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RESEARCH PAPER

The performance of the non-destructive tests in predicting the uniaxial compressive strength of the limestone: A Case study of Asmari Formation, Lorestan Province, western Iran

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Abstract
The uniaxial compressive strength (UCS) is one of the most important resistance parameters of rocks in the site investigation of a geotechnical project built upon or within the rock. The P–wave velocity (Vp) and Schmidt hardness (SH) are nondestructive tests that are frequently used for predicting the UCS due to their rapidity and easiness. The present paper aims to investigate the performance of Vp and SH to predict the UCS of the limestone using simple and multiple regression analyses. For this purpose, twenty limestone samples were collected from Asmari formation, Lorestan Province (western Iran) and their UCS, Vp, and SH were determined. The simple and multiple regression equations have been developed for predicting the UCS from Vp and SH. To check the accuracy and validity of the regression equations, the determination coefficient (R^2), standard error of estimate (SEE), the diagonal line (1:1), and analysis of variance (ANOVA) were used. In addition, the validity and performance of the regression equations in predicting the UCS were investigated using the raw data obtained from the experimental works of several researchers and statistical indices [including coefficient values accounted for (VAF) and the root mean square error (RMSE)]. Based on simple regression analysis, there are moderate correlations between UCS with Vp and SH with the R^2 values of 0.86 and 0.71, respectively, whereas there is a strong multiple correlation with an R^2 of 0.92 for predicting the UCS when both Vp and SH are considered. According to the results of R^2, SEE, diagonal line, and variance analysis, the multiple regression equation was more reliable than the simple regression equations for predicting the UCS. Overall, it was concluded that the multiple regression equation has acceptable performance for predicting the UCS of the limestone in other regions of the world and thus rapidly and indirectly assess the UCS. As a result, the multiple regression equation avoids the UCS test, which is cumbersome and time-consuming for determining the UCS of the rocks.

Keywords: Limestone, P–Wave Velocity, Regression Analyses, Schmidt Hardness, Uniaxial Compressive Strength

Introduction

The uniaxial compressive strength (UCS) of an intact rock is a key parameter in the design and

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construction of most geotechnical projects that interact with the rock, such as slope stability, underground excavation, dams, foundations on rock, and rock classification for engineering purposes (Lashkaripour, 2002; Heidari et al., 2012; Sadeghi et al., 2022). Commonly, the UCS of a rock is determined directly by laboratory tests following the standards of the American Society for Testing and Materials (ASTM) or the International Society for Rock Mechanics (ISRM). However, UCS testing is time-consuming, tedious, and expensive. In addition, performing the UCS test requires rock specimens with appropriate dimensions (i.e., size and shape). In many cases, such as laminated sedimentary rocks, metamorphic rocks with schistosity, and highly weathered rocks, it is difficult to obtain standard-sized rock specimens for UCS testing (Basu & Kamran, 2010; Akbay, 2023). For these reasons, some methods are often utilized for indirect assessment of UCS.

The P–wave velocity (Vp) and Schmidt hardness (SH) tests are among the most common indirect methods used by various researchers for predicting the UCS of the rocks (Tuğrul & Zarif, 1999; Fener et al., 2005; Diamantis et al., 2011; Azimian & Ajalloeian, 2015; Celik, 2019; Valido et al., 2024). Vp and SH tests are easy to apply, both for laboratory and site conditions. These tests are increasingly used in various engineering fields such as civil, geotechnical, mining, and geology due to their simplicity and rapid execution, low-cost, and non-destructiveness. Because of these advantages, Vp and SH tests provide a fast, low-cost, and effective way to predict UCS of rocks during the preliminary site investigation of geotechnical projects.

Several researchers have studied the correlations between UCS with Vp and SH. Tables 1 and 2 present the correlations developed for predicting the UCS from Vp and SH, respectively. In the following, some of these correlations are discussed in detail. Kahraman (2001) correlated the UCS of different rock types with Vp via a power equation and obtained a determination coefficient (R^2) of 0.69. Based on experimental tests on limestones, sandstones, marbles, and basalts, Yasar and Erdogan (2004b) established a positive correlation between UCS and SH with an R^2 of 0.79. Kilic and Teymen (2008) described a power correlation between UCS and Vp with an excellent R^2 equal to 0.94 for sedimentary, metamorphic, and igneous rocks. These researchers also found the same correlation with an R^2 of 0.94 between UCS and SH. Based on the results of the laboratory tests on sedimentary, metamorphic, and igneous rocks by Karaman et al. (2015), a linear correlation ($R^2 = 0.71$) was obtained between UCS and SH. Azimian and Ajalloeian (2015) developed a strong linear correlation between UCS and Vp with a good R^2 of 0.91 for marly rocks. Based on the results of Jamshidi et al. (2016) on the different types of travertines, a logarithmic correlation with an R^2 of 0.90 between UCS and Vp was established. Abdi and Khanlari (2019) reported a moderate linear correlation ($R^2 = 0.83$) between UCS and SH for sandstones. By experimental studies on the marbles, dolomites, limestones, and travertines, Celik (2019) recommended an exponential correlation between UCS and SH with an R^2 equal to 0.78. This researcher also obtained the same correlation between UCS and Vp with a moderate R^2 of 0.61. Song et al. (2020) tested for UCS and Vp of some coal with the aim to investigate the correlation between these parameters. The results indicated that there is a moderate exponential correlation ($R^2 = 0.64$) between the UCS and Vp. An exponential correlation with a moderate R^2 of 0.69 was obtained between UCS and SH of sedimentary, metamorphic, and igneous rocks in the study of Teymen (2021). Rahimi et al. (2022) conducted a study on gypsum for the same purpose and developed a positive power correlation between UCS and SH with an R^2 equal to 0.82. A linear correlation between UCS and Vp (R^2 of 0.72) for some marbles was established by Ahmad et al. (2023). Based on the experimental results, Abdi et al. (2024) reported a power correlation between UCS and Vp for sandstones with an R^2 equal to 0.64. Ajalloeian et al. (2024) described a weak linear correlation with an R^2 of 0.52 between UCS and SH for different granitic rocks. Valido et al. (2024) correlated UCS with Vp and SH for different types of ignimbrites. The results of these researchers revealed the linear correlations between UCS with Vp and SH with good R^2 values of 0.88 and 0.84, respectively.

Table 1. Correlation between UCS and Vp reported in the previous studies

References	Rock type	Equation form	Equation	R^2
Tuğrul and Zarif (1999)	Igneous	Linear	$\mathbf{UCS} = 35.54\mathbf{Vp} - 55$	0.80
Kahraman (2001)	Different rock types	Power	$\mathbf{UCS} = 9.95\mathbf{Vp}^{1.21}$	0.69
Yasar and Erdogan (2004a)	Limestone, marble, dolomite	Linear	$\mathbf{UCS} = (\mathbf{Vp} - 2.0195)/0.032$	0.66
Cobanoglu and Celik (2008)	Sandstone, limestone, cement mortar	Linear	$\mathbf{UCS} = 56.71\mathbf{Vp} - 192.93$	0.67
Kilic and Teymen (2008)	Sedimentary, metamorphic, igneous	Power	$\mathbf{UCS} = 2.304\ \mathbf{Vp}^{2.4315}$	0.94
Sharma and Singh (2008)	Sedimentary, metamorphic, igneous	Linear	$\mathbf{UCS} = 0.0642\mathbf{Vp} - 117.99$	0.90
Diamantis et al. (2011)	Peridotite	Linear	$\mathbf{UCS} = 0.14\mathbf{Vp} - 899.33$	0.83
Sarkar et al. (2012)	Different rock types	Linear	$\mathbf{UCS} = 0.038\mathbf{Vp} - 50$	0.93
Khandelwal (2013)	Different rock types	Linear	$\mathbf{UCS} = 0.033\mathbf{Vp} - 34.83$	0.87
Azimian and Ajalloeian (2015)	Marl	Linear	$\mathbf{UCS} = 0.026\mathbf{Vp} - 20.47$	0.91
Jamshidi et al. (2016)	Travertine	Logarithmic	$\mathbf{UCS} = 90.08 \ln(\mathbf{Vp}) - 709.65$	0.90
Jamshidi et al. (2018b)	Limestone	Logarithmic	$\mathbf{UCS} = 131.77 \ln(\mathbf{Vp}) - 1048$	0.82
Abdi and Khanlari (2019)	Sandstone	Linear	$\mathbf{UCS} = 0.041\mathbf{Vp} - 15.40$	0.88
Celik (2019)	Marble, dolomite, limestone, travertine	Exponential	$\mathbf{UCS} = 2.6837e^{0.5495\mathbf{Vp}}$	0.61
Saldana et al. (2020)	Travertine	Linear	$\mathbf{UCS} = -123.37 + 41.13\mathbf{Vp}$	0.60
Song et al. (2020)	Coal	Exponential	$\mathbf{UCS} = 3.21e^{1.04\mathbf{Vp}}$	0.64
Azadmehr et al. (2021)	Sandstone	Linear	$\mathbf{UCS} = 32.072\mathbf{Vp} - 76.896$	0.69
Cherifi et al. (2021)	Schist	Exponential	$\mathbf{UCS} = 61.857e^{0.187\mathbf{Vp}}$	0.92
Zhang et al. (2021)	Sandstone	Linear	$\mathbf{UCS} = 0.03\mathbf{Vp} - 23.778$	0.35
Fereidooni and Sousa (2022)	Limestone, sandstone	Power	$\mathbf{UCS} = 0.6376\mathbf{Vp}^{3.0447}$	0.87
Ahmed et al. (2023)	Marble	Linear	$\mathbf{UCS} = 0.0067\mathbf{Vp} + 26.567$	0.72
Abdi et al. (2024)	Sandstone	Power	$\mathbf{UCS} = 14.64\mathbf{Vp}^{1.742}$	0.64
Valido et al. (2024)	Ignimbrite	Linear	$\mathbf{UCS} = 0.058\mathbf{Vp} - 165.598$	0.88

