Journal of Computational Applied Mechanics 2024, 55 (2): 201-208

DOI:10.22059/JCAMECH.2024.373173.985

RESEARCH PAPER

Vibration Behaviour of Shear Deformable Laminated Plates Composed of Non-Homogeneous Layers

Abdullah Sofiyev [a, b, c,*], Tarkan Vergül [d], Isa Khalilov [e]

[a] *Department of Mathematics, Istanbul Ticaret University, Beyoglu, 34445 Istanbul, Turkey*
[b] *Scientific Research Department of Azerbaijan University of Architecture and Construction, Baku 1073, Azerbaijan*
[c] *Scientific Research Centers for Composition Materials of UNEC Azerbaijan State Economic University, Baku 1001, Azerbaijan*
[d] *Department of Structural Engineering, Engineering Faculty, Istanbul Ticaret University, 34445 Beyoglu/Istanbul, Türkiye*
[e] *Department of Machine Design and Industrial Technologies, Azerbaijan Technical University, H. Javid ave. 25, AZ1073, Baku, Azerbaijan*

Abstract

The free vibration behavior of laminated plates consisting of non-homogenous orthotropic layers is presented. First, the mechanical properties of laminated plates composed of non-homogenous (NH) orthotropic layers are modelled. After establishing the basic relations of laminated plates within shear deformation theory (SDT), governing equations are derived in the framework of Donnell type plate theory. The solution of the governing equations is carried out by the Galerkin method and the analytical expression is found for the linear frequency of plates composed of non-homogenous orthotropic layers. Finally, the influences of various factors such as shear stresses, non-homogeneity, number and arrangement of layers on the frequency of rectangular plates are examined.

Keywords: Non-homogeneity; shear deformations; laminated plates; vibration; frequency

1. Main text

With the increasing use of laminated composite plates in various engineering structures, their vibration behaviors are attracting more attention from researchers. The fact that the transverse shear modules of composite laminates are lower than the in-plane modules make the effect of transverse shear deformations more important as the plate thickness increases. Since classical plate theory, which neglects transverse shear deformation effects, can only predict the response of thin isotropic plates with reasonable accuracy, more improved theories need to be used for moderately-thick and thick plates. The first-order shear deformation theory proposed by Reissner [1] was extended to laminated plates by Yang et al. [2]. Some first-order theories have been developed to overcome the lack of a constant or uniform transverse shear stress distribution across the plate thickness [3]. Various higher order theories leading to parabolic distribution of transverse strain through the thickness have also been developed, and the shear correction factor is not used in these theories [4].

Although, in the early 2000s, the extensive use of non-homogeneous materials encouraged the development of more general and precise theories to provide a better representation of laminated non-homogeneous plate kinematics, these studies were limited to the response of homogeneous composite plates. There are few models that

* Corresponding author. *E-mail address:* aavey@ticaret.edu.tr

describe the structural response of non-homogeneous composite structural elements [5-7]. Some studies have been carried out in recent years on the combined effects of shear deformation and non-homogeneity on the frequencies of cross-ply laminated orthotropic structural elements [8-15].

Literature review indicates that the free vibrations of laminated plates consisting of non-homogeneous orthotropic materials within SDT have been studied a lot. The current paper is devoted to the solution of the free vibration problems of laminated non-homogeneous composite plates. The shear deformation theory of homogeneous laminated plates is extended to the nonhomogeneous ones. A wide variety of numerical results are presented for homogeneous and nonhomogeneous cross-ply laminates as per classical and shear deformation theories. The influences of the non-homogeneity, aspect ratio, thickness effect, number of layers and material anisotropy on the natural frequencies are studied in detail.

The remaining of present paper is arranged as follows: Section 2 presents the formulation of the problem. In Section 3, the governing equations of laminated plates consisting of non-homogeneous orthotropic materials are derived within SDT and the natural frequency is obtained using the Galerkin method. In Section 4, the convergence and accuracy of the solution are verified, and then the free vibration of laminated plates composed of non-homogeneous orthotropic materials in SDT is discussed. Finally, Section 5 provides conclusions.

2. Formulation of the problem

Consider a laminated rectangular plate of total thickness h, length a, width b composed of N orthotropic inhomogeneous layers. The geometry and coordinate system are shown in Fig. 1. The coordinate system $Oxyz$ is such that the mid-plane of the plate coincides with xy plane, and z axis is normal to the middle plane.

