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Delimitation of the final pit of open-pit mines using the maximum flow pseudoflow method

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ABSTRACT

The primary aim of this study was to define the final pit boundaries utilizing the maximum flow Pseudoflow method in an open-pit mining context. Our methodology encompassed exploratory data analysis (EDA), establishing geomechanical and economic factors, and assessing the final pit. The study was conducted using Python 3.11 and SGeMS 3.0. We discovered that our block model comprised 480,000 blocks of 10x10x10 m dimensions. We generated 20 pits with revenue factors between 0.1 to 2, increasing by increments of 0.1. The Study indicated that pit 20 was optimal, with an estimated NPV of 17855 MUSD, extracting 212 million tons of ore and 58 million tons of waste rock, achieving a stripping ratio based on block model and market conditions, and is subject to change with further block sequencing analysis. Nevertheless, pit 20 emerged as the most advantageous when considering economic feasibility, given its high estimated NPV and favorable stripping ratio.

Keywords: *Final pit, Estimated NPV, Pit, Pseudoflow, Stripping ratio.*

1. Introduction

Open-pit mining is a prevalent method in mineral extraction, which necessitates precise design of the pit's shape and final boundaries before operations begin. The primary aim of this design phase in open-pit mining is to determine the definitive configuration and dimensions of the mine [1 - 2]. This process takes into account various critical factors, including the geology and topography of the deposit, mineral distribution, geotechnical and slope stability constraints [3], environmental considerations, as well as extraction and processing costs, metallurgical recovery, and the price of the mineral [4 - 10].

The optimization of the final pit limits plays a crucial role in open-pit mining design. To achieve this precise delimitation, block models are used to represent the reserve as a combination of small blocks, employing methods of inverse distance or geostatistics [1]. In recent decades, two main approaches have been employed to delimit the final pit: maximization of the undiscounted profit and maximization of the Net Present Value (NPV) [11]. Each approach has specific methods and algorithms. In the first approach, the aim is to maximize undiscounted profit by initially setting the pit limits. Additionally, production scheduling is planned to obtain the highest NPV. Heuristic algorithms used include floating cone [12] and its improvements [13], Korobov [14], Boykov-Kolmogorov [15], and Ford-Fulkerson [16]. However, these algorithms do not guarantee mathematically optimal solutions. The Lerchs-Grossman algorithm [17] (LG), based on graph theory, and the network flow algorithm [18] also determine the final pit limits using mathematical approaches. Each method has its advantages and disadvantages. For example, when using a block model with a large amount of data, such as in generating final pits for an open-pit mine with over 100 million data points, applying the Lerchs-Grossman algorithm to obtain optimal pit limits may result in significant runtime delays [19].

The maximum flow problem aims to maximize the amount of flow that can pass from a source to a sink in a network with capacities on the arcs. To solve this problem, two types of algorithms have been developed: feasible flow algorithms, which increase the flow in each iteration using augmenting paths, and preflow algorithms, which allow for excesses in flow equilibrium [20]. The first feasible flow algorithm was proposed by Ford and Fulkerson in 1957, while the first known use of preflows was in 1955 by Boldyreff [21]. However, this technique did not guarantee optimal solutions. The Push-relabel algorithm by Goldberg and Tarjan in 1988 [22] [23] uses preflows and has been shown to be efficient both theoretically and empirically.

In this context, the pseudoflow algorithm is presented as a highly effective tool for determining the final pit limits in open-pit mining. The pseudoflow algorithm uses an optimality certificate based on the Lerchs and Grossman algorithm for the maximum closure problem in a weighted node graph. This approach demonstrates that the concept of mass can be generalized to capacity networks using the notion of pseudoflow [20]. Unlike the maximum flow problem, pseudoflow addresses the maximum blocking cut problem, which focuses on arc capacities and node weights without having source and sink nodes. Its goal is to find a subset of nodes that maximizes the sum of node weights minus the capacities of arcs leaving the subset [24]. Within this study, the "Final Pit Limit" is defined as the ultimate boundary derived from the optimization process, indicating the most economically viable limit over the mine's lifespan. This is accomplished through iterative determinations of "optimal pit limits', with each iteration representing a step towards the "Final Pit Limit", which is the culmination of maximizing economic value under given constraints [25, 26].