UCS: Uniaxial compressive strength, Vp: P–wave velocity

The results of various researchers indicated that the UCS of the rocks increases with increasing Vp and SH. In the previous studies, some simple and multiple equations were developed to predict UCS from the Vp and SH. However, comparative studies of Vp and SH accuracy in predicting the UCS considering the type of regression equation (i.e., simple or multiple) remain rare. Therefore, the present study investigates that when both the Vp and SH are considered together (through multiple regression analysis), more accurate correlations for predicting the UCS can be achieved or not?. In addition, the evaluating the UCS using the multiple regression analysis can be associated with prediction errors; thus, it is necessary to investigate the performance of the predictive equations.

In the present study, the UCS, Vp, and SH of twenty limestone samples were determined. Based on the obtained data, two main aims have been pursued: i) developing the correlations between UCS with Vp and SH using simple and multiple regression analyses, and comparing their prediction accuracy, ii) investigating the validity and performance of the multiple regression equation for predicting the UCS.

Materials and methods

Twenty limestone samples from outcrops of the Asmari formation located in the northern of Khorramabad city (Lorestan Province, western Iran), were collected. Fig.1 shows the geological map of the study area and some block samples. The Oligo-Miocene Asmari formation was deposited in a subtropical environment in a NE- SW oriented Zagros basin. This formation is mostly composed of limestone and marly limestone, lithic and limy sandstone (Vaziri-Moghaddam et al., 2006). Due to the widespread outcrops of the limestones, they are generally used as construction materials in some local geotechnical projects located in the study area.

Table 2. Correlation between UCS and SH reported in the previous studies

References	Rock type	Equation form	Equation	R^2
Singh et al. (1983)	Sedimentary	Linear	$\mathbf{UCS} = 2\mathbf{SH}$	0.72
Haramy and DeMarco (1985)	Different rock types	Linear	$\mathbf{UCS} = 0.994\mathbf{SH} - 0.383$	0.70
Gokceoglu (1996)	Marl	Power	$\mathbf{UCS} = 0.0001\mathbf{SH}^{3.2658}$	0.84
Katz et al. (2000)	Chalk, limestone, marble, granite	Exponential	$\mathbf{UCS} = 2.21e^{0.07\mathbf{SH}}$	0.96
Kahraman (2001)	Different rock types	Exponential	$\mathbf{UCS} = 6.97e^{0.014\,\mathbf{SH}\times\rho}$	0.61
Yilmaz and Sendir (2002)	Gypsum	Exponential	$\mathbf{UCS} = \exp(0.818 + 0.059\mathbf{SH})$	0.96
Yasar and Erdogan (2004b)	Limestone, marble, basalt, sandstone	Power	$\mathbf{UCS} = 0.000004\mathbf{SH}^{4.2917}$	0.79
Fener et al. (2005)	Different rock types	Exponential	$\mathbf{UCS} = 4.24e^{0.059\mathbf{SH}}$	0.66
Kilic and Teymen (2008)	Sedimentary, metamorphic, igneous	Power	$\mathbf{UCS} = 0.0137\mathbf{SH}^{2.2721}$	0.94
Cobanoglu and Celik (2008)	Sandstone, limestone, cement mortar	Linear	$\mathbf{UCS} = 6.59\mathbf{SH} - 212.63$	0.65
Yagiz (2009)	Carbonate, metamorphic	Power	$\mathbf{UCS} = 0.0028\mathbf{SH}^{2.584}$	0.85
Yurdakul et al. (2011)	Carbonate	Linear	$\mathbf{UCS} = 0.0682\mathbf{SH} + 57.973$	0.62
Minaeian and Ahangari (2013)	Conglomerate	Linear	$\mathbf{UCS} = 0.678\mathbf{SH}$	0.93
Karaman et al. (2015)	Sedimentary, metamorphic, igneous	Linear	$\mathbf{UCS} = 3.66\mathbf{SH} - 63$	0.71
Jobli et al. (2017)	Granite	Exponential	$\mathbf{UCS} = 6.31e^{0.057\mathbf{SH}}$	0.92
Jamshidi et al. (2018a)	Sandstone	Logarithmic	$\mathbf{UCS} = 58.35\ln(\mathbf{SH}) - 154.6$	0.73
Abdi and Khanlari (2019)	Sandstone	Linear	$\mathbf{UCS} = 3.615\mathbf{SH} - 42.57$	0.83
Celik (2019)	Marble, dolomite, limestone, travertine	Exponential	$\mathbf{UCS} = 4.2281e^{0.0547\mathbf{SH}}$	0.78
Teymen (2021)	Sedimentary, metamorphic, igneous	Exponential	$\mathbf{UCS} = 5.56e^{0.06\mathbf{SH}}$	0.69
Fereidooni and Sousa (2022)	Limestone, sandstone	Linear	$\mathbf{UCS} = 3.9184\mathbf{SH} - 77.063$	0.96
Rahimi et al. (2022)	Gypsum	Power	$\mathbf{UCS} = 0.0319\mathbf{SH}^{2.0748}$	0.82
Ahmed et al. (2023)	Marble	Logarithmic	$\mathbf{UCS} = 57.911\ln(\mathbf{SH}) - 181.44$	0.88
Ajalloeian et al. (2024)	Granitic rocks	Linear	$\mathbf{UCS} = 2.6432\mathbf{SH} - 49.299$	0.52
Valido et al. (2024)	Ignimbrite	Linear	$\mathbf{UCS} = 2.67\mathbf{SH} - 118.548$	0.84

