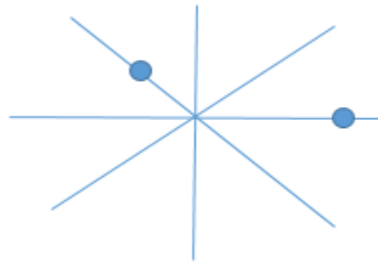


Figure 1. The Spokes Model, 2 Firms, $N = 8$ Spokes Each of Length of $\frac{1}{2}$

Source: Research findings.

These assumptions are just for the simplification of the model and its results. Here, all assumptions except for the equality of street lengths are considered fixed, which is one of the main and even the most critical simplification assumptions; however, the distance of all streets up to the city center is not equal in the real world. This model is called the unequal-length streets model (Explosion Model)¹.

Since the real world consists of cities with unequal streets lengths, this section seeks to tackle the weaknesses of Lijesen and Reggiani's work and propose a more appropriate solution in Enterprise location. Concerning the above-mentioned points, this study primarily aims to extend and generalize the competition in the spokes model to the unequal-length streets model through the following research questions: How will the results of the spokes model change for streets passing through the center that are of different lengths? Are the results of the spokes model true for the unequal-length streets model? Do Enterprises' location and profit change with the change in the length of streets? How does the optimal location of Enterprises change in such conditions? Do the Enterprises select a location farther from the center as the optimal location or are they inclined to the center? How does the increase in the number of streets affect the location and profit of Enterprises?

¹. The reason for selecting this name for the considered model is that all particles will not necessarily pass the same distance from the explosion center in an explosion. In other words, every particle will be at different distance from the explosion center such as big bang theory.

Model

This paper has used the spoke model (competitive location choice) proposed by Chen and Riordan (2007) and studied by Lijesen and Reggiani (2013; 2016), with the difference that the lengths of streets are not equal, or more specifically, not equal to $\frac{1}{2}$ here. The market is consisted of N streets of unequal lengths, and any street has a length of l_i ($i=1, \dots, N$). The consumers are uniformly distributed with density $\frac{2}{N}$ on every street. Following Chen and Riordan (2007), it is assumed that the utility any consumer derives from purchasing a product is v . It is assumed that the products are offered by two Enterprises potentially located on the same street where the consumer is located or on any of the other N streets. The street on which the consumer is located is denoted by s and shown by l_s . In addition, the Enterprises cannot locate on other streets. In the end, the strategy of Enterprise i is $y_i \in (-\infty, l_i)$, $i=1, 2$.

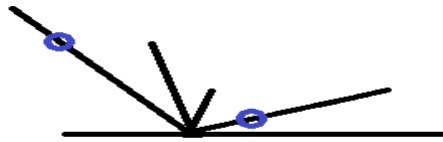


Figure 2. Streets with Unequal-Length Model with Six Streets of Unequal Lengths and Two Enterprises Shown on them

Source: Research findings.

Figure 2 shows the streets with unequal-length model with the highlighted points showing the situation of Enterprises. The consumer's position on street s is shown by x_s and defined as $x_s \in [0, 1]$. The distance between the consumer located on street s (x_s) and the Enterprise located on the street i (y_i) is shown by $d(y_i, x_s)$, defined based on whether the consumer and Enterprise are located on the same or different streets. If they are both located on street i , then:

$$d(y_i, x_s) = |y_i - x_s| \quad s = i \quad (1)$$

On the other hand, if the Enterprise and consumer are located on different streets, we will have:

$$d(y_i, x_s) = (l_i - y_i) + (l_s - x_s) = l_i + l_s - y_i - x_s \quad \forall s \neq i \quad (2)$$

In addition, it is assumed that if the consumer purchases from other Enterprises located in other streets, he/she is obliged to pass through the city center. Following Lijesen and Reggiani (2013 and 2016), it is assumed that transportation costs are proportional to the square of the distance between the consumer and Enterprise. In other words, transportation costs are defined as:

$$T_{is}(y_i, x_s) = \begin{cases} td^2(y_i, x_s) = t(y_i - x_s)^2 & s = i \\ td^2(y_i, x_s) = t(l_i + l_s - y_i - x_s)^2 & \forall s \neq i \end{cases} \quad (3)$$

Without loss of generality and for simplification, it is assumed that $t=1$, and the marginal production cost in all analysis procedures is zero ($c=0$). The Enterprises play a three-stage game with incomplete information:

1. The nature allocates a street $s=i$ to each Enterprise $i=1, 2$.
2. Enterprises simultaneously determine their location y_i on the street where they are located.
3. Enterprises simultaneously set a uniform price p_i (Lijesen and Reggiani, 2013).

In this section, the game will be solved by backward induction.

A Location in the Streets with Unequal-Length Model

We consider a two-stage game where the Enterprises decide to determine their location in the first round and the price in the second round. To solve this game, we begin with the pricing stage and define our profit function. The first step in determining the profit function is to determine the indifferent consumers. Following Chen and Riordan (2007) and Lijesen and Reggiani (2013; 2016), it is assumed that each consumer has preferences for just two brands, and there will be no problem with whether there is an Enterprise in the market or not (Lijesen and Reggiani, 2013; 2016). This assumption indicates that the consumer located on each street prefers the products produced by the Enterprise located on their street and as the second preferred brand, he might prefer each sign of $N-1$ other streets (the brand produced by Enterprise 2, or $N-2$ other Enterprises that don't supply) (Lijesen and Reggiani, 2013; 2016). Thus, there are two types of consumers, including those who prefer both existing signs and those just preferring one type of the presented signs. Moreover, some consumers prefer two signs that are not offered by any Enterprise, indicating that the whole market is not completely covered. Thus, the Enterprises' competition is on type 1 consumers, who are indifferent between purchasing from Enterprise i and Enterprise j whenever:

$$x_{ij} \text{ s.t. } T_{is}(y_i, x_{ij}) + p_i = T_{js}(y_j, x_{ij}) + p_j \quad (4)$$

Thus:

$$x_{ij} = \frac{1}{2}(l_i + l_s) - \frac{1}{2}(y_i - y_j) - \frac{p_j - p_i}{2(l_i + l_s - y_i - y_j)} \quad x_{ij} \in j \quad (5)$$

$$x_{ij} = \frac{1}{2}(l_i + l_s) + \frac{1}{2}(y_i - y_j) - \frac{p_j - p_i}{2(l_i + l_s - y_i - y_j)} \quad x_{ij} \in i \quad (6)$$

Following Lijesen and Reggiani (2013 and 2016), the first area of Chen and Riordan (2007) is focused, where v is sufficiently high so that all consumers could purchase on the streets and $x_{ij} = 1$; $i = 1, 2$; $j = 3 \dots N$. Consequently, other Enterprises will achieve equilibrium price by having a location.

Price Selection

Assuming that the location of Enterprise i is in y_i , we will begin from the last stage of the game. The profit function of Enterprise 1 will be as follows with respect to all consumers who can purchase from it:

$$\pi_1 = \frac{2}{N} \left(\frac{1}{N-1} p_1 (1 - x_{12}) + \frac{1}{N-1} \sum_{j=3}^N p_1 x_{1j} \right) \quad (7)$$

In the above equation, $\frac{2}{N}$ is the distribution of consumers on each street and $\frac{1}{N-1}$ is the ratio of consumers who have the second preferred brand and are located on each of S streets. X_{12} shows the indifferent consumer in purchasing from each Enterprise 1 and 2, and X_{1j} is the indifferent consumer in purchasing from Enterprises 1 and j , where the number of Enterprises j varies from 3 to N (all Enterprises except 1 and 2). After simplification and assuming that the indifferent consumer in purchasing from Enterprises 1 and 2 (X_{12}) is located on street 2, the profit function of Enterprise 1 will be achieved as follows:

$$\pi_1 = \frac{2}{N} \left(\frac{1}{N-1} p_1 \left(1 - \left(\frac{1}{2} (l_1 + l_2) - \frac{1}{2} (y_1 - y_2) - \frac{p_2 - p_1}{2(l_1 + l_2 - y_1 - y_2)} \right) \right) + \frac{N-2}{N-1} p_1 \right) \quad (8)$$

The following results could be obtained from the profit functions:

Theorem 1

There is a unit Nash equilibrium in prices, and this equilibrium price is increasing in N .