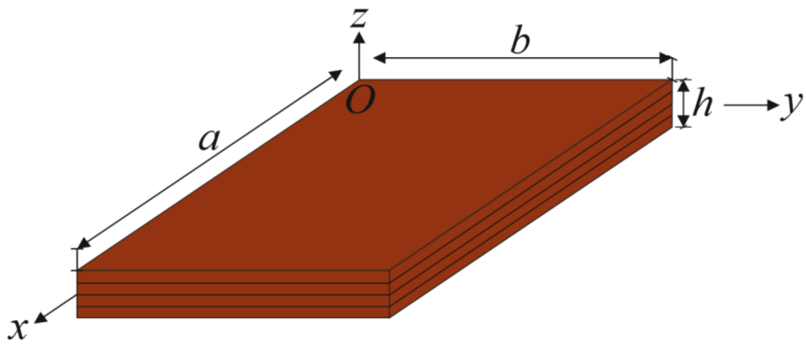

Fig.1. The geometry and coordinate system of the laminated plate

It is assumed that the layers of laminated plate are perfectly bonded to each other, they do not slip and remain elastic during deformation. The displacements in the x, y and z directions are indicated by u, w and w, respectively. Let $\Phi(x, y, t)$ be the Airy stress function for the stress resultants, so that [16],

$$N_{11} = \frac{\partial^2 \Phi}{\partial y^2},\ N_{22} = \frac{\partial^2 \Phi}{\partial x^2},\ \ N_{12} = -\frac{\partial^2 \Phi}{\partial x \partial y} \tag{1}$$

The Young moduli $\left(E_{ijz_1}^{(k)}, j = 1, 2\right)$ and shear modulus $\left(G_{ijz_1}^{(k)}, \imath = 1, 2, j = 2, 3\right)$ of lamina k^{th} is the power function of the thickness coordinate of layers and defined as follows [5, 6, 9]:

$$\begin{aligned} &E_{11z_1}^{(k)} = \left(1 + \mu z_1^{\ \alpha}\right) E_{11}^{0(k)},\ E_{22z_1}^{(k)} = \left(1 + \mu z_1^{\ \alpha}\right) E_{22}^{0(k)},\ G_{12z_1}^{(i)} = \left(1 + \mu z_1^{\ \alpha}\right) G_{12}^{0(k)}, \\ &G_{13z_1}^{(k)} = \left(1 + \mu z_1^{\ \alpha}\right) G_{13}^{0(k)},\ G_{23z_1}^{(k)} = \left(1 + \mu z_1^{\ \alpha}\right) G_{23}^{0(k)},\ \ z_1 = z / h \end{aligned} \tag{2}$$

where $\alpha = 1, 2$ is the material gradient exponent, μ indicate the non-homogeneity parameter for the elasticity moduli in the k^{th} layer of laminated plates, which characterizes its variation depending on the z_1 and $\mu \in [0,1]$. The symbols with "0" in the superscript indicate the mechanical properties of the homogeneous orthotropic material ($\mu = 0$). Since the Poisson ratio ($\nu_{12}^{(k)}$ and $\nu_{21}^{(k)}$) and density ($\rho_0^{(k)}$) in the layers vary little according to the thickness coordinate, they are considered constant and the following condition is satisfied: $\nu_{21}^{(k)} E_{11}^{0(k)} = \nu_{12}^{(k)} E_{22}^{0(k)}$.

3. Basic equations and solution method

Within SDT, the basic relations of the layer consisting of non-homogeneous orthotropic materials are expressed as follows [9]:

$$\begin{bmatrix} \tau_{11}^{(k)} \\ \tau_{22}^{(k)} \\ \tau_{12}^{(k)} \\ \tau_{13}^{(k)} \\ \tau_{23}^{(k)} \end{bmatrix} = \begin{bmatrix} D_{11z_1}^{(k)} & D_{12z_1}^{(k)} & 0 & 0 & 0 \\ D_{21z_1}^{(k)} & D_{22z_1}^{(k)} & 0 & 0 & 0 \\ 0 & 0 & D_{66z_1}^{(k)} & 0 & 0 \\ 0 & 0 & 0 & D_{55z_1}^{(k)} & 0 \\ 0 & 0 & 0 & 0 & D_{44z_1}^{(k)} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{bmatrix} \tag{3}$$