UCS: Uniaxial compressive strength, SH: Schmidt hardness

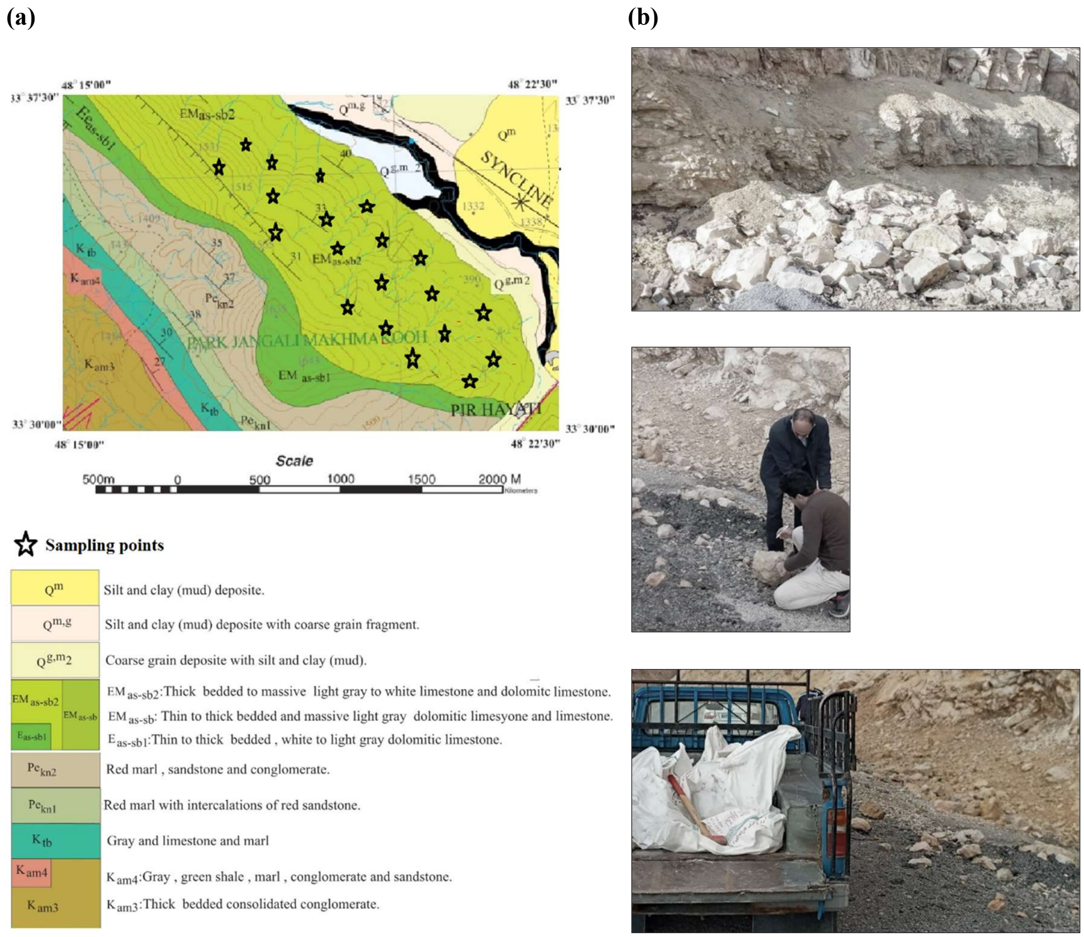

Figure 1. (a) Geological map of study area and sampling points, and (b) some of collected limestone blocks during the sampling

Also, limestones have been used as a heritage stone in rural areas for building farmland walls. The collected block samples were varied from ~ 20 × 25 × 25 to ~ 25 × 30 × 30 cm in size and taken at distances of ~ 150 to 200 m from each other. Next, the block samples were transferred to the Laboratory of the Engineering Geology of Lorestan University in Khorramabad, Iran. Then, cylindrical core specimens from block samples of each limestone type were prepared for the UCS, Vp, and SH tests. With the help of a polishing and lapping machine, the ends of the specimens were made flat and perpendicular to the axis of the specimens within 0.05 mm in 50 mm and their sides were smoothed and polished. The materials and methods of different experiments are presented in Table 3.

Test procedures

Uniaxial compressive strength (UCS) test

The UCS of the samples was determined following the method suggested by the ISRM (1981). Fig. 2 shows the setup of the UCS device and some of the specimens prepared to perform tests. During the UCS test, the stress applied to the specimens was controlled at a rate of approximately 0.5 MPa/s. The maximum load at the failure moment was recorded to calculate the UCS of the specimens. The average UCS values of the samples are presented in Table 4.

Table 3. Information about the materials and methods used to preform UCS, Vp, and SH tests

Property	Specimen shape	Specimen size			Specimen status	Specimen number	Source
		Diameter (mm)	Length (mm)	D to L			
UCS	Cylindrical core	54	108	2	Dry	5	ISRM (1981)
Vp	Cylindrical core	54	108	2	Dry	5	ISRM (1981)
SH	Cylindrical core	54	135	2.5	Dry	5	ISRM (1981)

Table 4. UCS, Vp, and SH of the limestone samples

Sample code	UCS (MPa)			Vp (m/s)			SH		
	Min.	Max.	Ave.	Min.	Max.	Ave.	Min.	Max.	Ave.
Sample 1	77.3	80.8	79.2	5102	5363	5245	53	58	55
Sample 2	75.4	80.6	78.0	4987	5232	5100	48	54	52
Sample 3	71.1	74.5	73.9	4707	5003	4890	48	52	49
Sample 4	67.0	71.3	68.5	4622	4844	4765	44	51	47
Sample 5	68.7	73.4	71.2	4651	4842	4700	41	47	45
Sample 6	64.2	69.0	66.9	4688	4876	4762	38	44	42
Sample 7	78.8	72.6	70.8	4509	4730	4653	39	46	42
Sample 8	75.4	81.1	78.3	5180	5432	5303	49	54	50
Sample 9	66.2	69.3	67.1	4653	4876	4732	33	38	36
Sample 10	57.6	63.7	61.1	4230	4455	4311	32	38	36
Sample 11	68.3	72.8	70.3	4840	5046	4904	37	43	39
Sample 12	62.3	66.4	65.3	4398	4566	4450	41	46	44
Sample 13	60.0	64.1	62.2	4298	4562	4412	36	43	38
Sample 14	69.4	74.7	72.9	4806	5069	4923	40	46	44
Sample 15	61.5	66.0	64.1	4465	4690	4567	32	37	34
Sample 16	65.3	70.2	68.6	4500	4701	4580	37	42	39
Sample 17	70.1	73.6	71.0	4707	4953	4810	46	52	50
Sample 18	66.6	70.0	67.8	4402	4678	4530	38	44	40
Sample 19	59.4	64.1	62.1	4374	4596	4423	35	40	37
Sample 20	69.6	73.2	70.2	4409	4685	4589	44	51	48

It can be seen from this table that samples 10 and 1 have the lowest and the highest UCS values with the 61.1 and 79.2 MPa, respectively. The samples were classified according to their UCS values as suggested by the IAEG (1979). According to Fig. 3, all samples fall into the rock class with strong strength (UCS 50–120 MPa).

P–wave velocity (Vp) test

The specimens were tested to determine their Vp in accordance with the ISRM (1981) (Fig. 2). To perform Vp tests, end surfaces of the specimens were covered with stiffer grease to provide a good coupling between the transducer face and the specimen surface to maximize the accuracy of the transit time measurement. The Vp of each specimen was calculated from the travel time from the generator to a receiver at the opposite end. The average values of the samples Vp are given in Table 4. According to this table, the sample 10 has the lowest Vp with 4311 m/s, while the highest Vp (5303 m/s) belongs to sample 8.

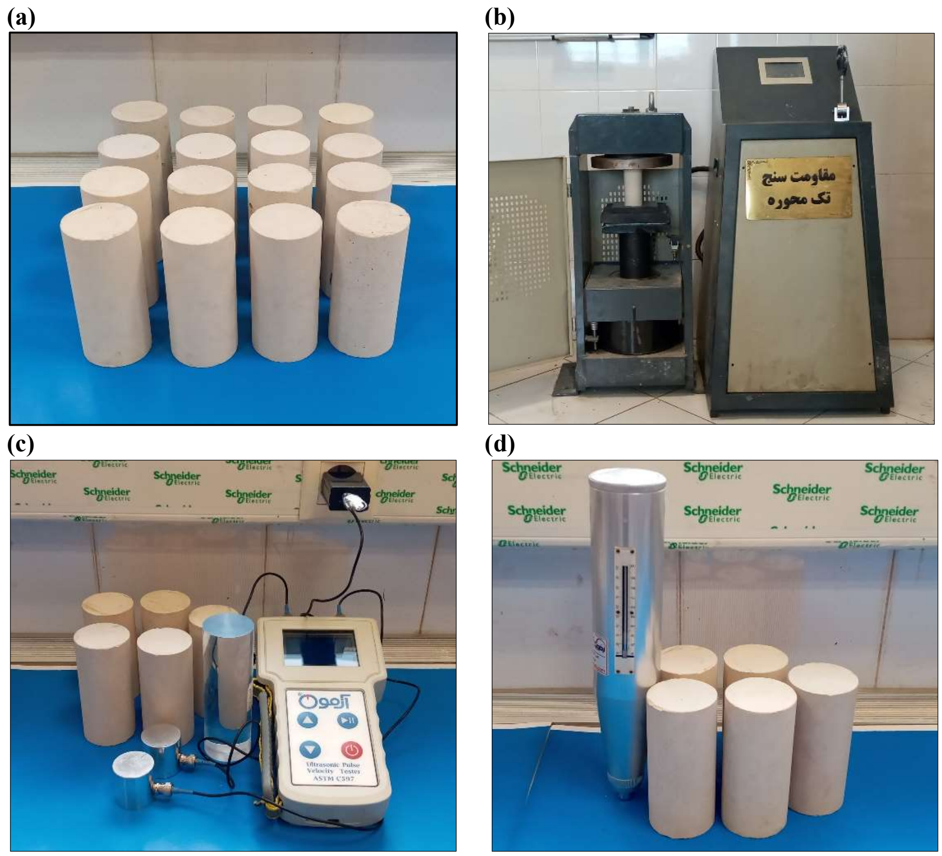

Figure 2. (a) Some specimens prepared for laboratory tests including (b) uniaxial compressive strength (UCS), (c) P–wave velocity (Vp), and (d) Schmidt hardness (SH)

