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Analysis of Hydraulic Jump Characteristics In U-Shaped Channel

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ABSTRACT: Typical supercritical flow characteristics like, sequent depth ratio (Y_2/Y_1), relative jump height (h_j/Y_1), relative energy loss (E_L/E_1), efficiency of jump (E_2/E_1), relative prejump depth (Y_1/E_1), relative postjump depth (Y_2/E_2), relative length of the roller and jump (L_r/Y_1 and L_j/Y_2) in the U-shaped channel are experimentally studied. Based on the experimental findings, physical theories for the variance in these characteristics with regard to the Froude number are presented. Empirical models are developed considering influence of inflow Froude number varying between 4 to 20 and Reynolds number between 1,638,009 to 3,394,784. Some models were also validated and yielded satisfactory results with good R^2 values. For comparison and a deeper comprehension of hydraulic jump characteristics, computational multivariate statistical techniques like principal component analysis (PCA) and factor analysis (FA) are applied. These techniques are used to identify patterns in an effort to explain the variation in a sizable set of closely related jump characteristics. Verifiers for the different principal components were analyzed and verifactor VF1 (with 64 %) had strong positive loadings on Y_2/Y_1, h_j/Y_1, E_L/E_1, L_r/Y_1 and L_j/Y_2, while VF2 (with 33 %) had strong positive loadings on Y_1/E_1 and Y_2/E_2 and moderate positive loading on E_2/E_1. These statistical methods resulted as valuable tools for identifying the key characteristics in a phenomenon and its relative significance.

Keywords: Super Critical Flow, U-Shaped Channel, Empirical Modeling, Principal Component Analysis, Factor Analysis

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Research Highlights

When the water falls over the spillway reaching to rectangular stilling basin which cannot expand laterally or vertically due to design constraints, a U-shaped channel may be one of the alternatives for hydraulic jump formation in energy dissipation. Present study has been made with the following objectives and research highlights:

1. First to understand the flow characteristics in this typical shaped channel, experimental work is carried out in U-shaped channel. Based on this, empirical modelling has been done to develop models for its direct application to the field. Reynold's number has been considered seeing the importance of it in energy dissipation (Jorge et al., 2022 and Ryugen and Masayuki, 2024) along with Froude number while developing empirical models (Eqn. 4 – Eqn. 11); such consideration were not made by other researchers while establishing such type of empirical models. These models were tested with experimental data which shows good R^2 value as mentioned in Table 2 and in Figure 3.
2. In order to check the accuracy of obtained experimental results and developed empirical models (Eqn. 4 - Eqn. 11), these were analyzed, tested and compared with other author's experimental results in section 5 and in section 6 respectively. Explanations were also given for its validation.
3. Further, for deeper comprehension of all the eight hydraulic jump characteristics, multivariate statistical techniques like factor analysis (FA) and principal component analysis (PCA) are applied to understand most influencing characteristics which is responsible for maximum energy dissipation. So by determining and hence by controlling this most influencing characteristics we can able to control safe energy dissipation with no need to provide appurtenances (sills and baffle blocks) to these hydraulic structures which otherwise required for high discharge ($F_{r1} > 9$) over the spillway during flood time.

1. Introduction

When the flow in a U-shaped channel changed from super-critical to sub-critical flow, there is a hydraulic jump formation. This phenomenon is vital for ensuring the stability of the channel and preventing undesirable consequences of erosion, scouring and dissipates excess momentum. U-shape channel beds were found to be more effective in reducing the jump length and sequent depth over other shaped channel (Hager, 1987, 89). Experimental observations by different authors were focused on surface profiles, shear stress and jump length. Even their results proved that different shaped bed (including corrugated) is insignificant on supercritical flow properties for lower range of Froude numbers. When the channel cannot expand laterally or vertically (Gandhi et al., 2024) and the tail water depth is insufficient to provide good jumps, a U-shaped channel may be one of the alternatives for hydraulic jump formation in energy dissipation. Additionally, it replaces the additional appurtenance arrangement to reduce the basin length (Bushra and Afzal, 2006; and Harshit and Bharat, 2023). Along with experimental and dimensionless approaches, methods of multivariate statistics (includes principal component analysis and factor analysis) are also useful to analyze possible factors which influence the hydraulic jump characteristics (Hamidreza et al., 2023 and Ogarekpe et al., 2022).

Authors (Belanger, 1849 and Bidone, 1819) were conducted the first substantial empirical study on the hydraulic jump. Using the momentum principle, they put up a theoretical explanation for the subsequent depth ratio. Hager (1987) published the initial experimental findings regarding free surfaces without dimensions. The author (Gandhi and Singh, 2016) discovered that as the Froude number increases, the length and subsequent depth of a classical hydraulic jump also increase.