where $\left[\tau^{(k)}\right]$ and $\left[\varepsilon^{(k)},\gamma^{(k)}\right]$ indicate the stress and strain tensors in the lamina k^{th}, respectively, and $D_{ijz_1}^{(k)}$ are coefficients of reduced material stiffness for non-homogeneous orthotropic lamina (k^{th}),

$$\begin{gathered} D_{11z_1}^{(k)} = \frac{E_{11z_1}^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}},\ D_{22z_1}^{(i)} = \frac{E_{22z_1}^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}}, D_{12z_1}^{(k)} = \nu_{21}^{(k)} D_{11z_1}^{(k)} = \nu_{12}^{(k)} D_{22z_1}^{(k)} = D_{21z_1}^{(k)}, \\ D_{44z_1}^{(k)} = G_{23z_1}^{(i)},\ D_{55z_1}^{(k)} = G_{13z_1}^{(k)},\ D_{66z_1}^{(k)} = G_{12z_1}^{(k)}. \end{gathered} \tag{4}$$

Using fundamental relations, the equations of motion of laminated plates consisting of non-homogeneous orthotropic materials can be expressed based on SDT as follows:

$$\begin{aligned} & L_{11}(\Phi) + L_{12}(w) + L_{13}(\psi_1) + L_{14}(\psi_2) = 0 \\ & L_{21}(\Phi) + L_{22}(w) + L_{23}(\psi_1) + L_{24}(\psi_2) = 0 \\ & L_{31}(\Phi) + L_{32}(w) + L_{33}(\phi_1) + L_{34}(\phi_2) = 0 \\ & L_{41}(\Phi) + L_{42}(w) + L_{43}(\psi_1) + L_{44}(\psi_2) = \rho_1 \frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{5}$$

where L_{ij} are differential operators, $\psi_1(x,y,t)$ and $\psi_2(x,y,t)$ are rotations of the normal to the middle plane relative to the y and x axes, and $\rho_1 = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \rho_0^{(k)} \mathrm{d}z$.

Since all edges of the laminated plate are assumed to be simply supported, the solution of basic equations is sought as follows [4, 9, 14]:

$$\begin{aligned} & w = C_1 \sin(\lambda x)\sin(\eta y)\cos(\omega \mathrm{t}),\ \Phi = C_2 \sin(\lambda x)\sin(\eta y)\cos(\omega \mathrm{t}), \\ & \psi_1 = C_3 \cos(\lambda x)\sin(\eta y)\cos(\omega \mathrm{t}),\ \psi_2 = C_4 \sin(\lambda x)\cos(\eta y)\cos(\omega \mathrm{t}) \end{aligned} \tag{6}$$

where $C_i,\ (i=1,2,...,4)$ are amplitudes, ω is the free vibration frequency, $\lambda = m\pi/a$ and $\mu = n\pi/b$ are the wave parameters in which (m, n) is the vibration mode.

Substituting (6) into the set of Eqs. (5), the following set of algebraic equations is obtained:

$$\begin{bmatrix} p_{11} & -p_{12} & p_{13} & p_{14} \\ p_{21} & -p_{22} & p_{23} & p_{24} \\ p_{31} & -p_{32} & p_{33} & p_{34} \\ p_{41} & -p_{42}\omega^2 & p_{43} & p_{44} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

where

$$\begin{aligned}
&p_{11}=\left[\left(c_{11}-c_{31}\right)\lambda^2\eta^2+c_{12}\lambda^4\right]h, \quad p_{12}=\left(c_{14}+c_{32}\right)\lambda^2\eta^2+c_{13}\lambda^4,\\
&p_{13}=c_{15}\lambda^3+c_{35}\lambda\eta^2+q_3\lambda, \;\; p_{14}=(c_{18}+c_{38})\eta\lambda^2,\\
&p_{21}=\left[c_{21}\eta^4+\left(c_{22}-c_{31}\right)\lambda^2\eta^2\right]h, \quad p_{22}=\left(c_{32}+c_{23}\right)\lambda^2\eta^2+c_{24}\eta^4,\\
&p_{23}=(c_{25}+c_{35})\lambda\eta^2, \;\; p_{24}=c_{28}\mu^3+c_{38}\lambda^2\eta+q_4\eta,\\
&p_{31}=\left[b_{22}\lambda^4+\left(b_{12}+b_{21}+b_{31}\right)\lambda^2\eta^2+b_{11}\eta^4\right]h,\\
&p_{32}=b_{23}\lambda^4+\left(b_{24}+b_{13}-b_{32}\right)\lambda^2\eta^2+b_{14}\eta^4, \;\; p_{33}=b_{25}\lambda^3+(b_{15}+b_{35})\lambda\eta^2,\\
&p_{34}=(b_{28}+b_{38})\lambda^2\eta+b_{18}\eta^3, \;\; p_{41}=0, \;\; p_{42}=\rho_1, \;\; p_{43}=q_3\lambda, \;\; p_{44}=q_4\eta.
\end{aligned} \tag{8}$$

