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RESEARCH PAPER

A Robust Model for Supplier Selection and Inventory Management in a Discrete Space

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Abstract

Uncertainty in the form of demand fluctuation has been determined as one of the most affecting sources of risk in supply chain management problems. This paper aims to determine the optimal supplier selection, order allocation, and synchronizing delivery process of a single product in a supply chain with several suppliers and a single buyer considering uncertain demand. The assumption regarding the inventory scheme contributes to the complexity of the model. The proposed model considers the total cost of the whole supply chain under demand sets related to different economic situations. A robust model is then proposed using a convex nonlinear scenario-based stochastic programming with the objective of minimizing the total system expected cost. Finally, we conduct several numerical studies to evaluate the performance of the proposed model and study the effect of variation of the scenario set data on the model outputs.

Keywords:
Lot sizing, Order allocation,
Robust optimization,
Supplier selection,
Uncertain demand

Introduction

A supply chain is a network of departments, such as suppliers, production, and distribution centers involved in all movements and storage of raw materials, work-in-process inventory, and finished goods from the supplier to the end customer (Simchi-Levi et al., 2004). Coordination of supply chain activities along with selecting the most appropriate suppliers is considered an important strategic management decision that impacts all areas of an organization (Jayaraman et al., 1999).

Most studies related to the JEIS problems have focused on the sale side of the supply chain but often neglect the importance of the supply side on the cost position of the supply chain. In many industries, outsourcing ratios have reached 60% or more, which illustrates that managing the supplier base can have a huge impact on the cost position of the company (C. Glock, 2012). Among the SC decision models, so-called Joint Economic Lot Size (JELS) models have received particularly high attention in recent years. JEIS models consider a situation where one or more vendors supply one or more products to a single or a group of buyers, with the objective of minimizing the total cost of the entire supply chain instead of the individual positions of selected supply chain members (Beck et al., 2017).

Many studies have described supplier relationship management as an essential measure to improve the competitive position of a company and consequently shipment consolidation in a multiple-vendor single-buyer integrated inventory model (Glock & Kim, 2014). The synthesis of supplier selection and integrated inventory models was researched by Glock (2011) who brought together both decision models as a unified coordination mechanism in a deterministic

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environment.

Moreover, supply chain decisions are prone to uncertainty which would intensely affect the SC performance. Demand fluctuation is undoubtedly one of the main sources of uncertainty in SC processes such as inventory replenishment, delivery scheduling merged with supplier selection, and order allocation. It seems necessary that the supplier selection lot sizing models should evolve to take into account the demand uncertainty in order to be in line with the new challenges faced by purchasing managers in the real world.

This paper considers a supply chain where a single buyer is sourcing one single product from several suppliers. A combination of the joint economic lot sizing model with supplier selection and order allocation under demand uncertainty is addressed. The proposed model determines the optimal value of the allocated order quantity to each supplier along with the delivery scheduling under demand uncertainty. A robust formulation is proposed to investigate the influence of unknown demand on the hybrid model in the form of a discrete scenario set. Robust Optimization is a branch of optimization theory that is used to obtain the optimal robust solution against the uncertainty or variability in the parametric value of the problem sought (Ashtiani et al., 2013).

The remainder of this paper is organized as follows: Section 2 reviews the literature related to supplier selection and the joint economic lot-sizing model. Sections 3 and 4 describe the problem and the robust approach for the proposed model. Section 5 provides several numerical experiments to examine the performance of the model and finally, section 6 summarizes the achievements of the research.

Literature Review

In this paper, two strands of research are considered as supplier selection and lot sizing models with a focus on uncertain models. Moreover, most of the literature related to lot sizing considers the sale side of the supply chain whereas this paper addresses integrating the inventory on the supply side. Thus, lot sizing literatures are confined to the suppliers' lot sizing model.

Suppliers selection models

Concerning the supplier selection problem, quantitative models mainly focus on the questions of which vendors to select and determine optimal consequent order allocation to the selected suppliers. Wadhwa and Ravindran (2007) classified supplier selection to enhance outsourcing operations into single or multi-objective mathematical programming methods, game theoretic methods, and artificial intelligence applications to supplier selection. Ghodsypour and O’Brien (2001) proposed a model for supplier selection under multiple sourcing policy and formulated a mixed-integer, nonlinear programming problem that explicitly takes into account material costs, ordering and inventory costs, and constraints on the buyer's and supplier's side. Considering the concept of risk, Ravindran et al. (2010) proposed a multicriteria optimization that offered four different alternatives via the value path approach. A scenario-based combination of supplier selection and inventory management was proposed by Hammami et al. (2014) where the currency exchange rate was assumed uncertain in a global supply. Zarindast et al. (2017) proposed a model of supplier selection under demand currency exchange rate uncertainty and provided a robust optimization approach to reach a solution. A game theory approach for supplier selection combined with a decision-making trial and evaluation laboratory (DEMATEL) was introduced, game theory was employed to combine the merits of subjective weight and objective weight, and DEMATEL was adopted to adjust the weight of the criteria to make the result more reasonable (Liu et al., 2018).