Proof

To prove the theorem, we need to derive the first and second derivatives from the profit function relative to prices:

$$\frac{\partial \pi_1}{\partial p_1} = - \frac{-2l_1 - 2l_2 + 2y_1 + 2y_2 - l_1^2 - 2l_1 l_2 + 2l_1 y_1 - l_2^2 + 2l_2 y_1 - y_1^2 + y_2^2 + p_2 - 2p_1 + 2N(l_1 + l_2 - y_1 - y_2)}{N(N-1)(y_1 + y_2 - l_1 - l_2)} = 0$$

$$\frac{\partial^2 \pi_1}{\partial p_1^2} = \frac{2}{N(N-1)(y_1 + y_2 - l_1 - l_2)} < 0 \quad \text{if } y_1 + y_2 < l_1 + l_2$$

Since $y_1 + y_2 < l_1 + l_2$ is always holds (except in certain conditions when both Enterprises are located at the city center), the Enterprises are never located in the city center. Considering symmetry, it is possible to easily calculate P_1 and P_2 values.

$$p_1 = \frac{1}{2}p_2 + y_1(1 + l_1 + l_2) + y_2 + N(l_1 + l_2 - y_1 - y_2) - \frac{1}{2}y_1^2 + \frac{1}{2}y_2^2 - \left(l_1 + l_2 + \frac{1}{2}l_1^2 + \frac{1}{2}l_2^2 + l_1l_2\right)$$

$$p_2 = \frac{1}{2}p_1 + y_1 + y_2(1 + l_1 + l_2) + N(l_1 + l_2 - y_1 - y_2) + \frac{1}{2}y_1^2 - \frac{1}{2}y_2^2 - \left(l_1 + l_2 + \frac{1}{2}l_1^2 + \frac{1}{2}l_2^2 + l_1l_2\right)$$

Solving the above equation, P_1 and P_2 Nash equilibrium values will be achieved as follows:

$$p_1 = \frac{4}{3} \left(\frac{3}{2} + l_1 + l_2 \right) y_1 + \frac{4}{3} \left(\frac{3+l_1+l_2}{2} \right) y_2 + 2N(l_1 + l_2 - y_1 - y_2) - \frac{y_1^2}{3} + \frac{y_2^2}{3} - 2 \left(l_1 + l_2 + \frac{1}{2}l_1^2 + \frac{1}{2}l_2^2 + l_1l_2 \right) \quad (9)$$

$$p_2 = \frac{4}{3} \left(\frac{3+l_1+l_2}{2} \right) y_1 + \frac{4}{3} \left(\frac{3}{2} + l_1 + l_2 \right) y_2 + 2N(l_1 + l_2 - y_1 - y_2) + \frac{y_1^2}{3} - \frac{y_2^2}{3} - 2 \left(l_1 + l_2 + \frac{1}{2}l_1^2 + \frac{1}{2}l_2^2 + l_1l_2 \right) \quad (10)$$

From Equations 9 and 10, we have:

$$\frac{\partial p_1}{\partial N} = 2(l_1 + l_2 - y_1 - y_2) = \frac{\partial p_2}{\partial N} \Rightarrow l_1 + l_2 - y_1 - y_2 > 0 \text{ if } l_1 + l_2 > y_1 + y_2 \text{ or } y_1 < (l_1 + l_2) - y_2 \quad (11)$$

Since:

$$\frac{\partial p_1}{\partial N} = \frac{\partial p_2}{\partial N} \geq 0$$

Notably, $l_1 + l_2 > y_1 + y_2$ is always holds, as Enterprises 1 and 2 with locations y_1 and y_2 are on the same streets with unequal lengths of l_1 and l_2 , established at the ending point of these streets that is city center (where the equality will be established).

The results obtained by replacing $l_1 = l_2 = \frac{1}{2}$ values are similar to the spokes model of Lijesen and Reggiani (2013; 2016).

Theorem 2

If the following conditions are established:

$$y_1 < -3N + 2l_1 + 2l_2 + 3, \quad y_1 > 3N - l_1 - l_2 - 3$$

$$y_2 < -3N + 2l_1 + 2l_2 + 3 \cdot y_2 > 3N - l_1 - l_2 - 3$$

An increase in y_1 and y_2 leads to an increase in the prices of Enterprises 1 and 2 and vice versa.

Proof

By taking derivatives of Equations 9 and 10, we have:

$$\frac{\partial p_1}{\partial y_1} = \frac{4}{3} \left(\frac{3}{2} + l_1 + l_2 \right) - 2N - \frac{2}{3} y_1 > 0 \Leftrightarrow y_1 < -3N + 2l_1 + 2l_2 + 3 \quad (12)$$

$$\frac{\partial p_1}{\partial y_2} = \frac{4}{3} \left(\frac{(3+l_1+l_2)}{2} \right) - 2N + \frac{2}{3} y_2 > 0 \Leftrightarrow y_2 > 3N - l_1 - l_2 - 3 \quad (13)$$

$$\frac{\partial p_2}{\partial y_1} = \frac{4}{3} \left(\frac{(3+l_1+l_2)}{2} \right) - 2N + \frac{2}{3} y_1 > 0 \Leftrightarrow y_1 > 3N - l_1 - l_2 - 3 \quad (14)$$

$$\frac{\partial p_2}{\partial y_2} = \frac{4}{3} \left(\frac{3}{2} + l_1 + l_2 \right) - 2N - \frac{2}{3} y_2 > 0 \Leftrightarrow y_2 < -3N + 2l_1 + 2l_2 + 3 \quad (15)$$

These results show that under the above conditions, the farther Enterprise 1 is from the center, the more it can increase its price. The price of Enterprise 1 also increases when Enterprise 2 becomes closer to the center and vice versa. The same condition is true for Enterprise 2. In the backward induction method, the second step is deriving the equilibrium location (by replacing the obtained equilibrium prices from the previous stage), which will be addressed in the next part. These results are in line with the results obtained by Shahbazi and Salimian (2017). They showed the price of firm 1 had an inverse relationship with its location, while this relation was direct in Hotelling's uniform distribution model, in which the more y_1 increased, the closer this firm became to the middle points, leading to higher prices. On the other hand, in a Bertrand game with differentiated products, the profit of firms increases with differentiation of products. This means that the differentiation of the products increases the monopoly power of firms producing trademarks by decreasing the price competition among the firms producing trademarks (Shy, 1996).

Location Choice

Substituting the obtained prices into the profit function of the Enterprise (Equation 8), it is possible to determine the situation of Enterprises:

$$\pi_1 = \frac{2}{N} \left(\frac{1}{N-1} p_1(y_1, y_2) \left(1 - \left(\frac{1}{2}(l_1 + l_2) - \frac{1}{2}(y_1 - y_2) - \frac{p_2(y_1, y_2) - p_1(y_1, y_2)}{2(l_1 + l_2 - y_1 - y_2)} \right) \right) + \frac{N-2}{N-1} p_1(y_1, y_2) \right) \quad (16)$$

The following results can be extracted from the profit function:

Theorem 3

There is a subgame perfect Nash equilibrium in location. This symmetric combination and optimal location decreases with respect to N .

Proof

From the obtained and simplified profit function (Equation 16), after replacing equilibrium prices and the first- and second-order conditions, we have:

$$\frac{\partial \pi_1(y_1, y_2)}{\partial y_1} = \frac{1}{9N(1-N)} (3y_1^2 - 2y_1(y_2 - 12N + 7l_1 + 7l_2 + 12) - y_2^2 + 2y_2(l_1 + l_2) + 3(2N - l_1 - l_2 - 2)(6N - 5l_1 - 5l_2 - 6)) = 0 \quad (17)$$

$$\frac{\partial^2 \pi_1(y_1, y_2)}{\partial y_1^2} = \frac{2(3y_1 - y_2 + 12N - 7l_1 - 7l_2 - 12)}{9N(1-N)} < 0 \Leftrightarrow \frac{1}{N(N-1)} > 0 \Rightarrow y_1 > \frac{y_2 - 12N + 7l_1 + 7l_2 + 12}{3} \quad (18)$$

The following results will be achieved for the optimal location of Enterprises by solving the above equations and the profit function of Enterprise 2 and differentiating and solving it, along with these equations:

$$\begin{aligned} 1- \quad y_1 &= \frac{l_1 + l_2}{2}, \quad y_2 = \frac{-12N + 7l_1 + 7l_2 + 12}{2} \\ 2- \quad y_1 &= \frac{-12N + 7l_1 + 7l_2 + 12}{2}, \quad y_2 = \frac{l_1 + l_2}{2} \\ 3- \quad y_1 &= \frac{-6N + 5l_1 + 5l_2 + 6}{4}, \quad y_2 = \frac{-6N + 5l_1 + 5l_2 + 6}{4} \end{aligned} \quad (19)$$

Considering the second rank condition, it is clear that states 1 and 2 show the situations where Enterprises 1 and 2 will unilaterally change their locations, respectively. Thus, neither of the first two states is a Nash equilibrium for Enterprises. State 3 shows a Nash equilibrium for the location. It is obvious that:

$\partial Y_1 / \partial N = \partial Y_2 / \partial N = -6/4$. Thus, the results of the above theorem will be proved. Considering $l_1 = l_2 = 1/2$, these results are consistent with the results obtained by Lijesen and Reggiani (2013). Moreover, considering the special case of $N=2$, the results of Hotelling's standard model will be achieved, in which the Enterprises are $[(5l_1 + 5l_2 - 6)/4, (5l_1 + 5l_2 - 6)/4]$, and the results will be as $[-1/4, -1/4]$ in special case of $l_2 = l_1 = 1/2$. Here, the results of Lijesen and Reggiani are again confirmed. In this state, where all streets are covered by Enterprises, the optimal location of Enterprises is out of the market area (where the consumers are distributed) (Lijesen and Reggiani, 2016). The Enterprises have maximum differentiation insofar as the equilibrium location is decreasing

with respect to N (Lijesen and Reggiani, 2016). Since equilibrium location is decreasing with respect to N , it is concluded that the Enterprises have maximum differentiation and differentiation. Lijesen and Reggiani (2016) showed that if the consumers were concentrated, the Enterprises would be inclined toward concentration. The consumers were supposed to be located on the empty streets at the center; i.e., there were consumers of the same number of empty streets at the center.