From Eq. (7), we obtain an expression for the frequency of laminated plates consisting of inhomogeneous orthotropic materials within SDT:

$$\Omega_{SDT}=\sqrt{\frac{p_{41}s_{41}+p_{43}s_{43}+p_{44}s_{44}}{s_{42}p_{42}}} \tag{9}$$

The dimensionless frequency parameter of laminated plates composed of nonhomogeneous orthotropic layers within SDT is defined as:

$$\Omega_{1SDT}=h\sqrt{\Omega_{SDT}\rho_0^{(k)}/E_{11}^{0(k)}} \tag{10}$$

Since the expressions (10) give the dimensional and dimensionless frequency values within the framework of classical plate theory (CPT) when the transverse shear stresses are eliminated in the basic relations, they are shown as Ω_{CPT} or Ω_{1CPT} in the table and graphs.

4. Results and Discussion

Three comparisons are made for the accuracy of the obtained formulas. When the calculations in the comparisons are made, vibration modes are not included in the tables for cases where the minimum values of the frequency parameter are obtained at $(m,n)=(1,1)$.

Example 1: In this example, the dimensionless frequency parameter of the laminated homogeneous orthotropic square plate with (0º/90º/0º)-sequence for CPT and SDT are compared with the results obtained in the study of Ref. [17] that used finite element method (See, Table 1). The dimensionless frequency parameter is expressed as, $\bar{\Omega}=\Omega\frac{a^2}{h}\sqrt{\frac{\rho_0^{(k)}}{E_{22}^{0(k)}}}$. The following material properties are used in the comparisons: $E_{11}^{0(k)}=175\text{GPa}$, $E_{22}^{0(k)}=7\,\text{GPa}$, $\nu_{12}^{(k)}=0.25, G_{12}^{0(k)}=G_{13}^{0(k)}=3.5\,\text{GPa}$, $G_{12}^{0(k)}=1.4\,\text{GPa}$, $\rho_0^{(k)}=1$ .It can be seen from Table 1 that the dimensionless frequency parameter obtained in this study are in good agreement with the results of Ref. [18] within SDT and CPT.

The new numerical results are performed for free vibration frequency of single-layer and laminated NH-orthotropic rectangular plates using Eq. (10). The $\mu=0$ corresponds to homogeneous case and is denoted as H. In the numerical analysis, the following layer arrangements were taken into account: The material characteristics are taken from the study of Reddy [4]:

$E_{11}^{0(k)}=2.069\times10^{11}$ Pa, $E_{22}^{0(k)}=2.069\times10^{10}$ Pa, $G_{12}^{0(k)}=G_{13}^{0(k)}=6.9\times10^{9}$ Pa, $G_{23}^{0(k)}=4.14\times10^{9}$ Pa, $\rho_0^{(k)}=1950\text{kg}/\text{m}^3$ and $\nu_{12}^{(k)}=0.3$. For subsequent examples use the following characteristics: $a/b=0.5,1.0,1.5,2.0,$ $a/h=15$ and $(m,n)=(1,1)$.

The variations of Ω_{1SDT} and Ω_{1CPT} for the laminated H and NH- orthotropic plates versus a/b are presented in Table 2 and Figs 2-5. As a/b increases, the values of Ω_{1SDT} and Ω_{1CPT} for single-layer and laminated plates increase. When the a/b ratio increases from 0.5 to 2.0, the effect of the NH-linear profile on the frequency of a laminated plate within SDT and CPT can be considered constant, although it changes slightly. While this effect is slightly erratic around (-3.83%) in the SDT framework, it is approximately (-4.3%) in the CPT framework. In laminated plates, although the non-homogeneity effect on the frequency reduces continuously as the a/b increases, this effect varies depending on the number and arrangement of layers.