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Despite the computational strengths of the pseudoflow algorithm and its robustness in handling large datasets, it is not without its disadvantages. The algorithm's efficacy is heavily dependent on the accuracy of the input data; inaccuracies in the block model can propagate, leading to suboptimal pit designs. Moreover, it predominantly focuses on the economic optimization of pit limits and does not inherently account for operational constraints, potentially necessitating further design adjustments. These shortcomings underscore the importance of comprehensive data collection and the integration of algorithmic outcomes with practical mining considerations [27].

This study primarily contributes to the field of open-pit mining by developing and implementing a rapid solution technique for determining the final pit limit via the pseudoflow algorithm, focusing on computational efficiency and robust data management. By enhancing the algorithm's speed and precision, the research bridges critical gaps in current methodologies, offering a practical tool for mine planners seeking to quickly adapt to varying economic scenarios and make informed decisions about pit designs. The application of this refined pseudoflow approach underscores its value not only in calculating economically optimized boundaries but also in providing a scalable solution that accommodates the complexities inherent in large-scale mining operations.

2. Materials and methods

Parametric analysis was conducted to define the final pit limits, incorporating key geomechanical and financial parameters. A series of final pit scenarios were generated by varying revenue factors, enabling a comprehensive assessment under different economic conditions. Each pit scenario was meticulously evaluated through the construction of comparative pit graphs and scatter plots to ascertain the optimal configuration. The visualizations and analyses were executed using Python 3.11 within the Jupyter Notebook interface [27] and SGeMS software version 3.0 [28], chosen for their analytical prowess and compatibility with mining optimization tasks.

2.1. Exploratory Data Analysis

Exploratory data analysis is a set of procedures followed by researchers to understand the overall structure of the data, identify anomalies, and gain insights that can be used in more complex analyses [29].

2.2. Graphical concepts and pit optimization

In optimizing the sequence of extraction for open-pit mining, a block model is utilized to represent the distribution and value of mineable materials [30]. Each block within the model is assigned a predefined value, which signifies its economic worth or cost, as depicted in Figure 1. The critical aspect of planning involves establishing the slope requirements, ensuring safe and efficient removal of material. The dependencies among blocks are determined by the need to maintain a consistent slope, typically set at 45° for operational safety and stability. Figure 2 presents a simplified two-dimensional representation, focusing on the critical dependency for a single block to elucidate the concept. For instance, the extraction of block “g” is contingent upon the prior removal of blocks “p”, “q” and “r” to preserve the integrity of the slope. This dependency graphically underscores the operational constraints that must be observed to realize the slope requirements [31].

A graph is a fundamental structure utilized in various fields for modelling relational data, consisting of a set of vertices (or nodes) connected by edges. In the domain of open-pit mine optimization, this abstract representation becomes a practical tool: each vertex corresponds to a discrete block of material within the mine, and the edges delineate the extraction sequence and slope constraints that govern the removal of these blocks. Such constraints are essential for maintaining the structural integrity of the mine and ensuring operational safety. Additionally, vertices are imbued with weights, which reflect the economic value or cost associated with each block's extraction. This graphical abstraction facilitates the computational assessment of the mine's design by allowing the application of optimization algorithms [32].

Figure 3, building on the block model presented in Figure 1, visually encapsulates this optimization problem. It demonstrates how the graph-based approach can be employed to determine an optimal excavation sequence that maximizes the overall value of the extracted materials while conforming to the technical constraints inherent in mine planning. This figure serves to translate the complex interdependencies and valuation of mine blocks into a comprehensible format, enabling the strategic planning that underpins efficient and profitable mining operations [31].

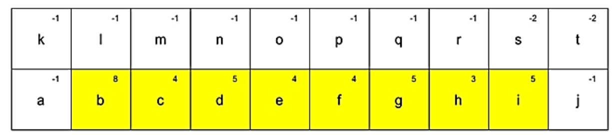

Figure 1. Block Model

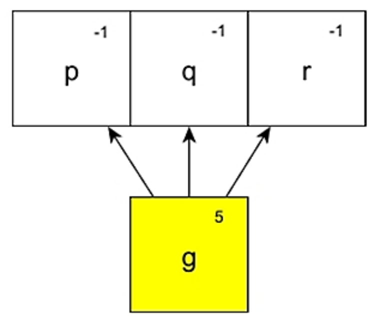

Figure 2. Dependency between the blocks to be mined.