Esmaeili and Ghobadi (2018) proposed a game theory for pricing and supplier selection in a closed-loop supply chain, where the end demand is affected by the selling price, and the competition between the suppliers is considered based on the Bertrand model. Moheb-Alizadeh and Handfield (2018) designed a sustainable supply network using a multi-objective mixed-integer nonlinear programming (MINLP) model for efficient and sustainable supplier selection and order allocation with stochastic demand. Hooshmandi Maher and Amiri (2018) developed a supplier selection lot sizing mode under demand and lead time uncertainty in a multi-period time for multiple products.

An integrated inventory, supplier selection model with a new supplier price break scheme, was introduced and suppliers had a specific delivery schedule (Duan & Ventura, 2019). In the context of the uncertain environment, Lamba et al. (2019) proposed a mixed-integer nonlinear program (MINLP) for supplier selection lot sizing under multi-periods, multi-products, and multi-suppliers setting with a view of an overall reduction in the supply chain cost as well as the related cost of carbon emissions. Delivery delay was considered by Thevenin et al. (2022) who investigated the use of robust optimization for the integrated lot-sizing and supplier selection problem under lead time uncertainty. Feng et al. (2022) proposed a robust multi-supplier multi-period inventory model with uncertainty in demand and carbon emission. To consider the reliability of production and transportation, a robust supplier selection model was proposed by Che et al. (2023) who also developed a model for two sources of uncertainty. Table 1 summarizes the supplier selection studies considering the source of uncertainty and lot sizing.

Table 1. Summary of Supplier Selection Studies

Reference	Approach/Model	Source of Uncertainty	Lot sizing
(Ghodsypour & O’Brien, 2001)	Multiple sourcing policy, mixed-integer nonlinear programming problem	-	✓
(Ravindran et al., 2010)	Multicriteria optimization, value path approach	Risk	-
(Hammami et al., 2014)	Scenario-based combination of supplier selection and inventory management, uncertain currency exchange rate	Currency exchange rate	-
(Zarindast et al., 2017)	Scenario-based supplier selection and order allocation	Demand, Currency exchange rate	-
(Liu et al., 2018)	Game theory approach for supplier selection, decision-making trial, and evaluation laboratory	-	-
Esmaeili and Ghobadi (2018)	Game theory for pricing and supplier selection, Bertrand model	-	-
Moheb-Alizadeh and Handfield (2018)	Multi-objective mixed-integer nonlinear programming (MINLP) model, sustainable supply network, stochastic demand	Stochastic demand	-
(Hooshmandi Maher & Amiri, 2018)	Multi-objective, multi-period, multi-product supplier selection and order allocation	Demand, lead time	-
(Duan & Ventura, 2019)	Integrated inventory, supplier selection model with a new supplier price break scheme, specific delivery schedule	-	✓
Lamba et al. (2019)	Mixed-integer nonlinear program, supplier selection lot sizing under multi-periods, multi-products, and multi-suppliers setting, reduction in supply chain and carbon emissions costs	-	✓
(Feng et al., 2022)	A robust multi-supplier multi-period inventory model	Demand, Carbon emission	-
Thevenin et al. (2022)	Robust optimization, integrated lot-sizing, and supplier selection problems under lead time uncertainty, delivery delay	Lead time uncertainty	✓

As it could be concluded from the summarized table, the supplier selection lot-sizing problem under demand uncertainty is yet to be studied, especially when robust optimization is applied. Demand fluctuation is a critical factor in supply chain planning, thus considering it helps industries to manage uncertainty. Moreover, the lot sizing scheme in this paper is closer to reality and makes the problem more complicated.

Joint economic lot sizing

Integrated inventory models and supplier selection decisions have frequently been discussed (Glock, 2011). C. H. Glock (2012) provided a broad literature review of integrated inventory models and the supplier selection problem.

Kim and Goyal (2009) were some of the first pioneers of modeling inventory replenishment in this type of supply chain which offered two different delivery structures – one model where all suppliers deliver their respective production lots simultaneously and one model where the suppliers deliver successively. Another model consisting of multiple suppliers, a manufacturer, and multiple buyers was proposed by Jaber and Goyal (2009) which assumed an identical order cycle for the buyers. A similar model was proposed by Sarker and Diponegoro (2009) which mostly focused on the ordering cycle of the buyers and the suppliers provided different raw materials which decreased the complexity of the calculation of average stock at the manufacturer. It employed a shortest-path approach to obtain the optimal solution.

An integrated inventory model with a variable number of vendors was developed which combined supplier selection and lot sizing decisions, a heuristic solution procedure was then introduced to derive a solution (Glock, 2011). An integrated inventory model is then proposed by C. Glock (2012) which assumes two different delivery structures, first a mechanism with overlapping production cycles with immediate delivery and second with overlapping production cycles with delayed delivery. The work was then followed by research by Glock and Kim (2014) which categorized the suppliers and assigned a delivery cycle to each group where suppliers in the same group deliver the orders simultaneously.