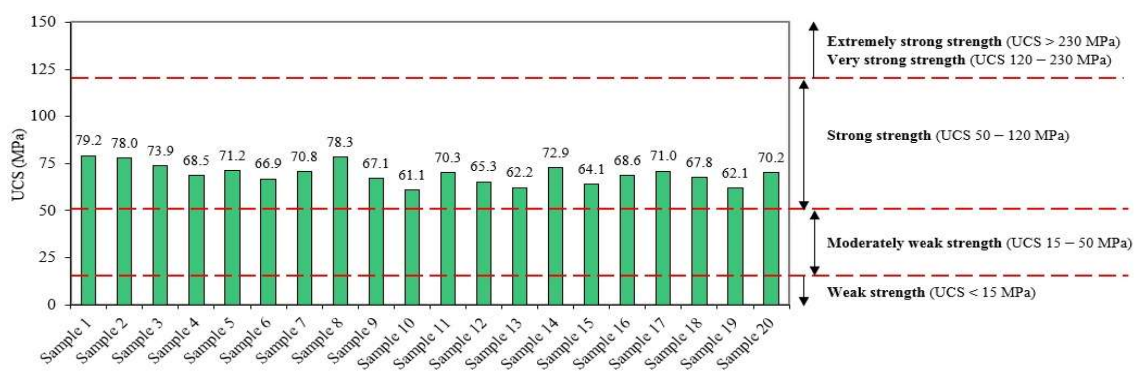

Figure 3. UCS classification of the limestone samples (IAEG, 1979)

One of the common classifications of the rocks based on Vp is that proposed by IAEG (1979). According to Fig. 4, rocks are classified into five Vp classes. As shown in this figure, samples 1, 2, and 8 fall into the rock class with very high Vp (Vp > 5000 m/s), whereas, other samples are classified as rocks with high Vp (Vp 4000–5000 m/s).

Schmidt hardness (SH) test

The SH test was performed with an N-type hammer having an impact energy of 2.207 Nm (ISRM, 1981). All tests were performed with the hammer held vertically downward and at a right angle to the horizontal faces of the specimens in a steel V-block with a weight of approximately 23 kg. The 20 rebound values from single impacts separated by at least one plunger diameter were recorded, and the upper ten values were averaged as SH value (rebound number). The average results of the SH tests are summarized in Table 4. As seen from this Table, the SH values of the samples are between 34 and 55. The samples were classified according to their SH values based on the classification suggested by ISRM (1978). Fig. 5 shows that the samples fall into the different rock classes with the strengths of slightly strong (SH 20–40), strong (SH 40–50), and very strong (SH 50–60).

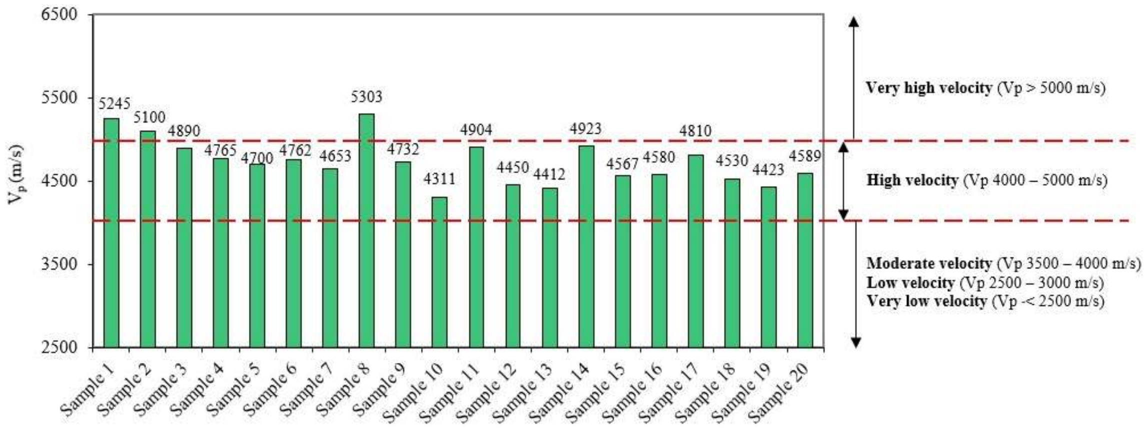

Figure 4. Vp classification of the limestone samples (IAEG, 1979)

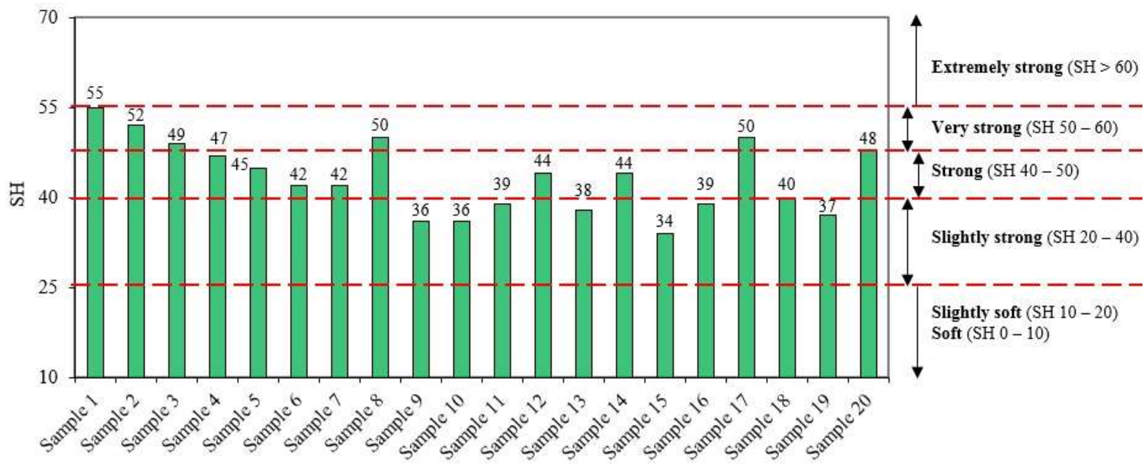

Figure 5. SH classification of the limestone samples (ISRM, 1978)

Results and discussion

Simple and multiple regression analyses

One of the most common methods for investigating the empirical correlations among the various parameters of the rocks such as UCS, Vp, and SH is regression analysis. The data given in Table 4 were used for the simple and multiple regression analyses with the aim of developing the correlation equations between UCS with Vp and SH.

The plot of the UCS as a function of the Vp and SH is shown in Fig. 6. It can be seen from this figure that with increasing Vp and SH, the UCS of the samples increases. Additionally, the best–fitted correlations between UCS with Vp and SH were found to be represented by linear regression. The equations for the correlations are as follows:

$$\mathbf{UCS} = 0.018\,\mathbf{Vp} - 15.45 \qquad R^2 = 0.86 \qquad (1)$$

$$\mathbf{UCS} = 0.736\ \mathbf{SH} + 37.55 \qquad R^2 = 0.71 \qquad (2)$$

There is a coefficient of determination (R^2) of 0.86 between UCS and Vp, and it is 0.71 for the correlation equation developed between UCS and SH. A comparison of R^2 values indicated that the correlation for predicting the UCS based on Vp is more reliable than that obtained using the SH.

For an in-depth insight into the reliability of Vp and SH in predicting the UCS of the samples, the results of the present study and the findings of the previous studies were investigated. In Fig. 7, the R^2 values of the correlation equations developed between UCS with Vp and SH in the present study and previous studies are illustrated. The results of the present study are in good agreement with the findings of Kahraman (2001), Cobanoglu and Celik (2008), Jamshidi et al. (2016), Abdi and Khanlari (2019), and Valido et al. (2024). According to the R^2 values of the correlation equations, the findings of these researchers showed a higher reliability of Vp than SH in predicting the UCS of the various rocks. Anyway, Celik (2019) and Fereidooni and Sousa (2022) reported contradictory results with those of the present study and previous studies. The findings of these researchers revealed that UCS has a stronger correlation (i.e., higher R^2) with the SH compared with the correlation between UCS and Vp (Fig. 7). It should be noted that Kilic and Teymen (2008) obtained an R^2 with the same value of 0.94 for both correlation equations between UCS with Vp and SH.