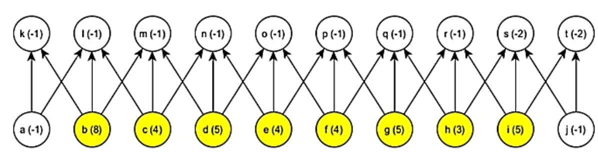

Figure 3. The graphical representation of optimization problem.

After establishing the fundamental graphical concepts and their application to pit optimization, it is pertinent to specify the algorithmic approach adopted for the maximum flow problem. For this study, we have implemented the Pseudoflow algorithm, a variant refined from the HI-PR (Highest-Label Preflow-Push) algorithm, due to its proven effectiveness in addressing the complexities of mining sequences while preserving operational safety through slope constraints. The algorithm's suitability for maximizing the Net Present Value within our mining project's geotechnical parameters confirms its selection over other maximum flow algorithms such as Push-relabel or Excess flow. The subsequent sections delve into the specifics of the Pseudoflow algorithm and its application to the mine block model [33].

2.3. Maximum Closure and Optimal Pit

In the optimization of open-pit mines, a “closure” or “closed set” refers to a subset of blocks that comply with the operational slope constraints, encapsulated within a directed graph $G = (V, E)$. Here, V represents the set of all blocks under consideration-both ore and waste-while E embodies the directional relationships indicative of mining sequence and slope adherence. A subset $S \subseteq V$ is defined as a “closure” if for any given block $i \in S$, all blocks that are to be mined subsequent to i, are also included in S [34]. Formally, this is denoted as:

$$S = \{i \in V | \forall (i, j) \in E, j \in S\} \tag{1}$$

The challenge of the "maximum closure" problem is to ascertain such a set S where the cumulative value of the blocks, as quantified by a weighting function $w: V \rightarrow R$, is at its pinnacle. This maximization reflects the balance between the economic gains from ore extraction against the costs of waste removal. Figure 4 provides a visual depiction of this concept, illustrating the maximum closure problem as it pertains to the strategic planning of open-pit mines. Nodes are assigned values, positive for economically beneficial ore and negative for non-valuable waste, thereby mirroring their extraction. The physical stability of mine dictated by geotechnical requirements, is represented by directional arcs. Solving the maximum closure problem identifies the optimal subset or blocks that maximizes the net present value, ensuring the venture´s economic viability and structural integrity [31].

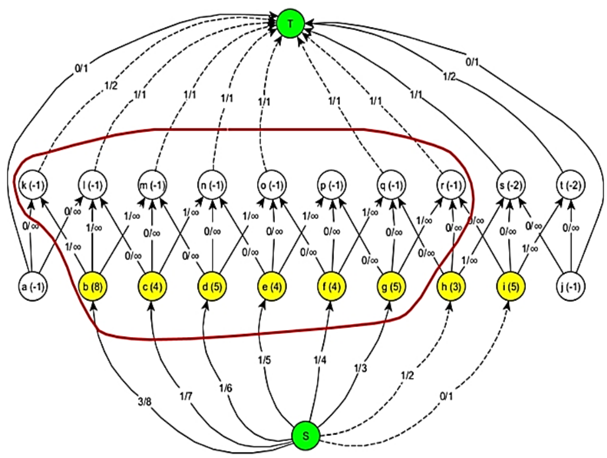

Figure 4. The flow graph with maximum closure limit.

The Pseudoflow procedure is designed to ensure an efficient allocation of flow throughout the network, leading to the precise identification of the block set that delineates the boundary of the final cut.