Study on hydrodynamics of flow in U-shaped channel was carried out by different authors (Lin et al., 2021; Hu et al., 2018; Parsaie et al., 2022 and Shi et al., 2022). For specific instances of jump formation over spillways, empirical modeling on energy dissipation has been carried out by Ghaderi et al. (2020). Author (Azimi and Shabanlou, 2019) simulated three dimensional hydraulic jumps in this typical shaped channel by focusing on variations of flow over free surface using volume of fluid method. Values of MAPE, RMSE and R^2 are calculated as 7.62, 0.022 and 0.99 respectively for sequent depth, relative length of jump and roller. Under the assumption that flow is two-dimensional, author (Bushra and Afzal, 2006) used the Reynolds equations to study the turbulent behavior of hydraulic jump to predict the subsequent depth ratio. In order to forecast the changes in the flow-free surface, it is suggested to use either smoothed particle hydrodynamics model to simulate the hydraulic jump as a two-dimensional flow or numerical technique (i.e volume of fluid technique). By analyzing the answers for sequent depth, roller length and jump length, author (Bushra and Afzal, 2006) have given the reasoned that the product of the depth-averaged axial velocity gradient and constant eddy viscosity gives the depth-averaged effective normal Reynolds stress.

According to the research works of Hager (1987, 89) the analysis for hydraulic jump in U-shaped channel has restrictions and boundary requirements. In a U-shaped channel, where the lower half has a semicircular section with a diameter D and the upper half has a constant width that is proportional to the diameter, author (Bushra and Afzal, 2006) expanded the model even further. Their results for the sequential depth ratio (Y_2/Y_1) and relative length of jump (L_j/Y_2) for different non-dimensional upstream flow depths (i.e., Y_1/D, where D is the diameter of the channel) ranging from 0.1 to 0.5 were in good agreement with those of (Hager, 1989). Authors (Stahl and Hager, 1999) have put forward the empirical relations for sequent depth ratio and length of jump and found nearly 20 % deviations from experimental results. Some authors (Bai, 2023 and Torkamanzad et al., 2019) state that tail water level often affects the location and height of jump formation. By ensuring that the flow width is consistent amongst the various flow areas, non-symmetric flow can be stabilized. Authors (Hager 1987, 89) studied the sequent depth ratio assuming that channel bottom consists of a semicircular section has constant width at top with 4% error. Circular closed channel has been considered by (Gargano et al., 2002; Hager, 1989; Maryami et al., 2021 and Stahl and Hager, 1999). The analysis for estimating the length of the jump is scarce according to many sources such as (Gandhi, 2024 and Yousefi et al., 2019).

Literature shows that studies are limited only on sequent depth and relative length of jump and roller characteristics of supercritical flow in U-shaped channel. Influence of other characteristics such as h_j/Y_1, E_L/E_1, E_2/E_1, Y_1/E_1, Y_2/E_2, L_r/Y_1 were not given due consideration for the variation of specific energies under supercritical flow conditions. Application of soft computing methodologies and importance of Reynolds number without sills and baffles were also lacked. Since U-shaped channel has uniform sides width and recommended to field engineers because it posses the hydraulic properties of semi-circular channel, it becomes the choice of design engineers. The depth-averaged values of the turbulent hydraulic jump profile in circular and U-shaped channels were approximately same (Bushra and Afzal, 2006).

In the present study hydraulic jump characteristics of U-shaped beds were analyzed and compared to identify the most effective conditions of the channel for reducing jump length and other flow characteristics. These were tested for a range of Froude numbers from 4 to 20. In order to cover the gaps in the literature, the present study focuses on developing and validating empirical models for distinct jump characteristics. Further, different multivariate statistical techniques viz, PCA/FA, were applied to offers a better understanding of different hydraulic jump characteristics in U-shaped channel. These methods allow to identifying possible factor that influence the phenomenon most. Many authors (Hamidreza et al., 2023, Ogarekpe et al., 2022 and Singh et al., 2004) stated that principal component analysis describes the entire experimental data set with minimal loss of original information and offers information on the most meaningful parameters. Most importantly, models so developed can be used directly in the field for analysing hydraulic characteristics of flow over spillway. This study is of practical applications to dams and spillways structures, where supercritical flow of high velocity in channel is used to dissipate excess kinetic energy through hydraulic jump formation.