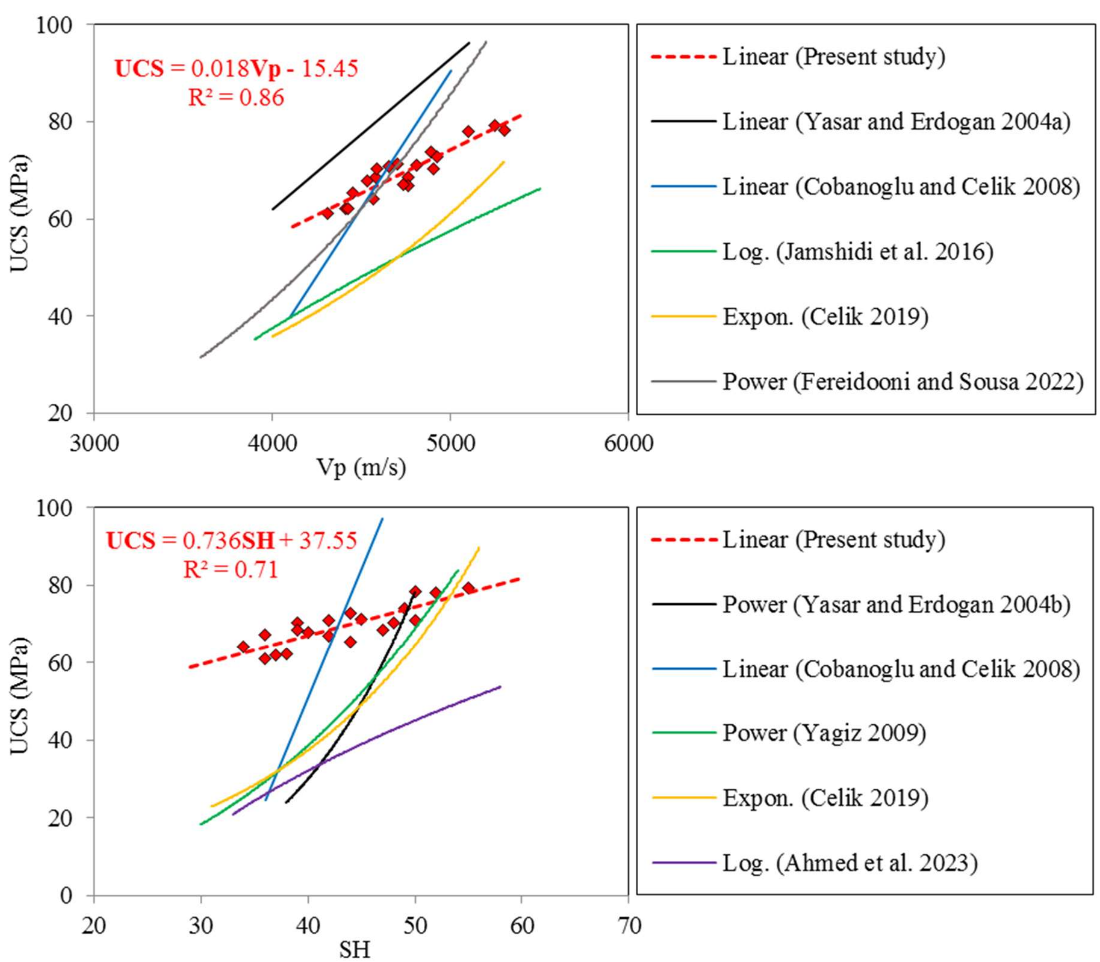

Figure 6. Correlations between UCS with Vp and SH in the present study and the previous studies

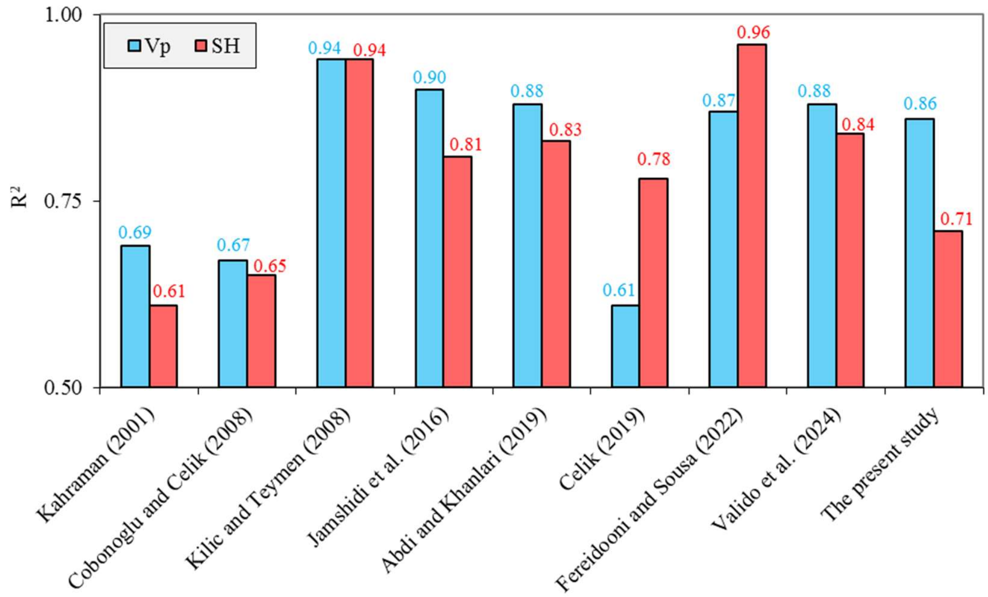

Figure 7. The R^2 values of the correlation equations developed for predicting the UCS from Vp and SH

The literature reports many equations to predict the UCS of the various rocks using the Vp and SH, which gives the correlations in different forms, including the linear (y = ax + b), power ($y = ax^b$), exponential ($y = ae^x$), and logarithmic (y = a + ln x). Some of these equations are graphically shown in Fig. 6. It can be seen from this figure that the there are differences among the correlation equations developed by the various researchers. In the present study and those by Yasar and Erdogan (2004a,b), Cobanoglu and Celik (2008), and Ahmed et al. (2023) the linear regression yields the strongest correlations between UCS with Vp and SH, with the most R^2 values, while researchers such as Yagiz (2009), Jamshidi et al. (2016), Celik (2019), and Fereidooni and Sousa (2022) revealed that strongest correlations (the most R^2) between UCS with Vp and SH were in power, exponential, and logarithmic forms. The differences in form and R^2 values of correlation equations developed by various researchers could be due to difference in the tested rock types, range of UCS, Vp, and SH, mineralogical composition and textural characteristics, the sample conditions used to test (i.e., air-dried or saturated states), number and dimensions of samples, and loading rate in the UCS test.

Multiple regression analysis is more amenable to ceteris paribus analysis because it allows researchers to explicitly control many other factors that simultaneously affect the dependent variable. Multiple regression models can accommodate many explanatory variables that may be correlated. Thus, researchers can hope to infer causality in cases where simple regression analysis is misleading (Tumac, 2015). For this reason, in the present study, multiple regression analysis was performed to determine the correlation between UCS as a function of both Vp and SH. In this analysis, UCS was considered as a dependent variable, and Vp and SH were regarded as independent variables as shown below:

$$\mathbf{UCS} = \alpha_0 + \alpha_1\,\mathbf{Vp} + \alpha_2\,\mathbf{SH} \tag{3}$$

Where α_0 is a constant, and α_1 and α_2 are the regression coefficients.

The data presented in Table 4 were analyzed using the SPSS®v.19 statistical software. Multiple regression analysis was undertaken at the 95% confidence level and the best–fit curve was obtained between variables using the least squares method. The results of multiple regression analysis are shown in Table 5. According to this Table, multiple regression equation for predicting the UCS using Vp and SH is as follows:

$$\mathbf{UCS} = -5.095 + 0.013\,\mathbf{Vp} + 0.310\,\mathbf{SH} \qquad R^2 = 0.92 \tag{4}$$

An R^2 equal to 0.92 was obtained for the multiple regression equation, which is an acceptable value. This result indicated that this equation can be accepted as a reliable model for predicting the UCS from Vp and SH.

Analysis of variance (ANOVA) is used to test the significance and global usefulness of simple and multiple regression equations. The F statistics test is widely used for analysis of variance. The null hypothesis for this test is H_0: $\alpha_1 = \alpha_2 = 0$. Additionally, the alternative hypothesis is H_1: at least one of α_1 or α_2 is not equal to zero. Table 5 shows the results of the analysis of variance for regression equations. At a significance level of 5%, the values of tabulated F-ratio for simple and multiple regression equations are 4.41 and 3.59, respectively. If the computed F-ratio is greater than the F-tabulated obtained from the F distribution table, the null hypothesis is rejected; therefore, the regression is significant (Stoodley et al. 1980). Since the computed F–ratio for the regressions is much greater than the tabulated F–ratio, the null hypothesis is rejected (Table 5). So, it can be concluded that simple and multiple regression equations are appropriate for predicting the UCS from Vp and SH.