Table 1. Comparison of dimensionless frequency parameter of laminated orthotropic square plates with (0°/90°/0°)-array within SDT and CPT

Arrangement of layer	(0°/90°/0°)	
Frequencies	$\bar{\Omega}_{SDT}$	$\bar{\Omega}_{CPT}$
a/h	20	100
Present study	14.422	15.193
Ref. [17]	14.004	15.041

When the a/b ratio increases from 0.5 to 2.0, the effect of the NH-quadratic profile on the frequency of laminated plates in the framework of SDT and CPT is more evident than the linear profile and can be considered constant, although it changes slightly. For example, in the SDT frame, the effect of the NH-quadratic profile on the frequency is slightly irregular, around +6.6%, while in the CPT frame, that effect is about +7.2%. In laminated plates, the non-homogeneity effect on frequency decreases continuously with the increase of the a/b ratio, but this effect varies depending on the number and arrangement of layers.

Within the framework of SDT, when comparing the laminated plate with the single layer plate, the most significant effect of the layer arrangement on the frequency occurs with (-74%) in the (90°/0°/90°)-aligned plate, while the weakest effect occurs with 6% in the (0°/90°/0°)-array plate. It is seen that the layer arrangement in symmetrical arrangements in square plates has little effect on the frequency compared to the single layer.

When the a/b ratio increases, the effect of shear deformations on the frequency increases significantly in NH-linear and NH-quadratic plates with all-layer arrangement, and that effect is more evident in the NH-quadratic profile. For example, for the NH-linear profile, in (0°), (0°/90°), (0°/90°/0°), (90°/0°/90°), (0°/90°/90°/0°) and (90°/0°/0°/90°)-aligned plates, the effects of shear deformations on frequency rise from 1.97% to 8.71%, from 5.29% to 6.65%, from 1.65% to 24.26%, from 4.76% to 8.8% and from 1.88% to 22.6%, while for the NH-quadratic profile, those effects rise from 1.93% to 10.43%, from 6.44% to 7.59%, from 2.11% to 27.93%, from 5.84% to 9.72% and from 2.26% to 26.24%, as the a/b ratio increases from 0.5 to 2 (see, Figs.2-5).

As can be seen from the ratios, when the NH-linear profile is compared with the homogeneous profile, the effect of SD on frequency is more evident in the homogeneous profile, while when the NH-quadratic case is compared with the homogeneous case, the effect of SD on frequency is more evident in the NH-quadratic profile.

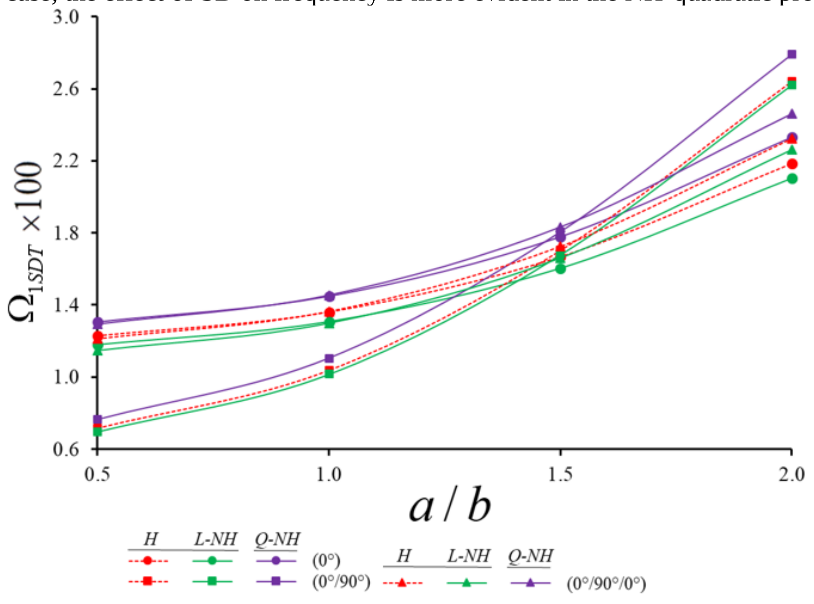

Fig. 2. Variation of the Ω_{1SDT} for (0°) single-layer and (0°/90°) and (0°/90°/0°)-array laminated plates with H and NH-linear and quadratic profiled layers versus the a/b

Table 2. Variation of the Ω_{1SDT} and Ω_{1CPT} for laminated plates with H and NH-linear and quadratic profiled layers versus the a/b