Complete Competition and Location

In this section, all consumers are considered on all streets. Similar to other location models, it is assumed that the consumer is indifferent among supplier Enterprises and purchases from the Enterprise with the minimum price and squared transportation costs. The optimization problem for Enterprises 1 and 2 will be solved when the number of streets (N) is ≥ 2 . The certain case of $N=2$ corresponds to the Hotelling's standard case. Demand for the streets where the Enterprise is located (with $\frac{2}{N}$ consumers on each street) will be defined as follows:

$$x_{12} = \frac{1}{2} (l_1 + l_2) - \frac{1}{2} (y_1 - y_2) - \frac{p_2 - p_1}{2(l_1 + l_2 - y_1 - y_2)} \quad (20)$$

Demand for consumers located on empty streets depends on the spatial combination of Enterprises and their symmetric or asymmetric positions. First, the case will be considered where the Enterprises are symmetric with respect to the center.

Demand if the Enterprises are Asymmetric

In this case, the only decision-making variable for consumers on empty streets is the commodity price, and the Enterprise with the lowest price will attract all consumers on the empty streets. For the same prices, it is assumed that the consumers on the empty streets are equally distributed between Enterprises; thus, we have:

$$q_1 = \frac{2}{N} \left(1 - \frac{1}{2} (l_1 + l_2) + \frac{1}{2} (y_1 - y_2) + \frac{p_2 - p_1}{2(l_1 + l_2 - y_1 - y_2)} \right) + \frac{N-2}{N} \quad (21)$$

if $p_1 < p_2$

$$q_2 = \frac{2}{N} \left(1 - \frac{1}{2} (l_1 + l_2) + \frac{1}{2} (y_2 - y_1) + \frac{p_1 - p_2}{2(l_1 + l_2 - y_1 - y_2)} \right) \quad \text{if } p_1 < p_2 \quad (22)$$

$$q_1 = q_2 = \frac{2}{N} \left(1 - \frac{1}{2} (l_1 + l_2) + \frac{1}{2} (y_1 - y_2) + \frac{p_2 - p_1}{2(l_1 + l_2 - y_1 - y_2)} \right) + \frac{N-2}{2N} x_{1j} \quad (23)$$

if $p_1 = p_2$

In the full symmetry condition ($p_1 = p_2$, $l_1 - y_1 = l_2 - y_2$), we have:

$$\frac{\partial q_1}{\partial p_1} = \frac{1}{Nt(y_1 + y_2 - l_1 - l_2)} - \frac{N-2}{2N} = \frac{\partial q_2}{\partial p_2}$$

Thus:

Theorem 4

Assuming we have the number of streets and Enterprise locations, an Enterprise could increase its production by increasing its price. Moreover, assuming we have the number of streets, it is possible to determine the location of Enterprises, and assuming we have the special location, it is possible to decide on the production increase or decrease having the number of streets.

Proof

Insofar as:

$$\frac{\partial q_1}{\partial p_1} = \frac{1}{Nt(y_1 + y_2 - l_1 - l_2)} - \frac{N-2}{2N} = \frac{\partial q_2}{\partial p_2}$$

Solving with respect to y_1 :

$$t = 1 \Rightarrow \frac{1}{N(y_1 + y_2 - l_1 - l_2)} - \frac{N-2}{2N} > 0 \quad (24)$$

if $y_1 < (l_1 + l_2) - y_2$, N

$$> 0, \text{ if } \left(N > 2, y_1 > \frac{y_2(N-2) - N(l_1 + l_2) + 2(l_1 + l_2 - 1)}{2 - N} \right)$$

Or,

$y_1 > (l_1 + l_2) - y_2$, N

$$> 0, \text{ if } \left(N > 2, y_1 < \frac{y_2(N-2) - N(l_1 + l_2) + 2(l_1 + l_2 - 1)}{2 - N} \right)$$

Solving in terms of N :

$$\frac{1}{N(y_1 + y_2 - l_1 - l_2)} - \frac{N-2}{2N} > 0$$

if $y_1 + y_2 - l_1 - l_2 < 0$, $N > 0$, if $(y_1 + y_2 - l_1 - l_2 < 0$, $N <$

$$\frac{2(y_1 + y_2 - l_1 - l_2 + 1)}{y_1 + y_2 - l_1 - l_2})$$

$y_1 + y_2 - l_1 - l_2 < 0, N > 0$, if $(y_1 + y_2 - l_1 - l_2 > 0$, $N >$

$$\frac{2(y_1 + y_2 - l_1 - l_2 + 1)}{y_1 + y_2 - l_1 - l_2})$$

$y_1 + y_2 - l_1 - l_2 > 0$, $N > 0$, if $(y_1 + y_2 - l_1 - l_2 < 0$, $N >$

$$\frac{2(y_1 + y_2 - l_1 - l_2 + 1)}{y_1 + y_2 - l_1 - l_2})$$

$$y_1 + y_2 - l_1 - l_2 > 0, \quad N > 0, \quad \text{if } \left(y_1 + y_2 - l_1 - l_2 > 0, \quad N < \frac{2(y_1 + y_2 - l_1 - l_2 + 1)}{y_1 + y_2 - l_1 - l_2} \right)$$

With these conditions, it is possible to show the decision-making terms for Enterprises on entering or exiting the market or changing the strategy of Enterprise location. In other words, assuming we have the number of streets, the location, and a certain location with the number of streets, it is possible to decide on production. Now, the next question is whether the results will change if the positions of Enterprises are not symmetric, which will be examined in the following.

Demand if the Enterprises' Location is Asymmetric

A consumer located on point x on the empty street j is indifferent between purchasing from Enterprises 1 and 2 when:

$$x_{12}^j = \frac{1}{2(y_1 - y_2 + l_2 - l_1)} [p_2 - p_1 + (y_2 - l_2)^2 - (y_1 - l_1)^2] + l_s \quad (25)$$

Without loss of generality, it is assumed that Enterprise 1 is closer to the center ($l_1 - y_1 < l_2 - y_2$). Given the convex transportation cost function, the consumer who is closer to the center than the indifferent consumer purchases from Enterprise 2, and the farther consumer will purchase from Enterprise 1. If $x_{12}^j < 0$, all consumers on the empty streets purchase from Enterprise 2 (Lijesen and Reggiani, 2013). The demand for all empty streets is obtained by multiplying the demand for one street by the total number of empty streets $\frac{2(N-2)}{N}$:

$$q_1 = \frac{2}{N} \left(1 - \frac{1}{2}(l_1 + l_2) + \frac{1}{2}(y_1 - y_2) + \frac{p_2 - p_1}{2(l_1 + l_2 - y_1 - y_2)} \right) - \frac{2}{2(y_1 - y_2 + l_2 - l_1)} (p_1 - p_2 + (y_1 - l_1)^2 - (y_2 - l_2)^2 + l_s) \frac{N-2}{N} \quad (26)$$

$$q_2 = \frac{2}{N} \left(1 - \frac{1}{2}(l_1 + l_2) + \frac{1}{2}(y_2 - y_1) + \frac{p_1 - p_2}{2(l_1 + l_2 - y_1 - y_2)} \right) + \left(1 + \frac{2}{2(y_1 - y_2 + l_2 - l_1)} (p_1 - p_2 + (y_1 - l_1)^2 - (y_2 - l_2)^2 - l_s) \frac{N-2}{N} \right) \quad (27)$$

Using Equations 26 and 27, it is possible to present theorem 5:

Theorem 5

If the positions of Enterprises 1 and 2 (y_1 and y_2) satisfy the following conditions, then Enterprise 1 increases its production by increasing its price. Moreover, the increasing number of streets leads to a decrease in the enterprise's production, and production will be zero when $N = \infty$.