Pseudoflow Procedure {Gst, f, T, S, W}
Begin
While (S, W) ∩ A_f ≠ ∅ do
Select (s', w) ∈ (S, W);
Let $r_{s'}$, r_w be the roots of the branches containing s' and w respectively.
Let δ = excess ($r_{s'}$) = $f_{r_{s'},ri}$
Merge T ←T\[r, $r_{s'}$] ∪ (s',w);
Renormalize {Push δ units of flow along the path [$r_{s'}$, ..., s', w, ..., r_w, r]:}
i=1;
Until v_{i+1}=r;
Let [v_i, v_{i+1}] be the ith edge on the path;
{Push flow} If $c^f_{vi,vi+1} \geq \delta$ augment flow by δ, $f_{vi,vi+1}$← $f_{vi,vi+1}+\delta$
Else, split {(vi, vi+1), δ - $c^f_{vi.vi+1}$};
Set $\delta \leftarrow c^f_{vi,vi+1}$;
Set $f_{vi,vi+1} \leftarrow c_{vi,vi+1}$;
i i+1
End
End
End
Division procedure {(a, b), M}
T T\(a,b) ∪ (a,r); excess (a) = far = M; {a is a root of a strong branch}
Af Af ∪ {(b,a)} \ {(a,b)};

2.4. Pseudoflow maximum flow method

The determination of the ultimate pit limit within the mine optimization process has been refined by implementing the Pseudoflow maximum flow method. This advanced algorithmic approach is underpinned by the network flow theory, which provides a robust framework for addressing the intricacies of pit design. The method is an evolution from traditional 'maximum closure' strategies, offering enhanced computational efficiency and precision. In this context, the mine is conceptualized as a 'flow graph,' where nodes represent discretized blocks of the mine, each with an associated economic value or cost, and directed edges delineate potential extraction sequences, adhering to mandatory slope constraints. To address the optimization challenge, the flow graph is augmented with a 'source' node, representing the initiation point of material extraction, and a 'sink' node, denoting the point of completion [35].

The optimization problem is formulated to maximize the net present value of extracted materials, constrained by the flow capacities of each block and the overall system, ensuring that the solution adheres to operational and geotechnical constraints. The objective function and constraints are mathematically expressed as follows: Maximize the total flow Z from the source to the sink [36]:

$$Z = \sum_{(s,i)\in E} F_{si} \tag{2}$$

Subject to capacity constraints on each edge (i,j):

$$F_{ij} \leq C_{ij} \quad \forall (i,j) \in E \tag{3}$$

Ensuring flow conservation at each node except for source and sink:

$$\sum_{(i,j)\in E} F_{ij} - \sum_{(j,i)\in E} F_{ji} = 0 \quad \forall i \in V \setminus \{source, sink\} \tag{4}$$

And non-negativity of flow:

$$F_{ij} \geq 0 \quad \forall (i,j) \in E \tag{5}$$

The Pseudoflow maximum flow method then systematically computes the optimal flow values that correspond to the selection of blocks to be excavated, while satisfying all constraints. The solution to this optimization problem yields a set of blocks that forms the ultimate pit limit, maximizing the economic return of the mining operation. The algorithm's performance and implementation for open-pit mining optimization are corroborated by the empirical findings of Picard and Smith (2004) [33], Hochbaum and Chen (2000) [34], and Thomas (1996) [37], whose research substantiates the efficacy of this approach.

To elucidate the application of the Pseudoflow maximum flow method in pit optimization, consider a simplified mine model composed of a series of blocks with varying economic values. Imagine three blocks, A, B, and C, where A is directly accessible and B and C are sequentially dependent on the extraction of A. Block A has an economic value of $10, B is worth $15, and C is worth $5, but cannot be accessed until A and B have been extracted. The flow graph is constructed by connecting a source node to block A, and then A to B, B to C, and finally C to a sink node, with arcs representing the extraction sequence. The capacities of the arcs are set to mirror the economic values of the blocks they connect. Applying the Pseudoflow algorithm, we initiate the flow from the source, respecting the capacities, and adjust the flow iteratively until the maximum flow from the source to the sink is achieved. The optimal solution, in this case, would identify the extraction of blocks A and B, with a total flow equivalent to their combined value, as the optimal sequence to maximize economic return while adhering to operational constraints.