	NH-linear profile											
a/b	(0°)				(0°/90°)				(0°/90°/0°)			
	CPT		SDT		CPT		SDT		CPT		SDT	
	H	NH	H	NH	H	NH	H	NH	H	NH	H	NH
0.5	1.306	1.250	1.227	1.180	0.731	0.711	0.718	0.697	1.286	1.211	1.211	1.147
1	1.445	1.384	1.360	1.308	1.070	1.044	1.039	1.016	1.445	1.370	1.363	1.299
1.5	1.771	1.695	1.668	1.604	1.807	1.771	1.703	1.679	1.831	1.755	1.725	1.661
2	2.334	2.235	2.187	2.103	2.926	2.872	2.640	2.622	2.501	2.423	2.325	2.262
a/b	NH-quadratic profile											
0.5	1.306	1.400	1.227	1.308	0.731	0.778	0.718	0.763	1.286	1.382	1.211	1.293
1	1.445	1.550	1.360	1.450	1.070	1.140	1.039	1.104	1.445	1.550	1.363	1.454
1.5	1.771	1.899	1.668	1.779	1.807	1.925	1.703	1.805	1.831	1.957	1.725	1.832
2	2.334	2.503	2.187	2.330	2.926	3.115	2.640	2.790	2.501	2.662	2.325	2.460
a/b	NH-linear profile				NH-quadratic profile							
	(90°/0°/90°)											
	CPT		SDT		CPT		SDT					
0.5	0.625	0.605	0.613	0.595	0.625	0.665	0.613	0.651	$a/h=15,\ (m,n)=(1,1)$			
1	1.445	1.370	1.323	1.264	1.445	1.550	1.323	1.406				
1.5	2.972	2.802	2.457	2.352	2.972	3.191	2.457	2.594				
2	5.146	4.846	3.798	3.656	5.146	5.528	3.798	3.984				

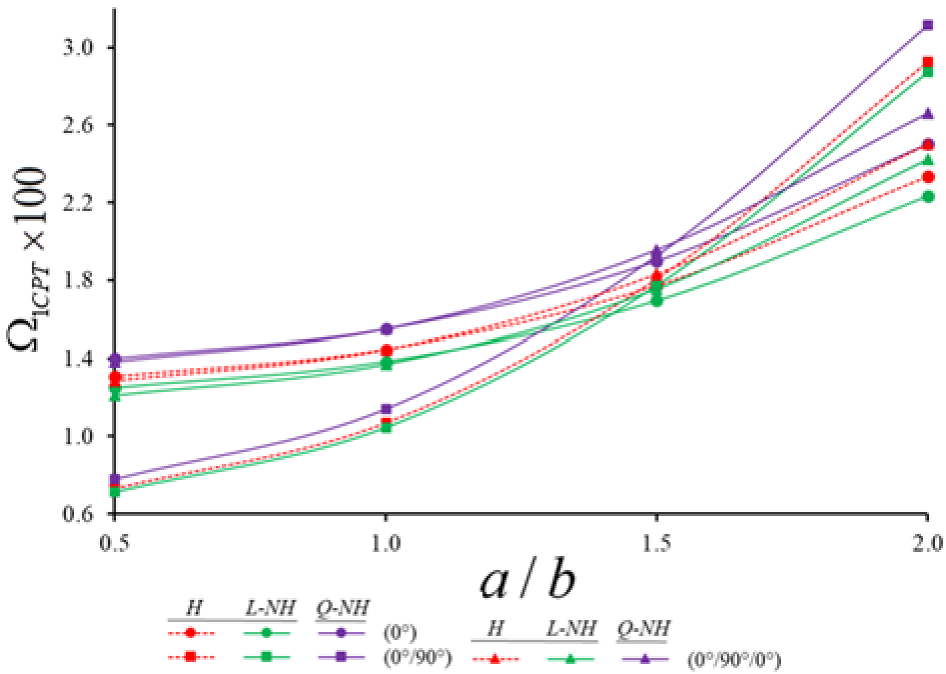

Fig. 3. Variation of the Ω_{1CPT} for (0°) single-layer and (0°/90°) and (0°/90°/0°)-array laminated plates with H and NH-linear and quadratic profiled layers versus the a/b