Proof

Differentiating Equation 26, we have:

$$\frac{\partial q_1}{\partial p_1} = -\frac{2}{N(2l_1 + 2l_2 - 2y_1 - 2y_2)} - \frac{N-2}{N(y_1 - y_2 + l_2 - l_1)} > 0 \Leftrightarrow$$

$$y_1 > -\frac{-y_2 + l_2 + 3l_1 - Nl_1 - Nl_2 + Ny_2}{-3 + N} \quad (28)$$

$$\frac{\partial q_1}{\partial p_1} = -\frac{2}{N(2l_1 + 2l_2 - 2y_1 - 2y_2)} - \frac{N-2}{N(y_1 - y_2 + l_2 - l_1)} > 0 \Leftrightarrow$$

$$y_2 > -\frac{-3y_1 + l_2 + 3l_1 - Nl_1 - Nl_2 + Ny_1}{-1 + N} \quad (29)$$

$$\frac{\partial q_1}{\partial p_2} = \frac{2}{N(2l_1 + 2l_2 - 2y_1 - 2y_2)} + \frac{N-2}{N(y_1 - y_2 + l_2 - l_1)} > 0 \Leftrightarrow$$

$$y_1 > -\frac{-y_2 + l_2 + 3l_1 - Nl_1 - Nl_2 + Ny_2}{-3 + N} \quad (30)$$

$$\frac{\partial q_1}{\partial p_2} = \frac{2}{N(2l_1 + 2l_2 - 2y_1 - 2y_2)} + \frac{N-2}{N(y_1 - y_2 + l_2 - l_1)} > 0 \Leftrightarrow$$

$$y_2 > -\frac{-3y_1 + l_2 + 3l_1 - Nl_1 - Nl_2 + Ny_1}{-1 + N} \quad (31)$$

$$\frac{\partial q_1}{\partial N} = (2y_2^2 - 2l_1^2 + 4l_1y_1 + 2l_2^2 - 4l_2y_2 - 2y_1^2 - 6l_2y_1y_2 + 6l_1l_2y_2 -$$

$$6l_1l_2y_1 + 2l_1y_1y_2 + 3l_1^3 - 3l_2^3 - 3y_1^3 + y_2^3 - 9l_1^2y_1 - l_1^2y_2 + 3l_1^2l_2 -$$

$$3l_1l_2^2 + 9l_1y_1^2 + 3l_2^2y_1 + 7l_2^2y_2 + 3l_2y_1^2 - y_1^2y_2 + 3y_2^2y_1 - 5y_2^2l_2 - 3y_2^2l_1 -$$

$$3p_1y_1 - p_1y_2 + p_1l_2 + 3p_1l_1 + 3p_2y_1 + p_2y_2 - p_2l_2 - 3p_2l_1 + 2l_5l_1 +$$

$$2l_5l_2 - 2l_5y_1 - 2l_5y_2) / (l_1 + l_2 - y_1 - y_2) N^2 (l_1 - l_2 + y_2 - y_1) \quad (32)$$

If $N = \infty$, then $\frac{\partial q_1}{\partial N} = 0$.

These results, indicating that the increasing number of streets leads to decreased production of enterprises, confirm Cournot's results in the N-seller game, where with an increase in the number of enterprises, the production of each enterprise tends toward zero, and the total production of the industry becomes closer to competitive production. The price selection in the symmetric case of enterprises will be investigated in the following.

Price Selection

We begin the price selection stage from the enterprises' symmetric model on streets. Forming the profit function and then differentiating Equation 23, it becomes possible to propose Theorem 6:

Theorem 6

If the location of Enterprises is as $y_1 = y_2$ or $l_1 - y_1 = y_2 - l_2$, there would be a Nash equilibrium in prices, where $p_1 = p_2$.

Proof

Forming the profit function and then the first-order conditions and solving and simplifying Equation 23, we have:

$$p_1 \left(\frac{1}{N(y_1 + y_2 - l_1 - l_2)} - \frac{N-2}{2N} \right) + \frac{2}{N} \left(1 - \frac{1}{2}(l_1 + l_2) + \frac{1}{2}(y_1 - y_2) + \frac{p_2 - p_1}{2(l_1 + l_2 - y_1 - y_2)} \right) + \frac{N-2}{2N} = 0 \quad (33)$$

Solving this equation and simplifying it would yield the following results:

$$p_1 = \frac{2p_2 + y_1(4l_1 + 4l_2 - 2) - 2y_2 + 2y_2^2 - 2y_1^2 + N(l_1 + l_2 - y_1 - y_2) - 2(l_1^2 + l_2^2 + 2l_1l_2) + 2l_1 + 2l_2}{4 - 2(l_1 + l_2 - y_1 - y_2) + N(l_1 + l_2 - y_1 - y_2)}$$

$$p_2 = \frac{2p_1 + y_2(4l_1 + 4l_2 - 2) - 2y_1 - 2y_2^2 + 2y_1^2 + N(l_1 + l_2 - y_1 - y_2) - 2(l_1^2 + l_2^2 + 2l_1l_2) + 2l_1 + 2l_2}{4 - 2(l_1 + l_2 - y_1 - y_2) + N(l_1 + l_2 - y_1 - y_2)}$$

$$\Rightarrow p_1 - p_2 = - \frac{y_2(4l_1 + 4l_2 - 2) - y_1(4l_1 + 4l_2 - 2) - 4y_2^2 + 4y_1^2 - 2y_1 + 2y_2}{N(l_1 + l_2 - y_1 - y_2) - 2l_2 + 6 - 2l_1 + 2y_1 + 2y_2}$$

This difference will be zero with respect to $y_1 = y_2$ or $l_1 - y_1 = y_2 - l_2$. Nicholas (1989) studied the presence of equilibrium and optimality in a market for the differentiation of products in terms of their diversity. It was shown that subgame perfect equilibrium was established in a three-stage game, where the enterprises entered in the first stage and diversity and price were chosen in the second and third stages, respectively. In equilibrium, the products are symmetrically distributed based on their features, and equal prices will be offered. These results confirm the results obtained by Shahbazi and Salimian (2017), who showed that in a three-stage game where the firms entered in the first stage, chose diversity in the second stage and prices in the third stage, there was a subgame perfect equilibrium. In equilibrium, the products are distributed based on their symmetric specifications, and the same prices are presented. Moreover, Lijesen and Reggiani (2013) achieved similar results and showed that

if the situations of two firms were symmetric, they would both gain the same market share and receive similar prices.

Now, the issue of price selection in the asymmetric positions of Enterprises will be studied. Theorem 7 is obtained by differentiating Equations 26 and 27.

Theorem 7

If $l_1 - y_1 < l_2 - y_2$, the price Nash equilibrium will be presented and explained as follows:

$$p_1 = - \left((y_1^3(N-3) + y_1^2(y_2(N-1) - N(3l_1 + l_2 + 1) + 11l_1 + 5l_2 - 6) - y_1(y_2^2(N-3) + 2y_2(N(l_1 - l_2) - l_1 + 3l_2) - N(3l_s + 3l_1^2 + 2l_1(l_2 + 1) - l_2^2) + 6l_s + 13l_1^2 + 2l_1(5l_2 - 6) - 3l_2^2) + y_2^3(1-N) + y_2^2(N(l_1 + 3l_2 + 1) - 5l_1 - 7l_2 + 6)) + y_2(N(3l_s + l_1^2 - 2l_1l_2 - l_2(3l_2 + 2)) - 6l_s - l_1^2 + 10l_1l_2 + l_2(11l_2 - 12)) - (l_1 + l_2)(N(3l_s + l_1^2 + l_1 - l_2(l_2 + 1)) - 6l_s - 5l_1^2 + 6l_1 + l_2(5l_2 - 6)) \right) / (3(y_1(N-3) + y_2(N-1) - N(l_1 + l_2) + 3l_1 + l_2)) \quad (34)$$

$$p_2 = \left((y_1^3(N-3) + y_1^2(y_2(N-1) - N(3l_1 + l_2 - 2) + 5l_1 - l_2 + 6) - y_1(y_2^2(N-3) + 2y_2(N(l_1 - l_2) - l_1 + 3l_2) + N(3l_s - 3l_1^2 + 2l_1(2 - l_2) + l_2^2) - 6l_s + l_1^2 + 2l_1(6 - l_2) - 3l_2^2) + y_2^3(1-N) + y_2^2(N(l_1 + 3l_2 - 2) + l_1 - l_2 - 6)) - y_2(N(3l_s - l_1^2 + 2l_1l_2 + l_2(3l_2 - 4)) - 6l_s + l_1^2 + 2l_1l_2 + l_2(l_2 - 12)) + (l_1 + l_2)(N(3l_s - l_1^2 + 2l_1 + l_2(l_2 - 2)) - 6l_s - l_1^2 + 6l_1 + l_2(l_2 - 6)) \right) / (3(y_1(N-3) + y_2(N-1) - N(l_1 + l_2) + 3l_1 + l_2)) \quad (35)$$

Proof

The profit functions of Enterprises according to Equations 26 and 27 are as follows:

$$\pi_1 = \left(p_1 \left(\frac{2}{N} \left(1 - \frac{1}{2}(l_1 + l_2) + \frac{1}{2}(y_1 - y_2) + \frac{p_2 - p_1}{2(l_1 + l_2 - y_1 - y_2)} \right) - \frac{1}{(y_1 - y_2 + l_2 - l_1)} \left((p_1 - p_2 + y_1^2 + l_1^2 - 2y_1 l_1 - y_2^2 - l_2^2 + 2y_2 l_2) + l_s \right) \frac{(N-2)}{N} \right) \right)$$

$$\pi_2 = p_2 \left(\frac{2}{N} \left(1 - \frac{1}{2}(l_1 + l_2) + \frac{1}{2}(y_2 - y_1) + \frac{p_1 - p_2}{2(l_1 + l_2 - y_1 - y_2)} \right) \right) + p_2 \left(1 + \frac{1}{(y_1 - y_2 + l_2 - l_1)} \left((p_1 - p_2 + y_1^2 + l_1^2 - 2y_1 l_1 - y_2^2 - l_2^2 + 2y_2 l_2) - l_s \right) \frac{(N-2)}{N} \right)$$

Considering the above functions and the **first-order condition**, p_1 and p_2 will be obtained after simplification, as explained in Theorem 7.