3. RESULTS

In applying the Pseudoflow algorithm to the block model for pit optimization, a succinct representation of the model's attributes is essential to demonstrate the effectiveness of the method. The block model was evaluated based on copper grades (see Figure 5), density distributions, and geomechanical properties, which were critical in delineating the final pit limit.

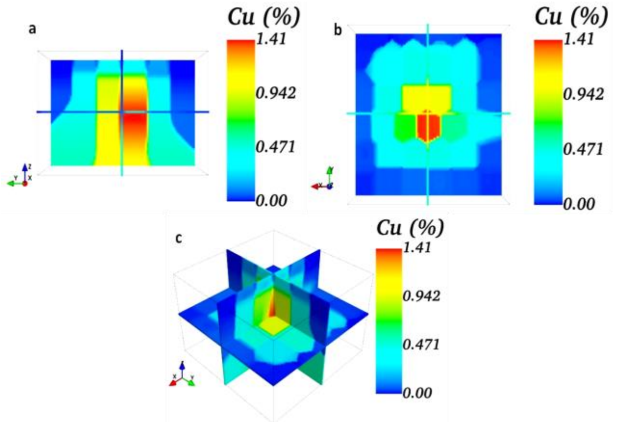

Figure 5. Cooper grades in the block model, a) frontal view, b) plan view, c) isometric view.

A summarized visualization of the block model highlighted the central concentration of copper grades, indicative of the region's economic potential. This facilitated the focus of the Pseudoflow algorithm on areas of higher value, aligning the excavation sequence with the most profitable blocks. A selective representation of the grade-tonnage relationship further emphasized the inverse correlation typically observed between these variables, which was instrumental in the algorithm's optimization process (see Figure 6).

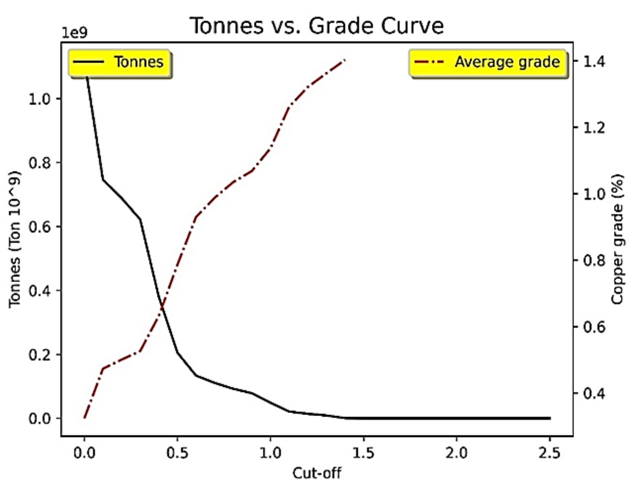

Figure 6. Tonnes vs. Grade curve.

The parameters of the block model were dimensions of 10x10x10 meters in length in the east (X), north (Y) and elevation (Z), with the average density set at 2.3 ton/m3. Geomechanical parameters, such as the slope angle, were set to 45°, conforming to standard safety practices. Operational and financial parameters, including costs and copper prices, were accounted for within the algorithm to ensure economic viability (see Table 1). The Pseudoflow algorithm effectively utilized these inputs to compute the optimal flow of value through the network, simulating the extraction process and determining the ultimate pit limit.

Table 1: The financial and operational parameters.

Scenario	Cu
Base Copper Price (US$/lb)	3.9
Smelting Cost (US$/lb)	0.4
Mining Cost (US$/ton)	2.3
Crushing and Grinding Cost (US$/ton)	11
Metallurgical Recovery (%)	90

In evaluating the scenarios for the final pit delineation, as depicted in Table 2, the economic viability was assessed using a range of revenue factors from 0.1 to 2. This range allowed for the analysis of the pit's financial performance under various market conditions, providing a comprehensive understanding of the pit's profitability across a spectrum of economic environments. The revenue factor, directly impacting the valuation of ore, was instrumental in understanding the economic thresholds that define the optimal pit limits.