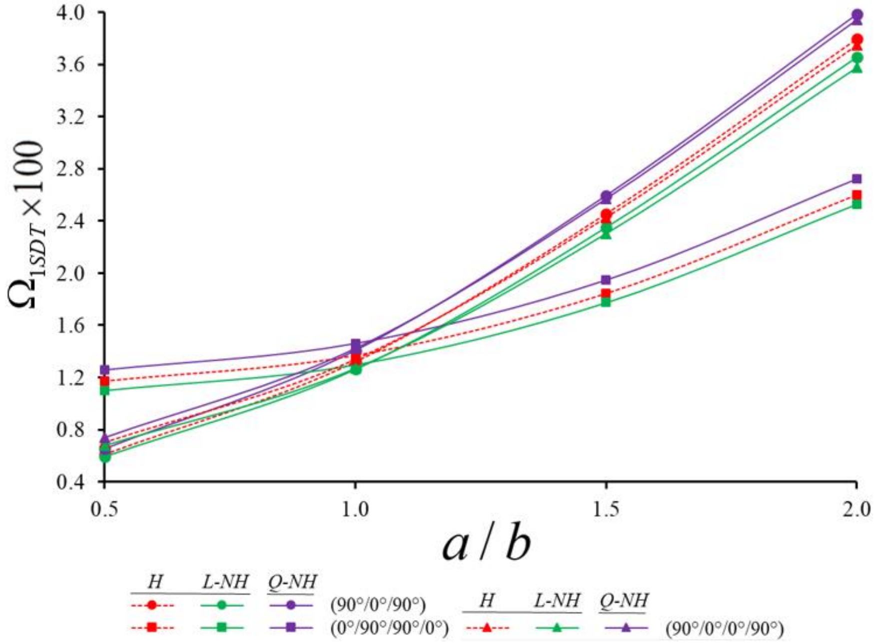

Fig. 4. Variation of the Ω_{1SDT} for (90º/0º/90º) single-layer and (0º/90º/90º/0º) and (90º/0º /0º/90º)-array plates with H and NH-linear and quadratic profiled layers versus the a/b

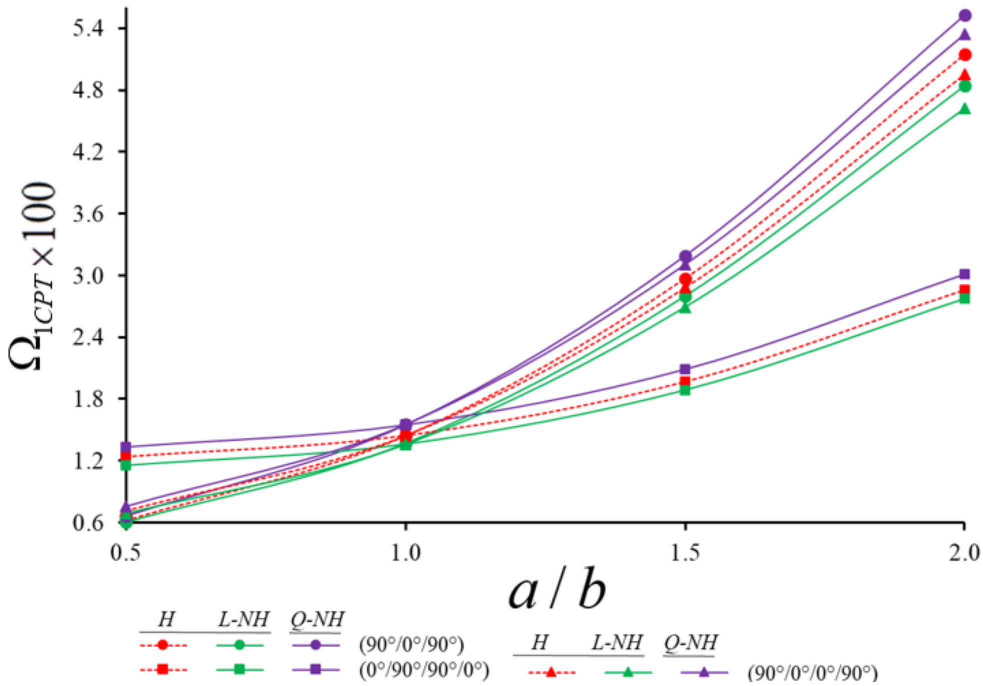

Fig. 5. Variation of the Ω_{1CPT} for (90º/0º/90º) single-layer and (0º/90º/90º/0º) and (90º/0º /0º/90º)-array plates with H and NH-linear and quadratic profiled layers versus the a/b

5. Conclusions

The free vibration behavior of laminated plates consisting of non-homogeneous orthotropic layers is presented within the framework of shear deformation theory. After the basic relationships of laminated plates are established according to the generalized Hooke's rule, the basic equations are derived within the framework of Donnell-type plate theory. The solution of the governing equations is carried out by the Galerkin method and the analytical expression for the linear frequency of plates consisting of non-homogeneous orthotropic layers is found. Finally, the effects of various factors such as shear stresses, inhomogeneity, number and arrangement of layers on the free vibration frequency of rectangular plates are examined.