Through analysis of a standard price-location game (first value and then price), Simon et al. (1997) showed a unique strategy equilibrium in a two-stage game by consumers' density as log-concave. If the density is neither "too asymmetric" nor "too concave", the equilibrium locations will be closer, equilibrium prices will be lower, and density higher. In a symmetric density that is too concave, there is no symmetric equilibrium, although it might be in an asymmetric state.

The above results are in line with the results obtained by Lijesen and Reggiani with respect to $l_1 = l_2 = \frac{1}{2}$. In the following, the location will be obtained solving the game through the backward induction method (after replacing equilibrium prices). First, the case where all enterprises have symmetric positions toward the center will be studied.

Location Selection

Now we step back one stage and calculate the Nash equilibrium of the first stage of the game concerning Nash equilibrium of the second stage. The decision variable in the first and second stages is location determination and price determination, respectively. Here, first the symmetric case of the enterprises' location will be initially examined, followed by their asymmetric case.

Theorem 8

If $N > \frac{4(l_1 + l_2 - 1)}{2l_1 + 2l_2 - 1}$, then there is no symmetric Nash equilibrium in the location.

Proof:

The profit function of Enterprise 1 according to Equation 23 will be as follows:

$$\pi_1 = p_1 \left(\frac{2}{N} \left(1 - \frac{1}{2}(l_1 + l_2) + \frac{1}{2}(y_1 - y_2) + \frac{p_2 - p_1}{2(l_1 + l_2 - y_1 - y_2)} \right) + \frac{(N-2)}{2N} \right) \quad (36)$$

Replacing the equilibrium prices and then the first-order conditions, the following results will be obtained:

$$y_1 = \frac{1}{8-4N} \left(16y_2^2(N-2)^2 + 8y_2(2-N)(N(4l_1+4l_2-1) - 8(l_1+l_2-1)) + N^2(16l_1^2 + 8l_1(4l_2-1) + 16l_2^2 - 8l_2 - 47) - 16N(4l_1^2 + l_1(8l_2-9) + l_2(4l_2-9)) + 64(l_1^2 + 2l_1(l_2-2) + l_2^2 - 4l_2 + 3) \right)^{\frac{1}{2}} - N$$

$$y_2 = \frac{1}{8-4N} \left(16y_1^2(N-2)^2 + 8y_1(2-N)(N(4l_1+4l_2-1) - 8(l_1+l_2-1)) + N^2(16l_1^2 + 8l_1(4l_2-1) + 16l_2^2 - 8l_2 - 47) - 16N(4l_1^2 + l_1(8l_2-9) + l_2(4l_2-9)) + 64(l_1^2 + 2l_1(l_2-2) + l_2^2 - 4l_2 + 3) \right)^{\frac{1}{2}} - N$$

After simplifying the above expressions, the Enterprises' equilibrium location will be obtained as follows:

$$y_1 = \frac{N(2l_1^2 + l_1(4l_2-1) + 2l_2^2 - l_2 - 6) - 4(l_1^2 + 2l_1(l_2-2) + l_2^2 - 4l_2 + 3)}{2(N(2l_1 + 2l_2 - 1) - 4(l_1 + l_2 - 1))} \quad (37)$$

$$y_2 = \frac{N(2l_1^2 + l_1(4l_2-1) + 2l_2^2 - l_2 - 6) - 4(l_1^2 + 2l_1(l_2-2) + l_2^2 - 4l_2 + 3)}{2(N(2l_1 + 2l_2 - 1) - 4(l_1 + l_2 - 1))} \quad (38)$$

These results show that $y_1 - y_2 = 0$. If $N > \frac{4(l_1+l_2-1)}{2l_1+2l_2-1}$, then $y_1 > l_1$, and the same condition will hold for Enterprise 2. In the Lijesen and Reggiani model and for the special case of $l_1 = l_2 = \frac{1}{2}$, which is the maximum value, the result will be $-\frac{5}{2}$, which is out of range $[0, l_i]$. These results indicate that the symmetric case of enterprises is not a Nash equilibrium when the number of streets is more than the above expression (and in the spokes model, more than two streets) since the enterprises should be located on the intended street. In this case, both enterprises will be located in the same place (center), and since each enterprise could increase its profit by deviating from its location, there is no symmetric Nash equilibrium. These results are consistent with the results obtained by Lijesen and Reggiani (2013).

Selection of Price and Location

In a game with sequential moves with two enterprises, the enterprises initially select their location, and then the price will be selected at the next stage by

observing the selected location. If enterprises select price and location strategies, the following results will be obtained:

Theorem 9

If there are just two streets of unequal lengths, and the location of Enterprises is symmetric, $l_1 - y_1 = l_2 - y_2$, there is a Bertrand Nash equilibrium in prices. However, symmetric Nash equilibrium doesn't exist in location, and one of the enterprises moves toward the center and the other moves away.

Proof

Concerning the demand functions of Enterprises 1 and 2 (according to Equations 9 and 10) and the assumption that one of the Enterprises is located on a street where the indifferent consumer is located, the following results will be achieved:

$$p_1 = \frac{4}{3}l_1y_1 - \frac{2}{3}l_1l_2 - \frac{2}{3}l_1y_2 + \frac{4}{3}l_2y_1 - \frac{2}{3}l_2y_2 - \frac{1}{3}l_1^2 - \frac{1}{3}l_2^2 - y_1^2 + y_2^2$$

$$p_2 = \frac{2}{3}l_1y_1 + \frac{2}{3}l_1l_2 - \frac{4}{3}l_1y_2 + \frac{2}{3}l_2y_1 - \frac{4}{3}l_2y_2 + \frac{1}{3}l_1^2 + \frac{1}{3}l_2^2 - y_1^2 + y_2^2$$

$$q_1 = \frac{1}{6}l_1 + \frac{1}{6}l_2 - \frac{1}{2}y_1 + \frac{1}{2}y_2$$

$$q_2 = \frac{1}{6}l_1 + \frac{1}{6}l_2 + \frac{1}{2}y_1 - \frac{1}{2}y_2$$

Thus,

$$\pi_1 = \frac{2}{9}l_1l_2y_2 - \frac{1}{18}l_1^3 - \frac{1}{18}l_2^3 - \frac{1}{6}l_1l_2^2 - \frac{1}{6}l_1^2l_2 + \frac{1}{9}l_1^2y_2 + \frac{1}{9}l_2^2y_2 \quad (39)$$

$$\pi_1 = \frac{2}{9}l_1l_2y_1 - \frac{1}{18}l_1^3 - \frac{1}{18}l_2^3 - \frac{1}{6}l_1l_2^2 - \frac{1}{6}l_1^2l_2 + \frac{1}{9}l_1^2y_1 + \frac{1}{9}l_2^2y_1 \quad (40)$$

$$\pi_2 = -\frac{2}{9}l_1l_2y_2 + \frac{1}{18}l_1^3 + \frac{1}{18}l_2^3 + \frac{1}{6}l_1l_2^2 + \frac{1}{6}l_1^2l_2 - \frac{1}{9}l_1^2y_2 - \frac{1}{9}l_2^2y_2 \quad (41)$$

$$\pi_2 = -\frac{2}{9}l_1l_2y_1 + \frac{1}{18}l_1^3 + \frac{1}{18}l_2^3 + \frac{1}{6}l_1l_2^2 + \frac{1}{6}l_1^2l_2 - \frac{1}{9}l_1^2y_1 - \frac{1}{9}l_2^2y_1 \quad (42)$$

Based on the obtained results, enterprise 1 is inclined to move closer to the center for a given location of it or enterprise 2, and enterprise 2 is inclined to move farther from the center with respect to a given location of it or enterprise 1. These results could be easily shown through the Hotelling linear model (1929). In the Hotelling linear model, if the length of streets is equal and the two enterprises are located at the same point, then $p_1 = p_2 = 0$ is the unique equilibrium. When the two enterprises are located very close to each other, they begin undercutting

each other's prices, and equilibrium will not be achieved as a result of this price-breaking process. In this case, the enterprises are inclined toward distancing from each other, which is the same as Hotelling's results that say:

In case some merchants make the transportation problematic instead of establishing road maintenance or enhancing the roads' reconstruction organizations, they have done a good job (Shy, 1995).