The R^2 and standard error of estimate (SEE) were used as the numerical measures to compare the accuracy of the simple and multiple regression equations in predicting the UCS of the sample. The degree of fit to a curve can be measured by R^2 and SEE. R^2 measures the proportion of variation in the dependent variable. On the other hand, SEE indicates how close the measured data points fall to the predicted values on the regression curve. The R^2 and SEE values of Eqs. (1), (2), and (4) are given in Table 5. As can be seen from this table, the R^2 values of these equations are higher than 0.71, which is in an acceptable level, however, the highest R^2 (0.92) was obtained for multiple regression equation (Eq. 4), and the lowest R^2 were obtained for simple regression equations (Eqs. 1 and 2) with R^2 of 0.86 and 0.71, respectively. In addition, the results of regression analyses showed that SEE values for simple regression equations (Eqs. 1 and 2) were 2.1 and 2.9, respectively; whereas it is 1.6 for multiple regression equation (Eq. 4). In the regression analyses, a greater R^2 corresponds to a lower SEE, indicating higher accuracy of correlation equation in predicting the UCS. Comparing the R^2 and SEE values of Eqs. (1), (2), and (4) shows that multiple regression equation is more accurate than simple regression equations for predicting the UCS.

Although the R^2 and SEE values showed that the multiple regression equation is more accurate for predicting the UCS than simple regression equations, plots of predicted versus measured values of UCS were also used to verify this result. For this, the predicted UCS values by Eqs. (1), (2), and (4) were plotted versus the measured UCS values using a diagonal line (1:1) (Fig. 8). The error in the predicted UCS is represented by the distance of each data from the diagonal line. A point on the line indicates an accurate prediction. For the multiple regression equation, the data points fall closer to the diagonal line and are less scattered than those for simple regression equations. A comparison of the scattering of data points around the diagonal line in Fig. 8, suggests that predicting the UCS using multiple regression equation is more accurate than that via simple regression equations. This finding is in good agreement with the results obtained based on the R^2 and SEE values.

Table 5. Results of the simple and multiple regression analyses

Equation no.	Regression equation	R^2	SEE	F Value		F Sig.
				Computed	Tabulated	
1	**UCS** = 0.0179 **Vp** – 15.454	0.86	2.1	107.6	4.41	0.000
2	**UCS** = 0.7364 **SH** + 37.552	0.71	2.9	43.8	4.41	0.000
4	**UCS** = –5.095 + 0.013 **Vp** + 0.310 **SH**	0.92	1.6	91.2	3.59	0.000

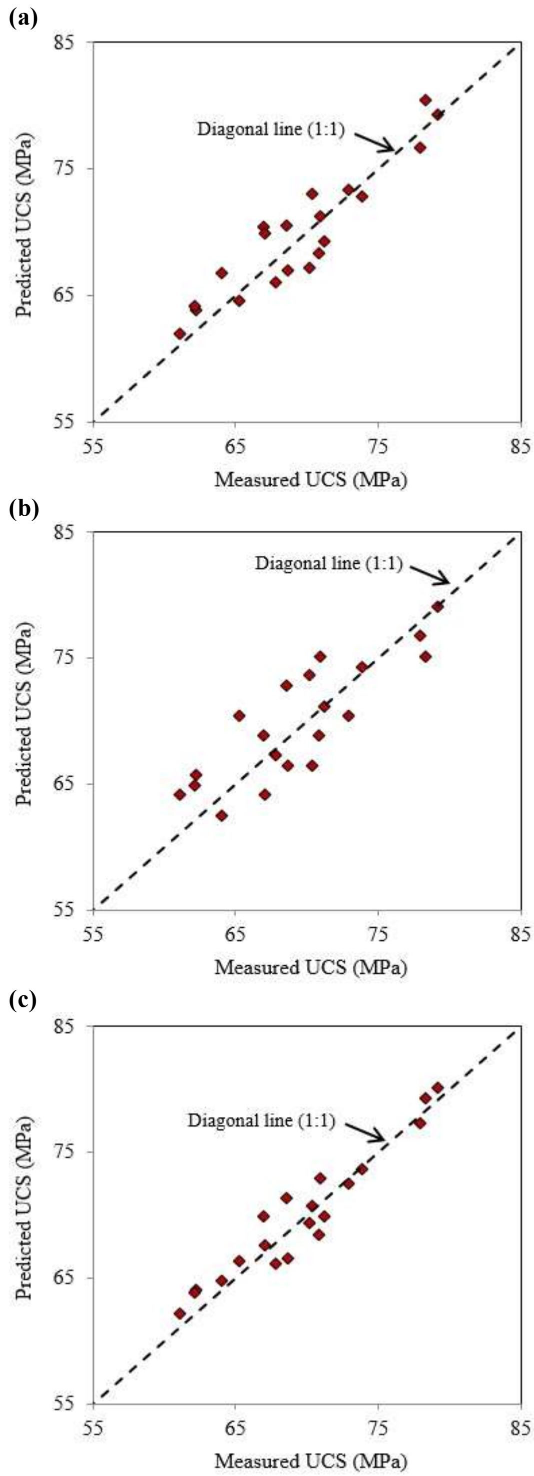

Figure 8. Measured UCS versus predicted UCS from (a) Eq. 1, (b) Eq. 2, and (c) Eq. 4

Validity and performance of the multiple regression equation

To validate of the multiple regression equation, the data published on limestone were collected from literature. After filtering the data values, those in the range of UCS, Vp, and SH of the samples tested in the present study (i.e., 61.1–79.2 MPa, 4311–5303 m/s, and 34–55, respectively) were extracted to investigate the validity of the multiple regression equation.

According to the data presented in Table 6, the UCS of the limestone in the studies of Cobanoglu and Celik (2008), Yavuz et al. (2008), Sengun et al. (2011), and Fereidooni and Sousa (2022) were predicted from their Vp and SH through multiple regression equation (Eq. 4) developed in the present study. The measured values of the UCS in these studies and those predicted from Eq. (4) are graphically illustrated in Fig. 9. It can be seen from this figure that the UCS predicted from Eq. (4) is in fair agreement with the UCS measured by abovementioned researchers. This result reveals that the multiple regression equation can be reliably used to predict the UCS of the limestone samples.

As quantitative measure for investigating the validity of the multiple regression equation, the prediction error of the UCS was calculated using Eq. (5):

$$\text{Prediction error (\%)} = \frac{\mathrm{UCS_m} - \mathrm{UC}\ {}_{\mathrm{p}}}{\mathrm{UCS_m}} \times 100 \tag{5}$$

where UCS_m and UCS_p are measured and predicted UCS values, respectively.

As shown in Table 6, the prediction errors of the samples UCS are between 0.9 and 13.0% with a prediction error mean equal to 6.3%. The findings showed that the prediction errors of the samples UCS are at an acceptable level, indicating the validity of the multiple regression equation for predicting the UCS of the limestone in other regions of world. However, the findings revealed that the prediction errors in some samples, including codes of CC1, CC2, and FS1, are greater value than those in other samples (Table 6). This difference can be attributed to the heterogeneity of limestone samples due to their porous nature, wide variety of mineralogical composition, and textural features, which cause them to behave differently.