Releasing the assumption of equal batch sizes for each supplier, Beck et al. (2017) proposed a lot sizing model, and the results of numerical studies show a significant reduction in the supply chain cost. Otrodi et al. (2019) proposed a joint pricing lot-sizing model for a perishable item under price-dependent demand regarding market segmentation. A Joint economic lot sizing was proposed by Gharaei et al. (2020) in multi-product multi-level integrated supply chains which considered a similar replenishment cycle for each product for all suppliers under deterministic demand. Utama et al. (2022) proposed a single-vendor single-buyer lot sizing model with maximizing total joint profit. Demand uncertainty was taken into account by Li et al. (2023) in the lot-sizing problem of raw material in a multi-item line balancing system.

To the best knowledge of the author, the multiple supplier models were mostly developed in the deterministic environment, while this study blends the suppliers' selection and delivery scheduling under demand uncertainty. Besides, different cycles on the supply side make the problem more complex as the inventory is accumulated by the buyer.

Robust Optimization

Dealing with real-world data has always been a concern for the academics of the realm of operations research. Robust optimization is a decision-making approach that aims to find solutions that are resilient to uncertainty and variability in the input data. The goal of robust optimization is to find a solution that performs well across a range of possible scenarios, rather than optimizing for a single scenario (Bertsimas & Thiele, 2006), (Marla et al., 2020), (Ben-Tal & Nemirovski, 1998). Robust optimization consists of two different components (Mulvey et al., 1995):

(1) design variables, $X \epsilon R^{n1}$, denote the vector of decision variables whose optimal value is not conditioned on the realization of the uncertain parameters. Variables in this set cannot be

adjusted once a specific realization of the data is observed.

(2) control variables, $Y \epsilon R^{n2}$, denote the vector of control decision variables that are subjected to adjustment once the uncertain parameters are observed. Their optimal value depends on the realization of uncertain parameters and the optimal value of the design variables.

The optimal solution of the mathematical program will be robust concerning optimality if it remains close to optimal for any realization of the scenarios and it is then termed solution-robust. The solution is also robust concerning feasibility if it remains almost feasible for any realization of S, it is then termed model-robust. The problem P is defined as follows:

$$Min\, Z = f(X, Y) \tag{1}$$

Subject to

$$\mathrm{g}(X) = \mathrm{b} \tag{2}$$

$$\mathrm{h}(X, Y) = \mathrm{e} \tag{3}$$

$$\mathrm{X}, \mathrm{Y} \geq 0,\ \mathrm{X} \in R^{n1}, \mathrm{Y} \in R^{n2} \tag{4}$$

to define a robust model a set of scenarios $S=\{s_1, s_2, \ldots, s_K\}$ is introduced, of which K is the number of scenarios. With each scenario, a set of control variables is defined as Y_s, and the probability of each scenario is p_s ($\sum_{s_1}^{s_K} p_s = 1$). In the proposed model, the RO model is defined as:

$$\mathrm{Min\ Z} = \sum_{s=1}^{K} p_s \varepsilon_s + \lambda \sum_{s=1}^{K} \left(\varepsilon_s - \sum_{s'=1}^{K} p_{s'} \varepsilon_{s'}\right)^2 \tag{5}$$

subject to

$$\mathrm{g}(X) = \mathrm{b} \tag{6}$$

$$\mathrm{h}(X, Y_s) = e_s \quad s=1,..,K \tag{7}$$

$$\mathrm{X}, Y_s \geq 0,\ \mathrm{X} \in R^{n1}, Y_s \in R^{n2} \tag{8}$$

where ε_s is a random variable taking the value $\varepsilon_s = \mathrm{f}(\mathrm{X}, Y_s)$ with probability p_s. The robust objective function is a linear combination of the weighted sum of objective functions of scenarios and the sum of the variances of objective functions under each scenario

Model Development

This article considers a single buyer who orders a single product from multiple suppliers. The buyer is also looking for the optimum lot sizing that benefits all the chain members, the demand of the product is not deterministic and depends on the economic situation with a limited number of scenarios. To illustrate the mathematical model, the following section describes the notations to illustrate the proposed model.