Now, the question arises whether the game will have equilibrium in the presence of streets of unequal lengths if the strategies of enterprises are the selection of both price and location.

Theorem 10

In the Hotelling linear city game (with two unequal streets), when the strategies of enterprises are selecting both price and location, there would be no equilibrium for the game.

Proof

To prove this theorem, we ask how Enterprise 1 would behave if the price and location of the competitive enterprise to enterprise 1 are given and enterprise 1 is allowed to choose its location again. Considering Equations 29 and 40, the results will be simply obtained:

$$\pi_1 = \frac{2}{9}l_1l_2y_2 - \frac{1}{18}l_1^3 - \frac{1}{18}l_2^3 - \frac{1}{6}l_1l_2^2 - \frac{1}{6}l_1^2l_2 + \frac{1}{9}l_1^2y_2 + \frac{1}{9}l_2^2y_2$$

$$\pi_1 = \frac{2}{9}l_1l_2y_1 - \frac{1}{18}l_1^3 - \frac{1}{18}l_2^3 - \frac{1}{6}l_1l_2^2 - \frac{1}{6}l_1^2l_2 + \frac{1}{9}l_1^2y_1 + \frac{1}{9}l_2^2y_1$$

The results show that the profit of enterprise 1 increases while moving toward Enterprise 2, which is the principle of minimum differentiation. The obtained results confirm Bulow, Geanakopolos, and Klemperer (1985), who showed that when the products become homogeneous, the profit level of all enterprises reduced to zero (the principle of minimum differentiation). On the other hand, if enterprises became so close to each other, there would be no equilibrium, and if Enterprise 1 was exactly located at the same point as Enterprise 2, its profit would be zero, indicating that it would be better for Enterprise 1 to distance itself from Enterprise 2.

Kreps and Scheinkman (1983) presented a special framework (a special two-stage dynamic game), where the enterprises would select their production quantity in the first period (accumulated inventory). Since the production quantity was fixed in the first period, and there was no possibility of its change, the enterprises would select the price in the second period. They showed that the

selected values by the enterprises in the first stage and the selected prices in the second stage were exactly equal to Cournot's results. They also showed that in some markets, in those games where two enterprises selected their production quantity in the first stage and then determined their prices in the second stage, SPE was equivalent to the production quantity and price in a single-stage game of Cournot structure, where the enterprises just determined their production quantity.

Nicholas (1986) showed that firstly, the principle of minimum differentiation did not hold in Hotelling price-location, since the subgame perfect equilibrium in the long term did not exist with the minimum differentiation of products. Secondly, it contradicts the principle of maximum differentiation and is totally incorrect. Although maximum product differentiation holds for most convex transportation cost functions, there is a wide range of utility functions whose complete equilibrium location is represented by inner points of product space. Now, we examine the effect of the length and number of streets on the profit and positions of enterprises.

The Length and Number of Streets

Since the length and number of streets are the main issues in determining the profit and location of enterprises, and specifically affect enterprises' profit, it is possible to achieve the following results from the profit function:

Theorem 11

As the length of streets increases, the enterprises' profit increases if:

$$y_1 + y_2 - \sqrt{p_1 - p_2} - l_2 \leq l_1 \leq y_1 + y_2 + \sqrt{p_1 - p_2} - l_2 \quad (43)$$

and

$$y_1 + y_2 - \sqrt{p_1 - p_2} - l_1 \leq l_2 \leq y_1 + y_2 + \sqrt{p_1 - p_2} - l_1 \quad (44)$$

Proof

According to Equation 36:

$$\frac{\partial \pi_1}{\partial l_1} = \frac{p_1 (y_1^2 + 2y_1(y_2 - l_1 - l_2) + y_2^2 - 2y_2(l_1 + l_2) - p_1 + p_2 + l_1^2 + 2l_1l_2 + l_2^2)}{N(1-N)(y_1 + y_2 - l_1 - l_2)^2}$$

The denominator is always negative for $N \geq 2$. For the fraction to be positive, it is sufficient to have a negative numerator, leading to the following result:

$$y_1 + y_2 + \sqrt{p_1 - p_2} - l_2 \leq l_1 \leq y_1 + y_2 - \sqrt{p_1 - p_2} - l_2$$

or

$$y_1 + y_2 - \sqrt{p_1 - p_2} - l_2 \leq l_1 \leq y_1 + y_2 + \sqrt{p_1 - p_2} - l_2$$

The following results will be obtained from differentiation from the profit function of Enterprise 1 with respect to l_2 :

$$y_1 + y_2 + \sqrt{p_1 - p_2} - l_1 \leq l_2 \leq y_1 + y_2 - \sqrt{p_1 - p_2} - l_1$$

or

$$y_1 + y_2 - \sqrt{p_1 - p_2} - l_1 \leq l_2 \leq y_1 + y_2 + \sqrt{p_1 - p_2} - l_1$$

It is obvious that only the second condition can hold. Thus, if these conditions hold, Enterprise 1 gains more profit concerning the increased length of streets. Assuming single-product enterprises, Maniez and Waterson (2001) investigated the outcomes of horizontal and vertical product differentiation in market structure and showed that:

- 1- The insider price elasticity is decreasing relative to quality,
- 2- The cross-price elasticity is decreasing relative to quality, and
- 3- Mark-ups are increasing relative to quality. These results could be expressed as the maximum differentiation where the distance from ideal feature is to be far from it. The last theorem is presented as follows:

Theorem 12

With an increase in the number of streets, the enterprises select a location farther from the center (maximum differentiation) unless under certain conditions, where the enterprises select the location closer to the center (minimum differentiation).

$$1 - l_1 \leq l_2 \leq \frac{3}{2} - l_1$$

Proof:

According to Equations 37 and 38, we have:

$$\begin{aligned} \frac{\partial y_1}{\partial N} &= - \frac{6(2l_1^2 + l_1(4l_2 - 5) + 2l_2^2 - 5l_2 + 3)}{(N(2l_1 + 2l_2 - 1) - 4(l_1 + l_2 - 1))^2} = \frac{\partial y_2}{\partial N} = 0 \\ \Rightarrow N &= \infty \quad 2l_1^2 + l_1(4l_2 - 5) + 2l_2^2 - 5l_2 = -3 \end{aligned} \quad (45)$$

$$\frac{\partial y_1}{\partial N} = - \frac{6(2l_1^2 + l_1(4l_2 - 5) + 2l_2^2 - 5l_2 + 3)}{(N(2l_1 + 2l_2 - 1) - 4(l_1 + l_2 - 1))^2} = \frac{\partial y_2}{\partial N} \leq 0$$

Since this equation will be positive just under the above conditions, with an increase in the number of streets, the enterprises select farther points from the center as their optimal location. In case the above conditions hold, with an increase in the number of streets, the enterprises select the closer points to the center as their optimal location. The results of this section are similar to vertical

differentiation and the results of Maniez and Waterson, as discussed in Theorem 12. Vandenbosch and Weinberg (1995) extended the vertical differentiation model of Shaked and Sutton (1982) and Moutry (1988), expanding it to two dimensions and analyzing price and quantity competition. They showed that unlike the vertical differentiation model, the enterprises were not inclined toward maximum differentiation. Although this solution might hold under certain conditions, when a wide range of locations in every dimension is equal, the max-min product differentiation will occur. In equilibrium, the enterprises are inclined to select the location, which shows maximum differentiation in one dimension and minimum differentiation in another.