The Net Present Value (NPV) calculations for each scenario were anchored by a discount rate of 10%, which is a standard figure reflecting the cost of capital and the associated investment risk within the mining industry. This rate was applied to the projected cash flows, composed of the revenue from ore sales and the costs of waste management and ore extraction. The ore sales revenue was calculated by multiplying the annual production rate, set at 10,616,800 tonnes, by the ore grade and the prevailing copper price, adjusted according to the specific revenue factor. The subsequent cash flows were then discounted to ascertain their present value, culminating in the NPV for each scenario.

An integral aspect of the evaluation process is the computational efficiency of the Pseudoflow algorithm used to delineate the final pit in each scenario. The solution times recorded for the generation of each pit configuration substantiate the algorithm's expeditious performance. For the initial pit (Pit 1), the algorithm required only 6.34 seconds to reach a solution. Subsequent pits demonstrated comparable efficiency, with Pit 2 through Pit 20 requiring 6.59, 6.5, 6.59, 6.58, 6.62, 6.66, 7.6, 7.75, 6.71, 6.83, 6.56, 6.8, 6.66, 6.91, 6.66, 6.95, 6.94, 6.61, and 6.76 seconds, respectively. This consistent rapidity in the generation of solutions, even as the complexity of the scenario increased, underscores the robustness of the algorithmic approach.

Figure 7 depicts the visualization of final pits 1, 5, 10 and 17, each accompanied by its respective views. These visual representations enhance the understanding of the shape and size of the final pits obtained in different scenarios.

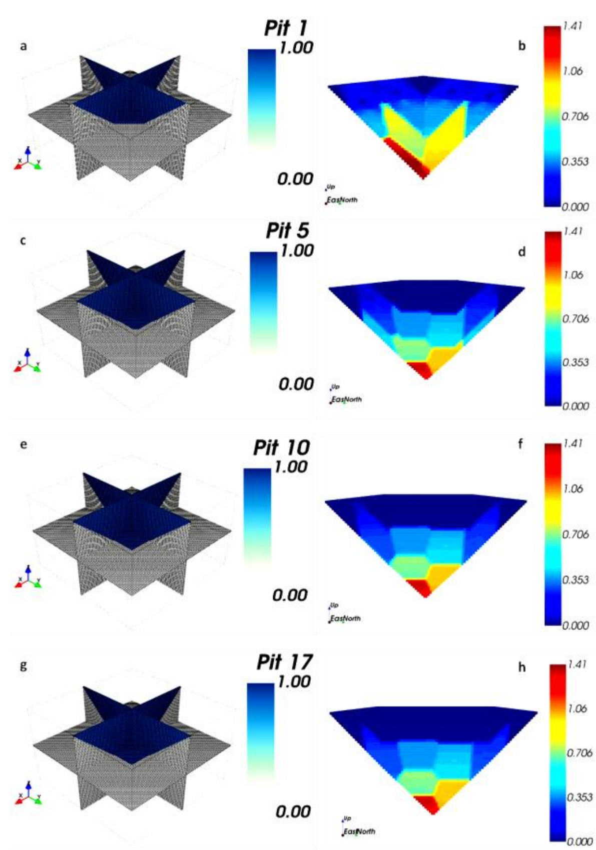

Figure 7. Final pit scenarios. a, b) Final pit 1. c, d) Final pit 5. e, f) Final pit 10. g, h) Final pit 17.

Table 2: Multiple final pit scenarios.