2. Experimentation and Data Acquisition

All the experiments for aforementioned eight characteristics (Y_2/Y_1, h_j/Y_1, E_L/E_1, E_2/E_1, Y_1/E_1, Y_2/E_2, L_r/Y_1 and L_j/Y_2) for Froude number varying between 4 to 20 and Reynold's number between 1,638,009 to 3,394,784 were conducted in a U-shaped channel. The experimental setup consists of six basic segments namely, inlet tank, stilling basin, sharp edge vertical regulating gates (both at upstream and downstream ends), main channel having perspex bottom and side wall, discharge tank and discharge channel with proper instrumentation for measurement and control. The depth of flow across the breadth and length of the channel can be measured at various places from the entrance gate using pointer gauges. Least count of pointer gauge is 1 mm.

Through a connecting pipe with a diameter of 10 cm and a regulating valve, water was able to enter the 0.43 X 31 X 0.80 m^3 constant head input tank, which was designed to minimize variation caused by changes in flow. Discharge is primarily regulated from minimum to maximum capacity of inlet tank to obtain the different data at various approach Froude numbers. Experiments were conducted in a U-shaped channel for numbers of runs at different discharges using sharp edged controlling gates (at both upstream and downstream), and the same parameters were recorded for each run (Table 1). Channels measuring 2.8 m in length, 0.3 m in width, and 0.4 m in height were used for the experiments. The generation of eddies and rollers is minimized by making the bottom surface flat for flow. The design of the side walls makes it easy to monitor the beginning and end of the jump and roller lengths during the experiments. At the very top of the side walls are affixed parallel rails that allow the pointer gauge to measure depth at various points throughout the length and width of the channel.

Total thirty five sets of runs were made in extracting experimental data with horizontal bed. Proper care would be taken to locate the end of the jump by finding immediate downstream positions of the roller were jump meets the downstream water surface. Tail water depth is ensured using downstream regulating gates installed in setup. With the help of upstream gate the discharge is changed and allowed jump to form by adjusting downstream gate. Once the jump position gets established at a particular discharge, all the measurement was taken. During the recording process, author paid close attention to the occurrence of surface rollers and extreme turbulence. Achieving lowest water losses, symmetric flow, depths at three locations throughout the main channel, and discharge at the downstream end have been the major design factors in order to get accurate and dependable experimental data. A schematic and a sectional view of the experimental setup are shown in Figures 1 and 2, respectively.

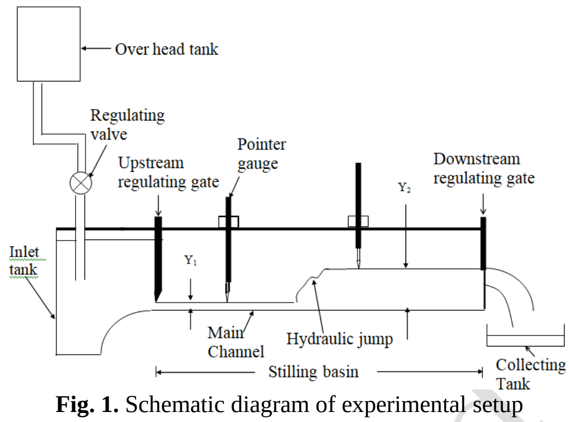


Fig. 1. Schematic diagram of experimental setup

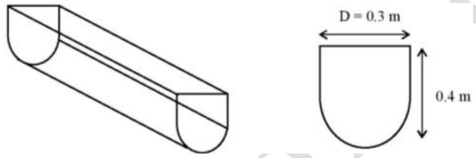


Fig. 2. Sectional view of a U-shaped channel

Table 1. Different flow characteristics and its range for F_{r1} = 4 to 20 and R_{e1} = 16,38,009 to 33,94,784

Measured Parameters	Y_1, Y_2, Q, L_j, L_r
Varied Parameters	F_{r1}, R_{e1}
Sequent depth ratio 'Y_2/Y_1'	1.5 - 5
Relative jump height 'h_j/Y_1'	0.7 - 3.5
Relative energy loss 'E_L/E_1'	0.5 - 0.95
Efficiency of jump 'E_2/E_1'	0.06 - 0.45
Relative prejump depth 'Y_1/E_1'	0.005 - 0.1
Relative postjump depth 'Y_2/E_2'	0.2 - 0.5
Relative length of roller 'L_r/Y_1'	4.5 - 22
Relative length of jump 'L_j/Y_2'	3 - 6

3. Development of Empirical Models

Several empirical models for flow properties are described in the literature (Bushra and Afzal, 2006 and Rajaratnam and Subramanya, 1968) which describes Reynolds' number is a key factor in determining flow behavior. The efficacy of these approaches is well defined in identifying the drag influence on hydraulic jump phenomena and was investigated by Hu et al. (2018). Parameters Y_1, Y_2, V_1, V_2, L_r, L_j, E_L, υ, g, ρ and ε are the variables that are involved and impact the flow characteristics in a U-shaped channel. As indicated in Eqn. (1), they can be expressed as functions of both dependent and independent variables.