Notations

Indices

i	Supplier index; 1,2, … , N
b	Buyer
s	Scenario index; 1,2,…,K

Decision variables

q_{is}	Order quantity of supplier i per shipment under scenario s – *Control variable*
m_s	Number of shipments per planning period under scenario s – *Control variable*
T_s	Buyer's replenishment cycle under scenario s – *Control variable*

Parameters and functions

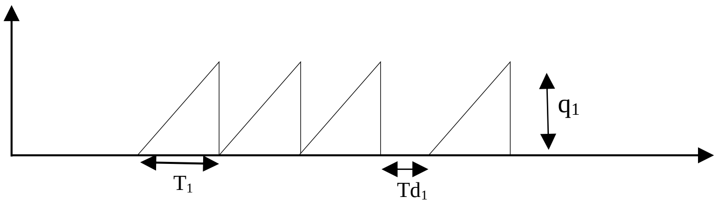
S	Set of scenarios (1,2,.., K)
N	Number of pre-selected suppliers
K	Number of scenarios
P_s	Probability of scenario s
D_s	Total demand rate under scenario s

Pr_i	Supplier i's production rate
h_i/h_b	Holding cost per unit per unit time
c_i	Variable cost of ordering from supplier i per unit of product
I_j^b	The buyer's inventory level before jth shipment
I_j^a	The buyer's inventory level after jth shipment
T_{ds}	Downtime during the production cycle of each supplier under scenario s
TC	Total cost
RO	Robust objective function
TC	Total cost
λ	Multiplier of robust function
τ_b	Transportation (shipment) cost of the buyer per shipment
ω_i	Transportation (shipment) cost of the supplier per one unit of product

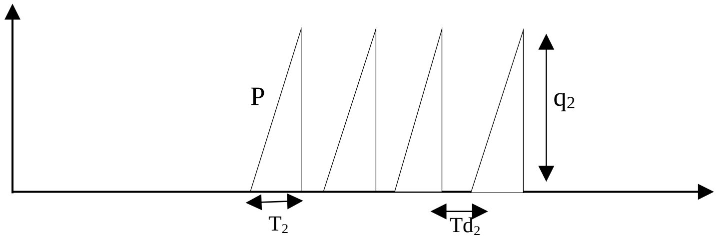
A two-echelon inventory/production system is considered where a buyer is sourcing one product from several suppliers. Suppliers deliver batches in equal sizes. The total production rate of suppliers is higher than the demand rate, therefore, the buyer does not face the shortage.

The proposed model consists of selecting a set of suppliers from a set of pre-selected suppliers, allocating order quantity, and finding the optimal delivery scheduling and lot sizing altogether while the demand rate is not deterministic. Fig 1 shows a schematic of the inventory level of the buyer and corresponding two suppliers.

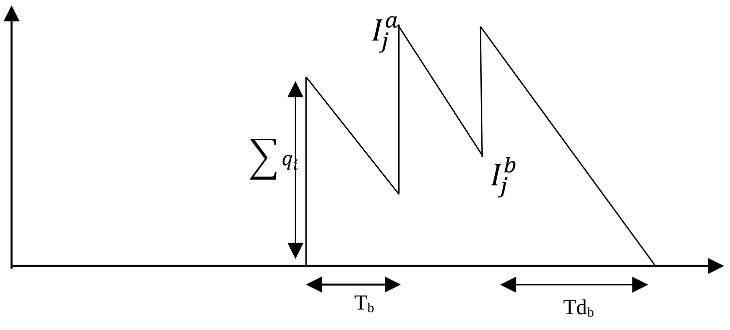
In this paper, the suppliers' holding cost is more than the buyer's; thus the inventory is prone to be accumulated at the buyer's warehouse. This situation is common in practice when the suppliers have limited storage space while the buyer is willing to receive orders regularly and make sure that shortage does not happen.



a) Supplier 1's inventory against time



b) Supplier 2's inventory against time



c) Buyer's inventory against time

Fig 1. A schematic of supply chain member's inventory level

The buyer’s model

The buyer’s objective is to determine the order quantity allocation and shipment scheduling which concludes to assess the downtime of the suppliers such that the total cost is minimized. The buyer’s cost function consists of two parts: the value related to order quantity allocation and the delivery policy. To determine the optimum quantity allocated to each supplier, the buyer’s cost function is composed of a fixed cost of supplier selection plus the variable cost of purchasing, inventory (holding), and shipment. To obtain the buyer’s holding cost, the average inventory is calculated at j$^{\text{th}}$ shipment as:

$$I_j^a = j\sum q_i - (j-1)\mathrm{D}T_b \tag{9}$$

$$I_j^b = (j-1)(\sum q_i - \mathrm{D}T_b\ ) \tag{10}$$

the average inventory of the buyer at j$^{\text{th}}$ shipment is then achieved as:

$$\left(I_j^a + I_j^b\right).T_b/2 \tag{11}$$

which is equal to:

$$\left(\mathrm{j}\sum_{i=1}^{N} q_i + \mathrm{j}\sum_{i=1}^{N} q_i - (\mathrm{j}-1)\mathrm{D}T_b\right).T_b/2 = \mathrm{j}\sum_{i=1}^{N} q_i T_b - (\mathrm{j}-1)\mathrm{D}T_b^2/2 \tag{12}$$