Pit	Ore (Ton)	Waste (Ton)	Total Ton	VAN (US$)	SR	Ore grade (%)
1	47 345 500	70 345 500	117 691 000	232 952 666	1.49	0.50
2	98 398 600	78 305 800	176 704 400	940 593 419	0.80	0.46
3	121 304 300	77 530 700	198 835 000	1 765 344 582	0.64	0.44
4	136 944 300	78 370 200	215 314 500	2 638 744 061	0.57	0.43
5	147 519 700	79 255 700	226 775 400	3 541 652 586	0.54	0.42
6	155 512 200	77 965 400	233 477 600	4 462 160 532	0.50	0.41
7	163 053 900	76 431 300	239 485 200	5 393 078 876	0.47	0.40
8	172 069 900	71 916 400	243 986 300	6 331 577 954	0.42	0.40
9	181 129 600	68 944 800	250 074 400	7 276 582 987	0.38	0.39
10	187 539 700	66 350 400	253 890 100	8 226 824 097	0.35	0.39
11	193 073 500	64 372 400	257 445 900	9 181 373 993	0.33	0.38
12	197 664 300	63 086 700	260 751 000	10 139 198 654	0.32	0.38
13	200 888 900	60 795 900	261 684 800	11 099 586 591	0.30	0.38
14	204 074 400	60 609 600	264 684 000	12 062 014 785	0.30	0.38
15	20 6068 500	60 112 800	266 181 300	13 025 177 993	0.29	0.37
16	207 892 400	59 560 800	267 453 200	13 990 289 538	0.29	0.37
17	209 079 200	59 121 500	268 200 700	14 955 546 199	0.28	0.37
18	210 424 700	59 576 900	270 001 600	15 921 877 589	0.28	0.37
19	211 390 700	58 610 900	270 001 600	16 888 844 623	0.28	0.37
20	212 336 000	58 355 600	270 691 600	17 855 973 282	0.27	0.37

To evaluate and determine the optimal final pit, a "pit by pit" graph was used. In this graph, each pit is displayed along with its corresponding ore and waste tonnages in relation to the associated NPV. It can be observed that pit 20 has the highest ore tonnage, totaling 212 million tons, and the lowest waste tonnage, totaling 58 million tons. Additionally, this pit has the highest NPV compared to the other generated pits (see Figure 8).

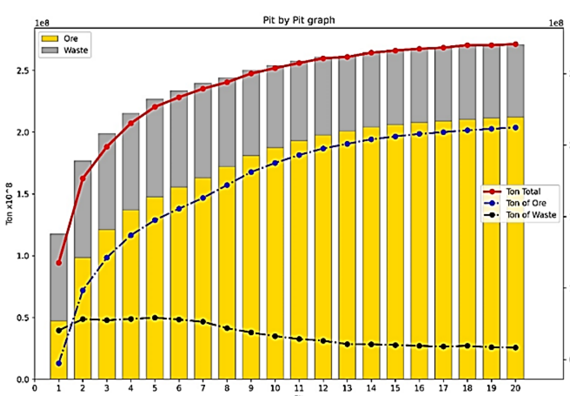

Figure 8. Pit by Pit graph.

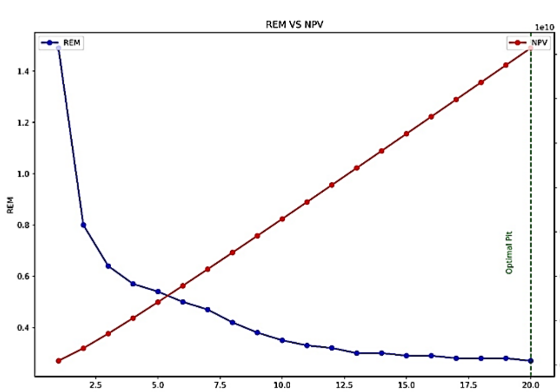

Figure 9. SR vs NPV graph.

In Figure 10, the visualization of the optimal pit (pit 20) is presented from different perspectives, showing its copper grade legend. These views allow for a better appreciation of the distribution and concentration of the copper grade in the pit selected as the optimum.

In a singular effort to validate the performance of the Pseudoflow algorithm within open-pit mining optimization, a comparative study was conducted against traditional Maximum Flow methods, including Boykov-Kolmogorov, Ford and Fulkerson, and Lerchs and Grossman. The results, encapsulated in Table 3, revealed that Pseudoflow significantly outperformed the alternatives in both computational speed and economic efficiency, achieving an NPV of 17,855.97 million USD across 486,000 blocks with an impressive execution time of just 6.76 seconds. This starkly contrasts with the lesser NPVs and longer processing times of the other methods, underscoring the Pseudoflow algorithm's robustness and its suitability for contemporary mining operations where time efficiency and maximizing economic return are paramount.

Table 3: Final Pit Comparison Using Different Methods.