Conclusion and Recommendation

One of the key and effective issues in the implementation and optimization of projects is to determine an appropriate location for them, which requires recognition and identification of the basic parameters and structures in the studied areas. The establishment of any urban element in a specific spatial-physical location of a city is subject to certain principles, rules, and mechanisms that if respected, the success and functional efficiency of that element will be greater at the exact location; otherwise, there will be many problems. Location selection is the selection of a location for one or more centers while considering other centers and existing constraints to optimize a specific objective. This objective could be transportation cost, gaining more profit, providing fair services to customers, taking the largest market share, etc. These issues and the review of literature on locating show that most studies have used simplifying assumptions for locating. One of the main assumptions is about circular cities and linear cities with streets of equal length. It is obvious that in the real world, the lengths of streets are not necessarily equal and may be affected by many factors. This paper expanded Chen and Riordan's initial model and its extended version by Lijesen and Reggiani considering streets of unequal lengths. One of the main results of this paper is keeping the results of Lijesen and Reggiani with respect to the streets' length of $\frac{1}{2}$ in all cases, indicating that Lijesen and Reggiani's model is a special case of a more general model of streets of unequal length. In addition to confirming all results of Lijesen and Reggiani for streets of unequal length, the following results have been obtained:

- 1- Concerning Theorem 1, the spokes model is a special form of the unequal-length streets model (just in special cases where all streets are of equal length and almost $\frac{1}{2}$).
- 2- Concerning Theorems 3, 4, 5, and 12, the "increasing number of streets affects the location and profit of enterprises".
- 3- Concerning Theorem 5, "with an increase in the number of streets, the production of an enterprise would decrease, and when the number of enterprises tends to infinity, it becomes closer to competitive outcome).
- 4- Concerning Theorem 1, "in the unequal-length streets model, there is a unique Nash equilibrium in prices, and this equilibrium price is directly related to the number of streets".
- 5- Concerning Theorem 6, "if the distance of enterprises from the center is equal, there is a Nash equilibrium in prices where the enterprises charge the same price".
- 6- Concerning Theorem 3, "the increase in the number of empty streets of enterprise, the enterprises select locations farther from the center".
- 7- Concerning Theorem 9, "if there are just two streets of unequal lengths and the location of enterprises is symmetric, a Bertrand Nash equilibrium would exist in prices, but symmetric Nash equilibrium doesn't exist".
- 8- Concerning Theorem 10, "if there are two streets of unequal lengths and the strategies of enterprises are the selection of both price and location, there would be no equilibrium for the game".

Briefly speaking, since cities consist of streets of unequal lengths in the real world, the enterprises could increase their prices with an increase in the number and length of streets, moving farther from the center, and increasing the price by the other enterprise. Moreover, the enterprises are recommended to move toward the center as the number of streets increases.

In the end, since there are many simplifying assumptions in locating models, and the assumption of uniform distribution of consumers is maintained, it is recommended to replace this assumption with more realistic assumptions, such as statistical distributions. In addition, various issues, such as current locations of enterprises, including future changes and revolutions, such as threats, opportunities, demand growth, lack of sufficient information by the enterprises, environmental factors such as political pressures, disturbance of balance, and many other factors could be effective in enterprises' locating. Thus, it is

recommended to use these factors within the framework of dummy variables in future studies.

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References

- Ahrens, A., & Lyons, S. (2021). Do rising rents lead to longer commutes? A gravity model of commuting flows in Ireland. *Urban Studies*, 58(2), 264-279. Retrieved from <https://doi.org/10.1177/0042098020910698>.
- Anderson, S. P., & Neven D. J. (1986). *Spatial Competition a la Cournot*, Charles WYLOSZ, Associate Dean for Research and Development. France: INSEAD.
- Anderson, S. P., & Neven, D. J. (1991). Cournot competition yields spatial agglomeration. *International Economic Review*, 32(4), 793–808. Retrieved from <https://doi.org/10.2307/2527034>.
- Baraklianos, I., Bouzouina, L., Bonnel, P., & Aissaoui, H. (2020). Does the accessibility measure influence the results of residential location choice modelling? *Transportation*, 47(3), 1147-1176. Retrieved from <https://doi.org/10.1007/s11116-018-9964-6>.
- Bautista, J., & Pereira, J. (2007). A GRASP Algorithm to Solve the Unicost Set Covering Problem. *Computers and Operation Research*, 34(10), 3162-3173. Retrieved from <https://doi.org/10.1016/j.cor.2005.11.026>.
- Behrens, K., Mion, G., Murata, Y., & Südekum, J. (2017). Spatial frictions. *Journal of Urban Economics*, 97, 40–70. Retrieved from <https://doi.org/10.1016/j.jue.2016.11.003>.
- Burdurla, E., & Ejder, E. (2003). Location Choice for Furniture Industry Firms by using Analytical Hierarchy Process (AHP) Method. *Journal of Science*, 16(2), 369-373.

Chen, Y., & Riordan, H. M. (2007). Price and Variety in the Spokes Model. *The Economic Journal*, 117(522), 897-921. Retrieved from <https://www.jstor.org/stable/4625536>.

Choo, S., Lee, T., & Mokhtarian, P. L. (2010). Do Transportation and Communications Tend to Be Substitutes, Complements, or Neither? U.S. Consumer Expenditures Perspective, 1984–2002. In *Transportation Research Record: Journal of the Transportation Research Board*, 2010, 121–132. Retrieved from <https://doi.org/10.3141/2010-14>.

Choo, S., & Mazzarol, T. (2003). A Study of the Factors Influencing the Operating Location Decisions of Small Firms. *Journal of Property Management*, 21(2), 190–208.

Currie, G., & Senbergs, Z. (2007). Exploring forced car ownership in metropolitan Melbourne. In *Australasian transport research forum (atr)*, Melbourne, Australia.

d'Aspremont, C., Gabszewicz, J. J., & Thisse, J. F. (1979). On Hotelling's "Stability in Competition". *Econometrica*, 47(5), 1145–1150. Retrieved from <https://doi.org/10.2307/1911955>.

Deka, D. (2015). Relationship between Households' Housing and Transportation Expenditures: Examination from Lifestyle Perspective. *Transportation Research Record*, 2531(1), 26-35. Retrieved from <https://doi.org/10.3141/2531-04>.

Dewita, Y., Burke, M., & Yen, B. T. (2020). The relationship between transport, housing and urban form: Affordability of transport and housing in Indonesia. *Case Studies on Transport Policy*, 8(1), 252-262. Retrieved from <https://doi.org/10.1016/j.cstp.2019.01.004>.

Drezner, Z. (1995). *Facility Location: A Survey of Applications and Methods* (1st Ed.). Retrieved from <https://link.springer.com/book/9780387945453>.

Duranton, G., & Puga, D. (2004). Micro-foundations of urban agglomeration economies. In J. V. Henderson and J. F. Thisse (Eds.), *Handbook of Regional and Urban Economics*, 4, 2063–2117. Retrieved from [https://doi.org/10.1016/S1574-0080\(04\)80005-1](https://doi.org/10.1016/S1574-0080(04)80005-1).

Fujita, M., Krugman, P., & Venables, A. J. (1999). *The Spatial Economy: Cities, Regions and International Trade*. Cambridge, MA: MIT Press.

Gelormino, E., Melis, G., Marietta, C., & Costa, G. (2015). From built environment to health inequalities: An explanatory framework based on evidence. *Preventive Medicine Reports*, 2, 737–745. Retrieved from <https://doi.org/10.1016/j.pmedr.2015.08.019>.

Haas, P., Morse, S., Becker, S., Young, L., & Esling, P. (2013). The Influence of Spatial and Household Characteristics on Household Transportation Costs. *Research in Transportation Business and Management*, 7(2013), 14–26. Retrieved from <https://doi.org/10.1016/j.rtbm.2013.03.004>.

Harrison, S., & Campbell, Jr. (2021). Income and cost of living: Are less equal places more costly? *Social Science Quarterly*, 102(6), 2689-2705. Retrieved from <https://doi.org/10.1111/ssqu.13017>.

- Hosun, R., Teck, H., & Karmarkar, S. (2003). Competitive Location, Production, and Market Selection. *European Journal of Operational Research*, 149(1), 211–288. Retrieved from [https://doi.org/10.1016/S0377-2217\(02\)00445-9](https://doi.org/10.1016/S0377-2217(02)00445-9).
- Hotelling, H. (1929). Stability in Competition. *Economic Journal*, 39(1929), 41–57.
- Hsiao, C., & Hansen, M. (2011). A passenger demand model for air transportation in a hub-and-spoke network, Contents lists available at Science Direct. *Transportation Research Part E*, 47(2011), 1112–1125. Retrieved from <https://doi.org/10.1016/j.tre.2011.05.012>.
- Hu, L., & Wang, L. (2017). Housing location choices of the poor: does access to jobs matter? *Housing Studies*, 34(2019), 1–25. Retrieved from <https://doi.org/10.1080/02673037.2017.1364354>.
- Isard, W. (1956). *Location and Space-Economy: A General Theory Relating to Industrial Location, Market Areas, Land Use, Trade, and Urban Structure*. Cambridge, MA: MIT Press.
- Jeremy, I., Bulow, J. D., Geanakoplos, P., & Klemperer, D. (1985). Multimarket Oligopoly: Strategic Substitutes and Complements. *the Journal of Political Economy*, 93(3), 488–511. Retrieved from <https://doi.org/10.1086/261312>.
- Jetzkowitz, J., Schneider, J., & Brunzel, S. (2007). Suburbanisation, Mobility and the Good Life in the Country: A Lifestyle Approach to the Sociology of Urban Sprawl in Germany. *Sociologia Ruralis*, 47(2), 148–171. Retrieved from <https://doi.org/10.1111/j.1467-9523.2007.00431.x>.
- Juan, A. M., & Waterson, M. (2001). Multiproduct Firms and Product Differentiation: A Survey. *Warwick Economic Research Papers*, 594(3), 1–56. Retrieved from <https://wrap.warwick.ac.uk/24/>.
- Kahneman, D., & Tversky, A. (1979). Prospect Theory: An Analysis of Decision under Risk. *Econometrica*, 47(2), 263–291. Retrieved from <https://doi.org/10.2307/1914185>.
- Kieron, J. M., Ernie, G., Teo, S., & Taojun, X. (2014). Socially-Optimal Locations of Duopoly Firms with Non-Uniform Consumer Densities. *Theoretical Economics Letters*, 4(6), 431–445. Retrieved from <https://doi.org/10.4236/tel.2014.46055>.
- Kreps, D., & Scheinkman, J. (1983). Quantity precommitment and Bertrand competition yield Cournot outcomes, *Bell Journal of Economics*, 14(2), 326–337. Retrieved from <https://doi.org/10.2307/3003636>.
- Lijesen, M., & Reggiani, C. (2016). Market Competition and Firms Specialization Choices. 56TH ERSA Congress 2016, Vienna. Retrieved from https://www.economia.unipd.it/sites/economia.unipd.it/files/spokes_Peter.pdf.
- (2013). Location Choice in the Spoke Model. *EARIE Conference 2013*. Retrieved from <https://doi.org/10.2139/ssrn.3246996>.
- Mark, B. V., & Charles B. W. (1995). Product and Price Competition in a Two-Dimensional Vertical Differentiation Model. *Marketing Science*, 14(2), 224–249. Retrieved from <https://doi.org/10.1287/mksc.14.2.224>.