As the inventory is accumulated at the buyer, the total average inventory is calculated as:

$$\sum_{j=1}^{m-1} j\sum_{i=1}^{N} q_i T_b - (j-1)DT_b^2/2 + (m.q - (m-1)DT_b)^2/2\mathrm{D} \tag{13}$$

The last term of Eq (13) demonstrates the value of the average inventory after the last shipment. Therefore, the buyer’s cost function is expressed as:

$$\begin{aligned}\mathrm{TC}_b &= m\sum_{i=1}^{N} c_i q_i + m.\tau_b + h_b(\sum_{j=1}^{m-1}(j\sum_{i=1}^{N} q_i T_b - (j-1)DT_b^2/2)\\ &+(m\sum_{i=1}^{N} q_i - (m-1)DT_b)^2/2D)\end{aligned} \tag{14}$$

Replacing the summations including the number of shipments at the upper bound, the total cost of the buyer is expressed as:

$$\begin{aligned}\mathrm{TC}_b &= m\sum_{i=1}^{N} c_i q_i + m.\tau_b\\ &+h_b(\frac{\sum_{i=1}^{N} q_i T_b m(m-1)}{2} - DT_b{}^2\left(\frac{(m-1)(m-2)}{4}\right) + (m\sum_{i=1}^{N} q_i - (m-1)DT_b)^2)/2D)\end{aligned} \tag{15}$$

Considering that shortage is not allowed on the buyer side, the following constraint is added to the buyer’s model:

$$m.\sum_{i=1}^{N} q_i = D \tag{16}$$

Supplier i’s model

The supplier’s objective is to determine the shipment quantity per cycle. Therefore, total cost is composed of holding cost and shipment cost, adding up these elements expresses the cost function of the supplier i. First, the average inventory at the supplier is calculated as:

$$(\tfrac{1}{2})q_i.(\tfrac{q_i}{Pr_i}) = \frac{q_i^2}{2Pr_i} \tag{17}$$

Then adding the supplier's holding cost to the transportation cost is equal to:

$$TC_{is}\left(q_i\right) = h_i/2\left(\frac{q_i^2}{Pr_i}\right) + mq_i\omega_i \tag{18}$$

The first term of (18) describes the holding cost of the supplier, and the second term is the transportation cost.

The joint optimum

Finding a joint optimum for a supplier selection lot sizing problem can help companies make better decisions about how to allocate their resources and manage their supply chains more effectively. Thus, the total cost of the supply chain consists of suppliers and the buyer's cost as (19) where constraint Eq (16) is satisfied :

$$\begin{aligned} &TC = m\sum_{i=1}^{N} c_i q_i + m\tau_b \\ &+h_b\left(\frac{\sum_{i=1}^{N} q_i T_b m(m-1)}{2} - DT_b{}^2\left(\frac{(m-1)(m-2)}{4}\right) + \left(m\sum_{i=1}^{N} q_i - (m-1)DT_b\right)^2\right)/2D) \\ &+\sum_{i=1}^{N}\left(h_i/2\left(\frac{q_i^2}{Pr_i}\right) + mq_i\omega_i\right) \end{aligned} \tag{19}$$

As the function is convex regarding the decision variables, the first derivative of it concerning T_b gives the optimum of TC.

$$\frac{\partial TC}{\partial T_b} = \frac{m(m-1)}{2}\sum_{i=1}^{N} q_i - \frac{(m-1)(m-2)}{2}DT_b - (m-1)\left(m\sum_{i=1}^{N} q_i - (m-1)DT_b\right) = 0 \tag{20}$$

Solving (19) leads to:

$$T_b = \frac{\frac{(m-1)(m-2)}{2}\sum_{i=1}^{N} q_i}{D\left((m-1)^2 - \frac{(m-1)(m-2)}{2}\right)} \tag{21}$$

To exploit the robust model of the joint optimum model, a set of scenarios as S:{s_1,s_2,…,s_k} are defined by demand uncertainty. As all decision variables are affected by demand uncertainty, we consider the variables as control variables. Regarding Eq (5) and replacing ε_s with TC, the robust model for the hybrid inventory–supplier selection problem is expressed as:

$$RO = \sum_{s=1}^{K} p_s TC_s + \lambda\sum_{s=1}^{K}\left(TC_s - \sum_{s'=1}^{K} p_{s'} TC_{s'}\right)^2 \tag{22}$$

$$m_s\sum_{i=1}^{N} q_{is} = D_s \qquad s = 1,..,K \tag{23}$$

$$q_{is} \le Pr_i T_s \qquad i = 1,..,N; s = 1,..,K \tag{24}$$

$$q_{is} \ge D_s T_s \qquad i = 1,..,N; s = 1,..,K \tag{25}$$

$$T_s, q_{is}, \alpha_{is} \ge 0 \qquad i = 1,..,N; s = 1,..,K \tag{26}$$

Eq (22) is the total cost of the supply chain, and Eq (23) ensures that the total demand of the buyer is met through the planning horizon. Eq (17) requires that the total order quantity does not exceed the production capacity and Eq (26) is the nonnegativity of the decision variables. The proposed mathematical model is a non-linear mixed integer programming model that has been solved with GAMS optimization software, and the following part presents the numerical results.