References

[1] E. Reissner, The effect of transverse shear deformation on the bending of elastic plates, 1945.

[2] P. C. Yang, C. H. Norris, Y. Stavsky, Elastic wave propagation in heterogeneous plates, *International Journal of Solids and Structures,* Vol. 2, No. 4, pp. 665-684, 1966/10/01/, 1966.

[3] M. E. Fares, M. N. M. Allam, Vibrations of an anisotropic rectangular plate including the effect of shear deformation and rotatory inertia, *Mechanics Research Communications,* Vol. 19, No. 3, pp. 209-218, 1992/05/01/, 1992.

[4] J. N. Reddy, 2003, *Mechanics of laminated composite plates and shells: theory and analysis*, CRC press,

[5] M. E. Fares, A. M. Zenkour, Buckling and free vibration of non-homogeneous composite cross-ply laminated plates with various plate theories, *Composite Structures,* Vol. 44, No. 4, pp. 279-287, 1999/04/01/, 1999.

[6] A. H. Sofiyev, E. Schnack, The buckling of cross-ply laminated non-homogeneous orthotropic composite conical thin shells under a dynamic external pressure, *Acta Mechanica,* Vol. 162, pp. 29-40, 05/01, 2003.

[7] R. M. J. Groh, P. M. Weaver, A computationally efficient 2D model for inherently equilibrated 3D stress predictions in heterogeneous laminated plates. Part I: Model formulation, *Composite Structures,* Vol. 156, pp. 171-185, 2016/11/15/, 2016.

[8] J.-Q. Tarn, Y.-B. Wang, Y.-M. Wang, Three-dimensional asymptotic finite element method for anisotropic inhomogeneous and laminated plates, *International Journal of Solids and Structures,* Vol. 33, No. 13, pp. 1939-1960, 1996/05/01/, 1996.

[9] A. H. Sofiyev, N. Kuruoglu, Combined influences of shear deformation, rotary inertia and heterogeneity on the frequencies of cross-ply laminated orthotropic cylindrical shells, *Composites Part B: Engineering,* Vol. 66, pp. 500-510, 2014/11/01/, 2014.

[10] F. G. Flores, S. Oller, L. G. Nallim, On the analysis of non-homogeneous laminates using the refined zigzag theory, *Composite Structures,* Vol. 204, pp. 791-802, 2018.

[11] V. C. Haciyev, A. H. Sofiyev, N. Kuruoglu, Free bending vibration analysis of thin bidirectionally exponentially graded orthotropic rectangular plates resting on two-parameter elastic foundations, *Composite Structures,* Vol. 184, pp. 372-377, 2018/01/15/, 2018.

[12] M. Bacciocchi, A. Tarantino, Natural Frequency Analysis of Functionally Graded Orthotropic Cross-Ply Plates Based on the Finite Element Method, *Mathematical and Computational Applications,* Vol. 24, pp. 52, 05/19, 2019.

[13] M. Li, R. Yan, L. Xu, C. Guedes Soares, A general framework of higher-order shear deformation theories with a novel unified plate model for composite laminated and FGM plates, *Composite Structures,* Vol. 261, pp. 113560, 01/09, 2021.

[14] C. Kumar, A. Kumar, Initial Buckling and Free Vibration Analysis of Elastically Supported Laminated Plates Using the Meshless Technique, *Journal of Multiscale Modelling,* Vol. 13, No. 03, pp. 2250004, 2022/09/01, 2022.

[15] A. H. Sofiyev, A new approach to solution of stability problem of heterogeneous orthotropic truncated cones with clamped edges within shear deformation theory, *Composite Structures,* Vol. 304, pp. 116411, 11/01, 2022.

[16] A. d. S. Vol′mir, *The nonlinear dynamics of plates and shells,* pp. 1974.

[17] M. Rastgaar, M. Mahinfalah, R. Jazar, Natural frequencies of laminated composite plates using third order shear deformation theory, *Composite Structures,* Vol. 72, pp. 273-279, 03/01, 2006.