Table 6. Measured values of UCS, Vp, and SH by various researchers and UCS predicted by multiple regression equation developed in the present study

Researcher/s	Rock type (Code)	Measured parameters			Predictive equation	Predicted UCS (MPa)	*Prediction error (%)
		UCS (MPa)	Vp (m/s)	SH			
Cobanoglu and Celik (2008)	Limestone (CC1)	63.0	4753	43	**UCS** = –5.095 + 0.013 **Vp** + 0.310 **SH**	70.0	– 11.2
	Limestone (CC2)	63.7	4799	44	**UCS** = –5.095 + 0.013 **Vp** + 0.310 **SH**	70.9	– 11.4
	Limestone (CC3)	74.1	4866	44	**UCS** = –5.095 + 0.013 **Vp** + 0.310 **SH**	71.8	3.1
	Limestone (CC4)	74.1	4869	45	**UCS** = –5.095 + 0.013 **Vp** + 0.310 **SH**	72.2	2.6
	Limestone (CC5)	74.9	5109	46	**UCS** = –5.095 + 0.013 **Vp** + 0.310 **SH**	75.6	– 0.9
Yavuz et al. (2008)	Limestone (Y)	68	4984	34	**UCS** = –5.095 + 0.013 **Vp** + 0.310 **SH**	70.2	– 3.3
Sengun et al. (2011)	Limestone (S)	62.3	4740	35.8	**UCS** = –5.095 + 0.013 **Vp** + 0.310 **SH**	67.6	– 8.5
Fereidooni and Sousa (2022)	Limestone (FS1)	67.4	4800	39	**UCS** = –5.095 + 0.013 **Vp** + 0.310 **SH**	69.4	– 3.0
	Limestone (FS2)	73.7	4370	40	**UCS** = –5.095 + 0.013 **Vp** + 0.310 **SH**	64.1	13.0
							$\lvert Error\ mean$ (%)| = 6.3%

* Calculated using Eq. 5: a positive sign indicates that the measured value was higher than the predicted value, and a negative sign indicates that the predicted value was higher than the measured value

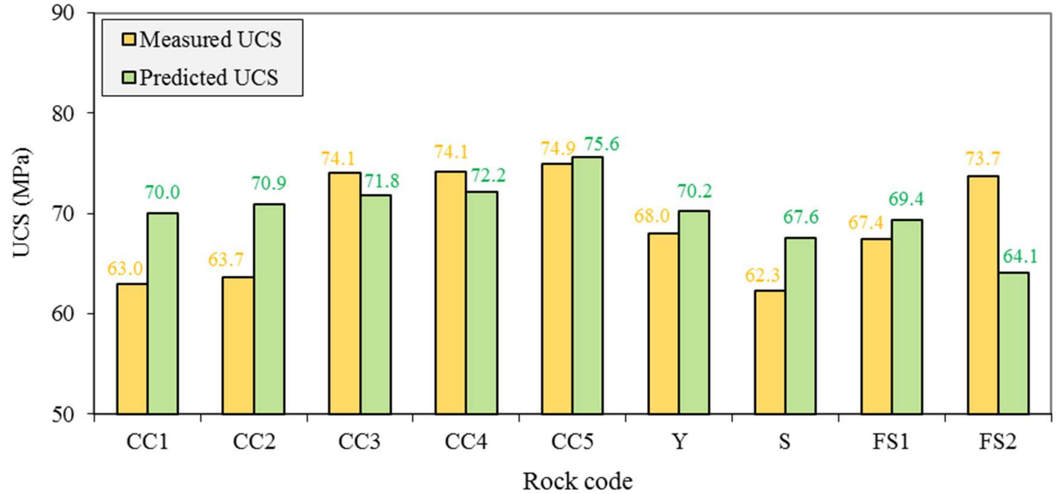

Figure 9. Measured UCS by various researchers versus predicted UCS (data from Table 6)

To assess the prediction performance of multiple regression equation, the statistical indices, including coefficient values accounted for (VAF) and the root mean square error (RMSE), were calculated using Eqs. (6) and (7), respectively:

$$\mathrm{VAF} = \left[1 - \frac{\mathrm{var}\,(y-y')}{\mathrm{var}\,y}\right] \times 100 \quad (6)$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y-y')^2} \quad (7)$$

where y and y′ are the measured and predicted values of the UCS, respectively, $\bar{y}$ and $\bar{y}'$ are the mean values of y and y′, respectively, and N is the number of the dataset.

A correlation equation is excellent for predicting the unknown variable from the one that is known (in the present study: UCS, and Vp and SH, respectively) if the VAF = 100% and RMSE = 0. The values of these indices for multiple regression equation were calculated using Eqs. (6) and (7). The VAF and RMSE of multiple regression equation are 84.50% and 4.89, respectively. These values are at good levels, suggesting the high performance of multiple regression equation in predicting the UCS of the limestone in other regions of the world. As a result, multiple regression equation developed in the present study is efficient and accurate for indirect assessment of the UCS of the limestones when measured data are not available.

Conclusions

In the present study, uniaxial compressive strength (UCS), P–wave velocity (Vp), and Schmidt hardness (SH) tests were carried out on the twenty limestone samples. The correlations between UCS with Vp and SH were investigated via simple and multiple regression analyses. The accuracy of these equations was compared through determination coefficient (R^2), standard error of estimate (SEE), diagonal line (1:1), and analysis of variance (ANOVA). In addition, the validity and performance of the multiple regression equation for predicting the UCS of the limestone from other regions of the world were evaluated. Based on the data analyses, main conclusions drawn from this study are as follows:

According to simple regression analyses, correlation equation between UCS and Vp (R^2 = 0.86, SEE = 2.1) is more reliable for predicting the UCS of the samples than correlation equation between UCS and SH (R^2 = 0.71, SEE = 2.9).

The results indicated that multiple regression equation is more appropriate and accurate than simple regression equations for predicting the UCS. This result was verified based on the R^2, SEE, diagonal line (1:1), and analysis of variance.

A good multiple regression equation with an R^2 of 0.92 was obtained between the UCS with Vp and SH. The performance of this equation was evaluated using statistical indices including coefficient values accounted for (VAF) and the root mean square error (RMSE). The results indicated that the multiple regression equation is significant for accurately predicting the UCS from the Vp and SH.

Based on the data analysis, the multiple regression equation can be utilized as an efficient and accurate practical tool in the indirect assessment of the UCS of the limestone from other regions of the world with ranges of UCS, Vp, and SH similar to those of the samples in the present study (i.e., 61.1–79.2 MPa, 4311–5303 m/s, and 34–55, respectively). As a result, in some cases where preparing test specimens from a limestone for the direct measurement of UCS is not possible, multiple regression equation can be used to avoid performing UCS tests.

Limestones are notoriously variable and heterogeneous in their UCS, Vp, and SH, which depend on the nature of their porous media, mineralogical composition, and textural features. Therefore, new predictive equations for the UCS can be developed for various limestones with a wide range of mineralogical compositions, textural features, and physico-mechanical characteristics. In this regard, further studies need to be undertaken by researchers in the future.

References

Abdi, Y., Khanlari, G.R., 2019. Estimation of mechanical properties of sandstones using P-wave velocity and Schmidt hardness. New Finding Applied Geology, 13: 33-47.

Abdi, Y., Khanlari, G.R., Jamshidi, A., 2024. Correlation between mechanical properties of sandstones and P-wave velocity in different degrees of saturation. Geotechnical and Geological Engineering, 42: 665-674.

Ahmed, W., Ahmad, N., Janjuhah, H.T., Islam, I., Sajid, M., Kontakiotis, G., 2023. The evaluation of non-destructive tests for the strength and physical properties of granite, marble, and sandstone: a case study from North Pakistan. Quaternary, 6: 4.

Ajalloeian, R., Jamshidi, A., Khorasani, R., 2024. Evaluating the effects of mineral grain size and mineralogical composition on the correlated equations between strength and Schmidt hardness of granitic rocks. Geotechnical and Geological Engineering, 42: 675-685.

Akbay, D., 2023. Investigating the accuracy of specimen shape for point load index test in predicting the uniaxial compressive strength for rocks using regression analysis and machine learning. Mining, Metallurgy and Exploration, 40: 2107-2115.

Azadmehr, A., Saffarian, M., Kazemi, S.M., 2021. Proposing regression models to estimate uniaxial compressive strength and elastic modulus of the sandstones based on physical properties and compressional wave velocity. Civil Infrastructure Researches, 6: 149-170.

Azimian, A., Ajalloeian, R., 2015. Empirical correlation of physical and mechanical properties of marly rocks with P wave velocity. Arabian Journal of Geosciences, 8: 2069-2079.

Basu, A., Kamran, M., 2010. Point load test on schistose rocks and its applicability in predicting uniaxial compressive strength. International Journal of Rock Mechanics and Mining Sciences, 47: 823-828.

Celik, S.B., 2019. Prediction of uniaxial compressive strength of carbonate rocks from nondestructive tests using multivariate regression and LS-SVM methods. Arabian Journal of Geosciences, 12: 193.

Cherifi, H., Chaouni, A.A., Fattah, G., Jalouni, A., Jabri, I., El-Asmi, H., Raini, I., 2021. Physico-mechanical characterization of schists in Tazzeka complex [Taza Province, Eastern Morocco]. Case Studies in Construction Materials, 15: e00692.

Cobanoglu, I., Celik, S.B., 2008. Estimation of uniaxial compressive strength from point load strength, Schmidt hardness and P-wave velocity. Bulletin of Engineering Geology and the Environment, 67: 491-498.

Diamantis, K., Bellas, S., Migiros, G., Gartzos, E., 2011. Correlating wave velocities with physical, mechanical properties and petrographic characteristics of peridotites from the Central Greece. Geotechnical and Geological Engineering, 29: 1049-1062.