Marval, A., & Silva, A. E. (2023). City affordability and residential location choice: A demonstration using agent based model. *Habitat International*, 136(2023), 102816. Retrieved from <https://doi.org/10.1016/j.habitatint.2023.102816>.

Melaniphy, J. (1999). *Food Courts–The Food Operator's Perspective*. Retrieved from <http://www.melaniphy.com/ARTIC3.htm>.

Michael, J., Teitel, H., Granskog, J., & James E. (1998). Production facility site selection factors for texas value_added wood producers. *Forest Products Journal*, 48(7), 27-32. Retrieved from <https://search.proquest.com/openview/e9b4fe87b905eb8232c4394fbcd1218f/1?pq-origsite=gscholar&cbl=25222>.

Murata, Y. (2003). Product diversity, taste heterogeneity, and geographic distribution of economic activities: Market vs. non-market interactions. *Journal of Urban Economics*, 53(1), 126–144. Retrieved from [https://doi.org/10.1016/S0094-1190\(02\)00507-7](https://doi.org/10.1016/S0094-1190(02)00507-7).

Neven, D. J. (1986). On Hotellings competition with non-uniform customer distributions. *Economics Letters*, 21(2), 121-126. Retrieved from [https://doi.org/10.1016/0165-1765\(86\)90049-2](https://doi.org/10.1016/0165-1765(86)90049-2).

Nicholas, E. (1989a). Quality Variations and Maximal Variety Differentiation. *Regional Science and Urban Economics*, 19(1989), 21-29. Retrieved from [https://doi.org/10.1016/0166-0462\(89\)90031-8](https://doi.org/10.1016/0166-0462(89)90031-8).

----- (1989b). Symmetric Equilibrium Existence and Optimality in Differentiated Product Markets. *Journal of Economic Theory*, 47(1), 178-194. Retrieved from [https://doi.org/10.1016/0022-0531\(89\)90108-7](https://doi.org/10.1016/0022-0531(89)90108-7).

----- (1986). Minimal and Maximal Product Differentiation in Hotellig's Duopoly. *Economics Letters*, 21(1986), 67-71. Retrieved from [https://doi.org/10.1016/0165-1765\(86\)90124-2](https://doi.org/10.1016/0165-1765(86)90124-2).

O'Kelly, M. E., & Bryan, D. L. (1998). Hub Location with Flow Economies of Scale. *Transportation Research Part B: Methodological*, 32(8), 605–616. Retrieved from [https://doi.org/10.1016/S0191-2615\(98\)00021-6](https://doi.org/10.1016/S0191-2615(98)00021-6).

Pereira, H. M., Schwanen, T., & Banister, D. (2017). Distributive justice and equity in transportation. *Transport Reviews, Taylor and Francis Journals*, 37(2), 170-191. Retrieved from <https://doi.org/10.1080/01441647.2016.1257660>.

Qin, L., Feng, H., & Gang, L. (2010). A Model of Regional System with Multi-level Hub-and-spoke Network Based on Regional Economy. *2010 3rd IEEE International Conference on Computer Science and Information Technology (ICCSIT 2010)*, 1(2), 1 – 727. Retrieved from [https://doi.org/10.1061/41139\(387\)124](https://doi.org/10.1061/41139(387)124).

Redondo, J. L., Fernandez, J., Garcia, I., & Ortigosa, P. M. (2008). Parallel Algorithms for Continuous Competitive Location Problems. *Optimization Methods and Software*, 50(2), 557-573. Retrieved from <https://api.semanticscholar.org/CorpusID:35018964>.

- Reggiani, C. (2009). Optimal Differentiation and Spatial Competition: The Spoke Model with Product Delivery. *Discussion Paper*, Retrieved from <https://ideas.repec.org/p/yor/yorken/09-13.html>.
- Salimian, S., Shahbazi, K., & Movahedi, B. M. (2023). Locating the Sale Agents in Spoke Model through Uniform Distribution of Consumers. *Journal of Economic Sciences: Theory and Practice*, 80(1), 21-34. Retrieved from <https://www.ecosciences.edu.az/view.php?lang=en&menu=journal&id=436>.
- Salop, S. C. (1979). Monopolistic competition with outside goods. *Bell Journal of Economics*, 10(1), 141–156. Retrieved from <https://doi.org/10.2307/3003323>.
- Shahbazi, K., & Salimian, S. (2017). The Expansion of Location Theories of Firms and Products' Consistency Using Triangular Distribution Approach. *Iranian Economic Review*, 21(3), 497-518. Retrieved from <https://doi.org/10.22059/IER.2017.62937>.
- Shaked, A., & Sutton, J. (1982). Relaxing price competition through product differentiation. *Review of Economic Studies*, 49(1), 3-13. Retrieved from <http://hdl.handle.net/10.2307/2297136>.
- Shy, O. (2014). *Industrial Organization: Theory and Applications*. Cambridge, MA: MIT Press.
- Simon, P. A., Jacob, K. G., & Roald, R. (1997). Location, Location, Location. *Journal of Economic Theory*, 77(1), 102-127. Retrieved from <https://doi.org/10.1006/jeth.1997.2323>.
- Sumetee, W. (2011). *Logistics Strategy for Agricultural Products in Thailand, Case Study: Hub and Spoke System Applied within the Shrimp Industry* (1st Ed.). Göttingen: Cuvillier Verlag.
- Tartavulea, R. I. (2015). Model for Determining the Optimum Location for Performance Improvement in Supply-Chain Strategies. *European Journal of Interdisciplinary Studies*, 7(1), 39–54. Retrieved from <https://ejist.ro/files/pdf/392.pdf>.
- Viggers, H., & Howden-Chapman, P. (2011). Urban form, public transport and mortgage sales. Growth misconduct? *Avoiding sprawl and improving urban intensification in New Zealand*, 159–177.
- Vinay, S., & Chakravorty, V. (2005). Industrial Location and Spatial Inequality: Theory and Evidence from India, *Review of Development Economics*, Wiley Blackwell, 9(1), 47–65. Retrieved from <https://doi.org/10.1111/j.1467-9361.2005.00263.x>.
- von Thünen, J. H. (1966). *The Isolated State* (Trans. by P. Hall). Oxford: Pergamon Press (Original work published 1826).
- Weber, A. (1929). *Theory of the Location of Industries* (Trans. by C. J. Friedrich). Chicago: University of Chicago Press (Original work published 1909).
- Yang, J., & Lee, H. (1997). An AHP decision model for facility location selection. *Facilities*, 15(9), 241- 254. Retrieved from <https://doi.org/10.1108/02632779710178785>.

Zhao, C., Nielsen, T. A. S., Olafsson, A. S., Carstensen, T. A., & Meng, X. (2018). Urban form, demographic and socio-economic correlates of walking, cycling, and e-biking: Evidence from eight neighborhoods in Beijing. *Transport Policy*, 64, 102–112. Retrieved from <https://doi.org/10.1016/j.tranpol.2018.01.018>.

Zi, L., Sisi, W., Xiaonan, L., Lihua, Y., Dong, Y., & Dianshuang, W. (2012). Online shop location optimization using a fuzzy multi-criteria decision model—Case study on Taobao.com. *Knowledge-Based Systems*, 32(4), 76–83. Retrieved from <https://doi.org/10.1016/j.knosys.2011.08.015>.



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