Numerical Example

This section provides several numerical experiments for evaluating the performance of the proposed model.

Numerical example 1 – A deterministic case: firstly, a deterministic case is considered as the demand of the product is equal to D={1500}, the buyer's parameters are h_b= 15, τ_b=120 for each example, and other input data regarding the suppliers are shown in Table 2.

Table 2. Input data of numerical example1

Supplier ID	Pr_i	h_i	ω_i	c_i
1	200	100	150	50
2	250	50	170	100
3	550	50	175	100
4	1000	80	150	50
5	1200	100	50	100

In this case, as the only scenario's probability is equal to 1, the robust function is equal to TC. The output of the model is as TC= 248,292 and the selected suppliers are suppliers 1, 4, and 5 who received the order quantities as $q_1 = 3.026, q_2 = 18.911$ and $q_5 = 87.745$ in $m = 14$ shipments. In this example, the problem is also solved with single scenario cases D={1200} and D={1800}, and the relevant output data of each case is summarized below:

Table 3. Output result of three cases as separate problems

Scenario	1	2	3
TC	187,062	248,292	309,580
q_1		3.026	5.737
q_4		18.911	35.860
q_5	90.268	87.745	83.194
m	13	14	14

Numerical example 2 – Uncertain demand with medium variance: In this case, three scenarios are introduced as S ={1,2,3} with equal probability for each scenario, where s=1 represents a weak economy with the lowest demand, s=2 is for the normal mode, and s=3 symbolizes a good prosperity with the highest demand, the corresponding probability is Ps={0.25,0.5,0.25} which represents as Ds:{1200,1500,1800} and λ=0.1 for all experiments and other input data are as

Table 2. The output results are shown in Table 4 and the value of the robust function is RO= 309,577.

Table 4. Output result of the numerical example 1

Scenario	1	2	3
q_1	58.192	32.353	5.743
q_2	72.740	7.411	
q_3	160.029	35.164	
q_4	58.192	106.925	35.897
q_5		60.793	83.283
m	3	6	14

In the second scenario, all the suppliers received orders from the buyer and the number of shipments increased in comparison to the first scenario which supplier 5 was not selected and $m_1 = 3$ and $m_2 = 6$ respectively. In the third scenario which represents a growing economy, suppliers 1, 4, and 5 are selected and receive orders in $m = 14$ shipments.

Comparing the total cost of separate scenarios of example 1 with the robust function's value, the value of the robust function is almost equal to the total cost of the third scenario. As the robust function value remains near-optimal and the output value of decision variables is a feasible solution for the problem, we could say that it is model robust (regarding the feasibility), Although the optimality is not achieved and the model is not solution robust (Ashtiani et al., 2013).

Numerical example 3 – Uncertain demand with different variance: The three scenarios and input data from numerical example 2 are considered here but the demand set variance is assumed to be different from the last experiment. Therefore the problem is solved with different scenario sets and the output results versus variation are shown in Fig 2 and Fig 3. Moreover, the number of the selected suppliers versus the demand variation is illustrated in Fig 4.