Fener, M., Kahraman, S., Bilgil, A., Gunaydin, O., 2005. A comparative evaluation of indirect methods to estimate the compressive strength of rocks. Rock Mechanics and Rock Engineering, 38: 329-343.

Fereidooni, D., Sousa, L., 2022. Predicting the engineering properties of rocks from textural characteristics using some soft computing approaches. Materials, 15: 7922.

Gokceoglu, C., 1996. An evaluation on the reliability of uniaxial compressive strength data estimated by using the Schmidt hammer hardness. Jeoloji Muhendisi, 48: 78-81.

Haramy, K.Y., DeMarco, M.J., 1985. Use of Schmidt hammer for rock and coal testing. 26th US Sympusim on Rock Mechanics, pp. 549-555.

Heidari, M., Khanlari, G.R., Torabi-Kaveh, M., Karegarian, S., 2012. Predicting the uniaxial compressive and tensile strengths of gypsum rock by point load testing. Rock Mechanics and Rock Engineering, 45: 265-273.

IAEG., 1979. Classification of rocks and soils for engineering geological mapping part 1: Rock and Soil Materials. Bulletin of the International Association of Engineering Geology, 19: 364-371.

ISRM., 1978. Suggested methods for determining hardness and abrasiveness of rocks. International journal of rock mechanics and mining sciences & geomechanics abstracts, 15: 89-98.

ISRM (1981) Rock characterization testing and monitoring. ISRM suggested methods. Pergamon Press, Oxford.

Jamshidi, A., Nikudel, M.R., Khamehchiyan, M., Zarei Sahamieh, R., 2016. The effect of specimen diameter size on uniaxial compressive strength, P-wave velocity and the correlation between them. Geomechanics and Geoengineering, An International Journal, 11: 13-19.

Jamshidi, A., Yazarloo, R., Gheiji, S., 2018a. Comparative evaluation of Schmidt hammer test procedures for prediction of rocks strength. International Journal of Mining and Geo-Engineering, 52: 199-206.

Jamshidi, A., Zamanian, H., Zarei Sahamieh, R., 2018b. The effect of density and porosity on the correlation between uniaxial compressive strength and P-wave velocity. Rock Mechanics and Rock Engineering, 51: 1279-1286.

Jobli, A.F., Hampden, A.Z., Tawie, T., 2017. The role of ultrasonic velocity and Schmidt hammer hardness - The simple and economical non-destructive test for the evaluation of mechanical properties of weathered granite. AIP Conference Proceedings, 1875:030005.

Kahraman, S., 2001. Evaluation of simple methods for assessing the uniaxial compressive strength of rock. International Journal of Rock Mechanics and Mining Sciences, 38: 981-994.

Karaman, K., Kesimal, A., Ersoy, H., 2015. A comparative assessment of indirect methods for estimating the uniaxial compressive and tensile strength of rocks. Arabian Journal of Geosciences, 8: 2393-2403.

Katz, O., Reches, Z., Roegiers, J.C., 2000. Evaluation of mechanical rock properties using a Schmidt hammer. International Journal of Rock Mechanics and Mining Sciences, 37: 723-728.

Khandelwal, M., 2013. Correlating P-wave velocity with the physicomechanical properties of different rocks. Pure and Applied Geophysics, 170: 507-514.

Kilic, A., Teymen, A., 2008. Determination of mechanical properties of rocks using simple methods. Bulletin of Engineering Geology and the Environment, 67: 237-244.

Lashkaripour, G.R., 2002. Predicting mechanical properties of mudrock from index parameters. Bulletin of Engineering Geology and the Environment, 61: 73-77.

Minaeian, B., Ahangari, K., 2013. Estimation of uniaxial compressive strength based on P-wave and Schmidt hammer rebound using statistical method. Arabian Journal of Geosciences, 6: 1925-1931.

Rahimi, M.R., Mohammadi, S.D., Beydokhti, A.T., 2022. Correlation between Schmidt hammer hardness, strength properties and mineral compositions of sulfate rocks. Geotechnical and Geological Engineering, 40: 545-574.

Sadeghi, E., Nikudel, M.R., Khamehchiyan, M., Kavussi, A., 2022. Estimation of unconfined compressive strength (UCS) of carbonate rocks by index mechanical tests and specimen size properties: Central Alborz Zone of Iran. Rock Mechanics and Rock Engineering 55: 125-145.

Saldana, M., Gonzalez, J., Perez-Rey, I., Jeldres, M., Toro, N., 2020. Applying statistical analysis and machine learning for modeling the UCS from P-wave velocity, density and porosity on dry travertine. Applied Sciences, 10: 4565.

Sarkar, K., Vishal, V., Singh, T.N., 2012. An empirical correlation of index geomechanical parameters with the compressional wave velocity. Geotechnical and Geological Engineering, 30: 469-479.

Sengun, N., Altindag, R., Demirdag, S., Yavuz, H., 2011. P-wave velocity and Schmidt rebound hardness value of rocks under uniaxial compressional loading. International Journal of Rock Mechanics and Mining Sciences, 48: 693-696.

Sharma, P.K., Singh, T.N., 2008. A correlation between P-wave velocity, impact strength index, slake durability index and uniaxial compressive strength. Bulletin of Engineering Geology and the Environment, 67: 17-22.

Singh, R.N., Hassani, F.P., Elkington, P.A.S., 1983. The application of strength and deformation index testing to the stability assessment of coal measures excavations. The 24th U.S. Symposium on Rock Mechanics (USRMS), College Station, Texas, 599-609.

Song, H., Zhao, Y., Noraei Danesh, N., Jiang, Y., Li, Y., 2020. Uniaxial compressive strength estimation based on the primary wave velocity in coal: considering scale effect and anisotropy. Energy Sources, Part A: Recovery, Utilization, and Environmental Effects, 1-120.

Stoodley, K.D.C., Lewis, T., Stainton, C.L.S., 1980. Applied statistical techniques. England: Ellis Horwood.

Teymen, A., 2021. Statistical models for estimating the uniaxial compressive strength and elastic modulus of rocks from different hardness test methods. Heliyon, 7: e06891.

Tuğrul, A., Zarif, I.H., 1999. Correlation of mineralogical and textural characteristics with engineering properties of selected granitic rocks from Turkey. Engineering Geology, 51: 303-317.

Tumac, D., 2015. Predicting the performance of large diameter circular saws based on Schmidt hammer and other properties for some Turkish carbonate rocks. International Journal of Rock Mechanics and Mining Sciences, 75: 159-168.

Valido, J., Caceres, J.M., Sousa, L., 2024. Mechanical properties of ignimbrites of Tenerife Island employed as building stone and their correlation with some physical properties. Journal of Building Engineering, 82: 108222.

Vaziri-Moghaddam, H., Kimiagari, M., Taheri, A., 2006. Depositional environment and sequence stratigraphy of the Oligo-Miocene Asmari Formation in SW Iran. Facies, 52: 41-51.

Yagiz, S., 2009. Predicting uniaxial compressive strength, modules of elasticity and index properties of rocks using Schmidt hammer. Bulletin of Engineering Geology and the Environment, 68: 55-63.

Yasar, E., Erdogan, Y., 2004a. Correlating sound velocity with the density, compressive strength and Young's modulus of carbonate rocks. International Journal of Rock Mechanics and Mining Sciences, 41: 871-875.

Yasar, E., Erdogan, Y., 2004b. Estimation of rock physicomechanical properties using hardness methods. Engineering Geology, 71: 281-288.

Yavuz, H., Ugur, I., Demirdag, S., 2008. Abrasion resistance of carbonate rocks used in dimension stone industry and correlations between abrasion and rock properties. International Journal of Rock Mechanics and Mining Sciences, 45: 260-267.

Yilmaz, I., Sendir, H., 2002. Correlation of Schmidt hammer rebound number with unconfined compressive strength and Young's modulus in gypsum from Sivas (Turkey). Engineering Geology, 66: 211-219.

Yurdakul, M., Ceylan, H., Akdas, H., 2011. A predictive model for uniaxial compressive strength of carbonate rocks from Schmidt hardness. Proc. 45th US Rock Mechanics/Geomechanics Symposium. ARMA, American Rock Mechanics Association. At: San Francisco, CA ABD.

Zhang, J., Shen, Y., Yang, G., Zhang, H., Wang, Y., Hou, X., Sun, Q., Li, G., 2021. Inconsistency of changes in uniaxial compressive strength and P-wave velocity of sandstone after temperature treatments. Journal of Rock Mechanics and Geotechnical Engineering 13: 143-153.