Method	Block count	NPV (MUS$)	Execution Time (Seconds)
Pseudoflow	486000	17855.97	6.76
Boykov-Kolmogorov [16]	40947	0.293	362.01
Push-Relabel [16]	40947	0.293	434.61
Ford and Fulkerson [23]	2366	2881.78	25
Lerchs and Grossman [23]	2366	0.880	45

4. CONCLUSIONS

In concluding our exploration of the open pit mine optimization, the Pseudoflow maximum flow method proved to be an effective computational tool for delineating the final pit, showcasing its methodological robustness and optimization capacity. The algorithm's adept handling of the block model and its ability to navigate economic variables demonstrate a significant advancement over traditional approaches. This study's findings underscore the efficacy of the Pseudoflow method in identifying the most economically viable pit configuration, with optimal pits showcasing favorable ore to waste ratios and solid financial prospects, as exemplified by the highest NPV in the

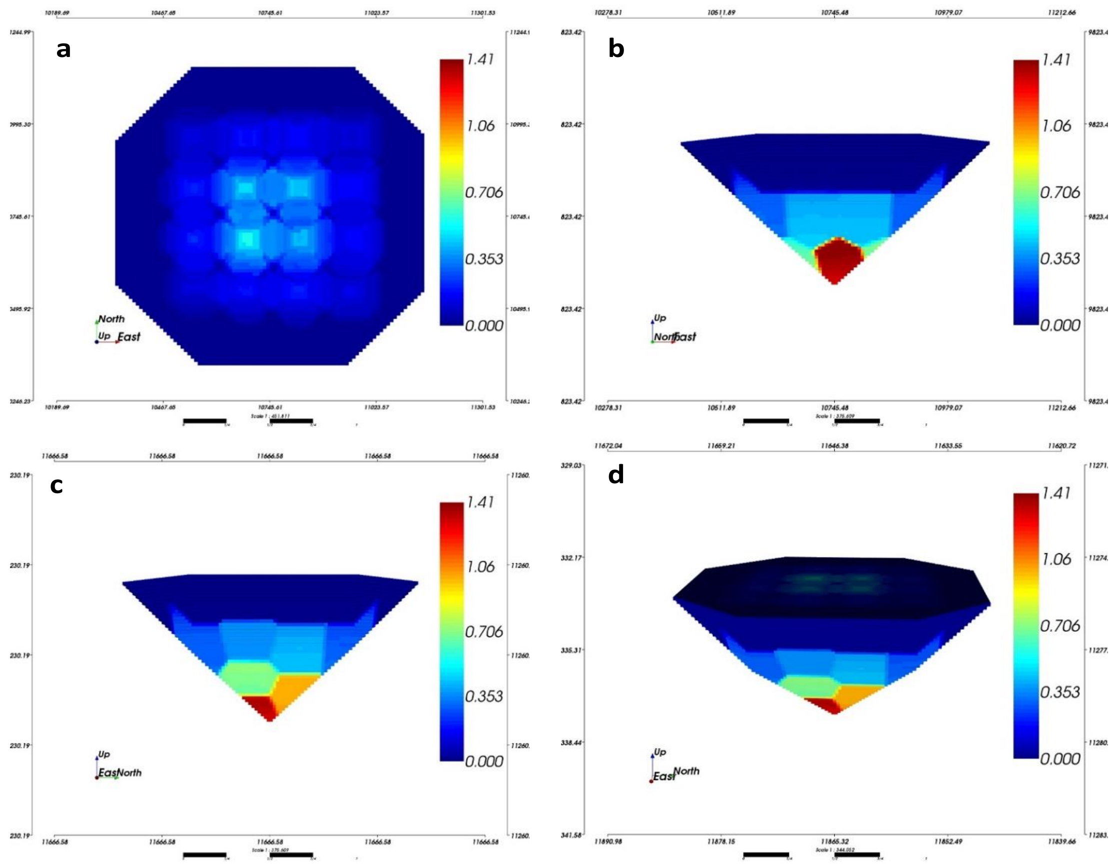

Figure 10. Pit 20. a) Plan view. b, c) Front views. d) Isometric view.

series of scenarios examined. Looking ahead, comparative analyses with classic algorithms, such as Lerchs and Grossman are proposed to further validate the Pseudoflow method's performance, which promises to refine the strategic decision-making process in the mining industry.

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