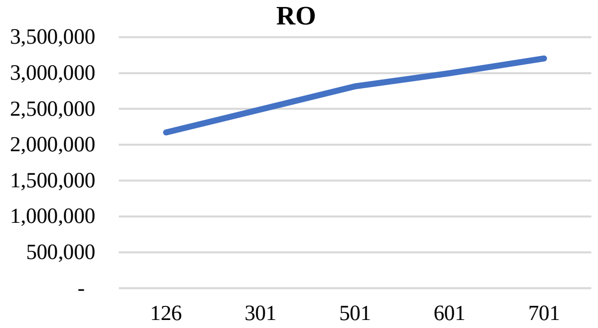


Fig 2. robust function value vs demand variation

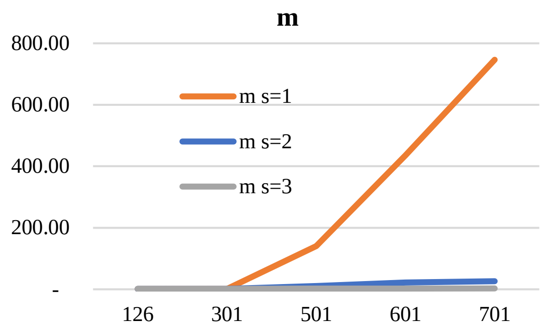


Fig 3. number of shipments vs demand variation

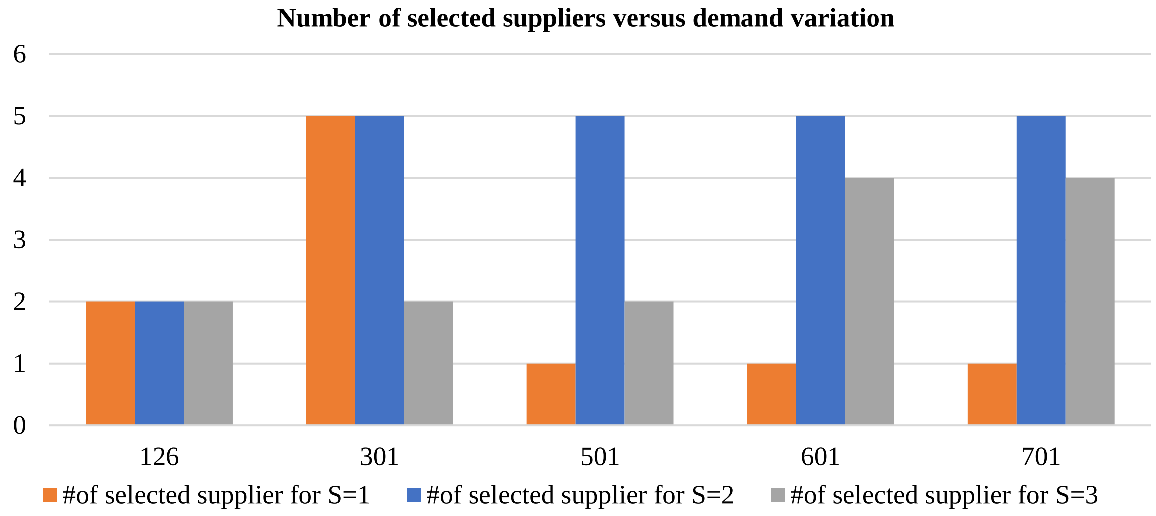


Fig 4. number of selected suppliers vs demand variation

It could be concluded from the above figures that the robust function value and shipment number increase as the variation in demand sets rises. A jump in the first scenario is observable, it means that with lower value of demand, holding inventory is not economic and the buyer receives orders in small lot sizes. Moreover, as the demand in the middle scenario is fixed, more variation in demand data leads to lesser demand in the first scenario which causes a smaller number of selected suppliers as Fig 4 shows. The number of suppliers has a rising trend in the third scenario as the demand variation leads to higher demand.

Sensitivity Analysis

In order to evaluate the model behavior under various circumstances, the effect of changes in the input parameters on the outcomes is investigated as follows:

The effect of changes in suppliers' production rate

To explore the variation in suppliers' production rate impact on the model outputs, we have changed the production rates from -25% to +25% of the current production rates of the suppliers.

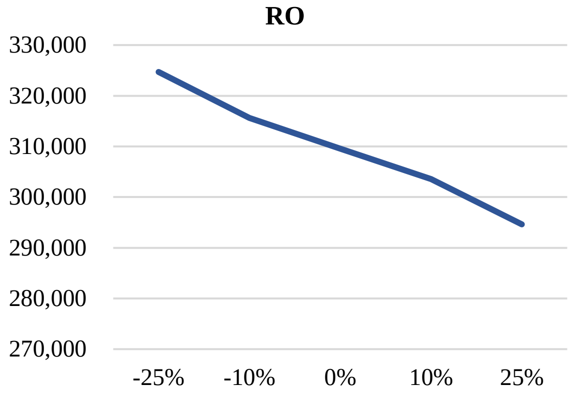


Fig 5. The effect of changes in production rates on RO value

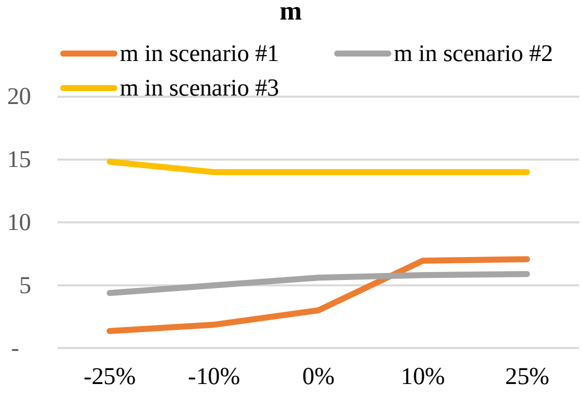


Fig 6. The effect of changes in production rates on number of shipments

As it could be concluded from Fig 5 increasing the production rate of suppliers leads to a decline in RO function value. Moreover, in Fig 6 the number of shipments has an upward trend versus the increase of production rates as we expected due to the total supply and demand equality conditions. As the most weight in the RO function belongs to scenario #2 and the number of shipments did not face a sharp increase in this scenario and scenario #3, the effect of shipment cost on the RO function is negligible.

The effect of changes in the buyer holding cost

The ratio of the buyer's holding cost has been alternated between [-50%,+50%] and the effect of this change on the outcome is presented as follows:

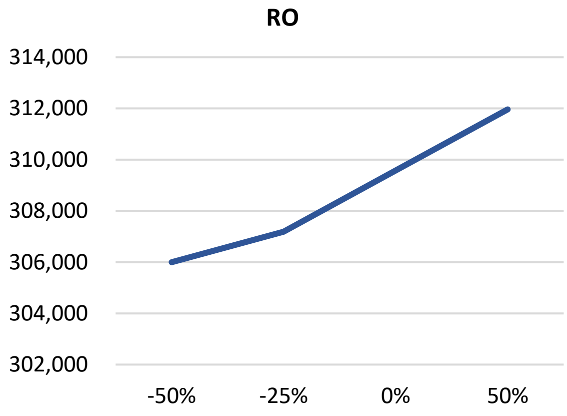


Fig 7. RO function value versus changes in the buyer's holding cost ratio

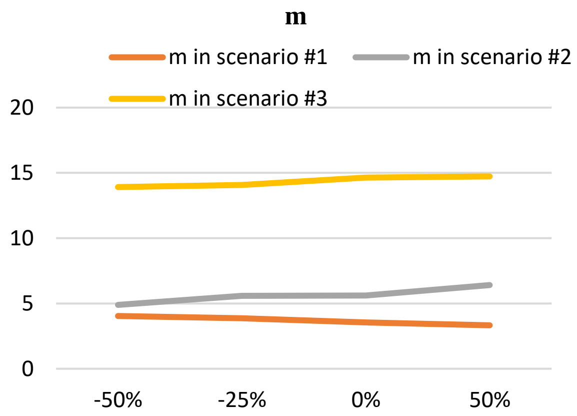


Fig 8. Number of shipments versus the changes in the buyer's holding cost

Increasing the buyer's holding cost while the suppliers' holding costs remain unchanged leads to an escalation in RO function value. **Error! Reference source not found.** shows the behavior of shipments versus the increase in the buyer's holding cost. As it was expected the number of shipments decreased while the holding cost increased.

The effect of changes in the buyer's transportation costs

Transportation cost plays a crucial role in lot sizing models, therefore the effects of changing this factor on the models' outputs are presented here.

The increase in the RO function as the transportation cost rises was expected as we could conclude from **Error! Reference source not found.**. Moreover, the reduction of shipment number is observed in **Error! Reference source not found.** as the transportation cost increases.

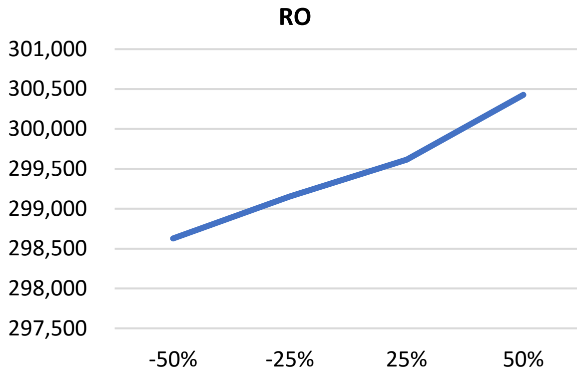


Fig 9. The effect of changes in number RO function versus changes in the transportation cost

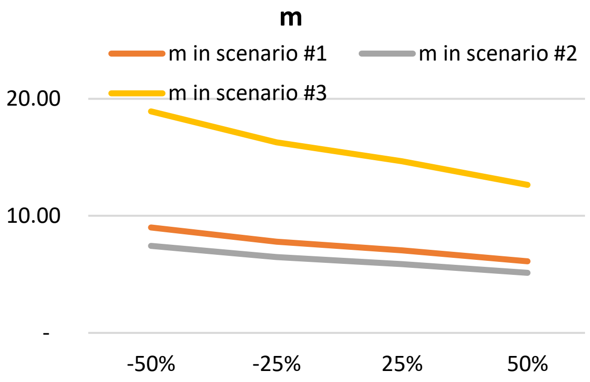


Fig 10. the effect of changes in the number of shipments versus the changes in transportation cost

Conclusions

This paper investigated the supply chain coordination between multiple suppliers and a single buyer as a unified model that integrates supplier selection, order allocation, and inventory replenishment altogether. To incorporate the uncertain nature of the demand, we assumed different demand scenarios and proposed a robust model to minimize the total cost of the supply chain. Several numerical experiments were carried out to evaluate the proposed model performance, and the results indicated that more variation in the demand data set would lead to the selection of more suppliers from the supplier pool and a higher total cost.

To increase the scope of the analysis, the models proposed in this paper could be extended in various ways. For example, in addition to uncertain demand, the production rate of the suppliers could be assumed stochastic. Also, it would be interesting to include several products in the model and relax the assumption of an equal number of shipments for the suppliers. Moreover, as the deliveries are assumed simultaneous, different delivery schemes could be a possible subject for future research. Besides, since the focus of this paper is on the system's total cost, considering a revenue-sharing program would also be motivating for future research ideas.

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