$$f\,(Y_1, Y_2, V_1, V_2, L_r, L_j, E_L, \mu, g, \rho, \varepsilon) = 0 \qquad (1)$$

were 'Y_1' is prejump depth, 'Y_2' is post jump depth, 'V_1' is prejump velocity , 'V_2' is postjump velocity, 'L_r' is length of roller, 'L_j' is length of jump, 'E_L' is energy loss, 'ρ' is density of water (Kg/m^3), 'υ' is kinematic viscosity (m^2/s), 'ε' is surface roughness (m) and 'μ' is dynamic viscosity of water (Ns/m^2). The dimensionless groups can be represented as:

$$\left(\frac{Y_2}{Y_1}, \frac{h_j}{Y_1}, \frac{E_L}{E_1}, \frac{E_2}{E_1}, \frac{Y_1}{E_1}, \frac{Y_2}{E_2}, \frac{L_r}{Y_1}, \frac{L_j}{Y_2}, \frac{V_1^2}{gY_1}, \frac{\rho V_1 Y_1}{\mu}, \frac{\varepsilon}{Y_1}\right) = 0 \qquad (2)$$

All variables involved in the phenomenon are considered and represented as a function in Eqn. (1) as per the dimensional approach. Different dimensionless groups (parameters) are formed using these variables as mentioned in Eqn. (2). Among these Y_2/Y_1, h_j/Y_1, E_L/E_1, E_2/E_1, Y_1/E_1, Y_2/E_2, L_r/Y_1 and L_j/Y_2 are dependent parameters as they depends upon varying velocity (discharge) and F_{r1} and R_{e1} are independent parameter. A relationship between the approach Froude number and the incoming Reynold's number is found to hold for all eight hydraulic jump characteristics in a U-shaped channel. As an example, sequent depth ratio in terms of dynamic viscosity can be expressed as follows:

$$\frac{Y_2}{Y_1} = f\left(\frac{V_1^2}{gY_1}, \frac{\rho V_1 Y_1}{\mu}\right) \qquad (3)$$

The remaining flow parameters can be represented in a manner analogous to Eqn. (3). The influence of surface roughness was not taken into account when creating these groupings because of experimental constraints. Here, from Eqn. (4) to Eqn. (11), developed empirical models for all flow characteristics are provided. Figure 3 (a to h) displays the best-fitting model along with its R^2 value.

Table 2. R^2 values for developed empirical models

Y_2/Y_1	h_j/Y_1	E_L/E_1	E_2/E_1
0.9881	0.9881	0.9785	0.9785
Y_1/E_1	**Y_2/E_2**	**L_r/Y_1**	**L_j/Y_2**
0.8493	0.7478	0.9889	0.9633

$$\frac{Y_2}{Y_1} = 17549\left(\frac{F_{r1}^2}{R_{e1}}\right) + 1.6 \qquad (4)$$

$$\frac{h_j}{Y_1} = 17549\left(\frac{F_{r1}^2}{R_{e1}}\right) + 0.6 \qquad (5)$$

$$\frac{E_L}{E_1} = 13.13\left(\frac{F_{r1}^{0.02}}{R_{e1}^{0.004}}\right) - 12 \qquad (6)$$

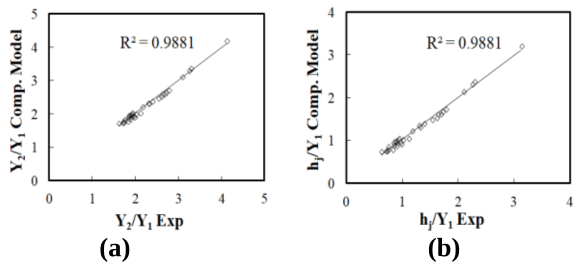
$$\frac{E_2}{E_1} = -13.13\left(\frac{F_{r1}^{0.02}}{R_{e1}^{0.004}}\right) + 13 \qquad (7)$$

$$\frac{Y_1}{E_1} = -1.9\left(\frac{F_{r1}^{0.03}}{R_{e1}^{0.001}}\right) + 2 \qquad (8)$$

$$\frac{Y_2}{E_2} = 340\left(\frac{F_{r1}^{0.05}}{R_{e1}^{0.4}}\right) - 1 \qquad (9)$$

$$\frac{L_r}{Y_1} = 74.5\left(\frac{F_{r1}^2}{R_{e1}^{0.5}}\right) + 4 \qquad (10)$$

$$\frac{L_j}{Y_2} = 611\left(\frac{F_{r1}^{0.8}}{R_{e1}^{0.5}}\right) + 2 \qquad (11)$$



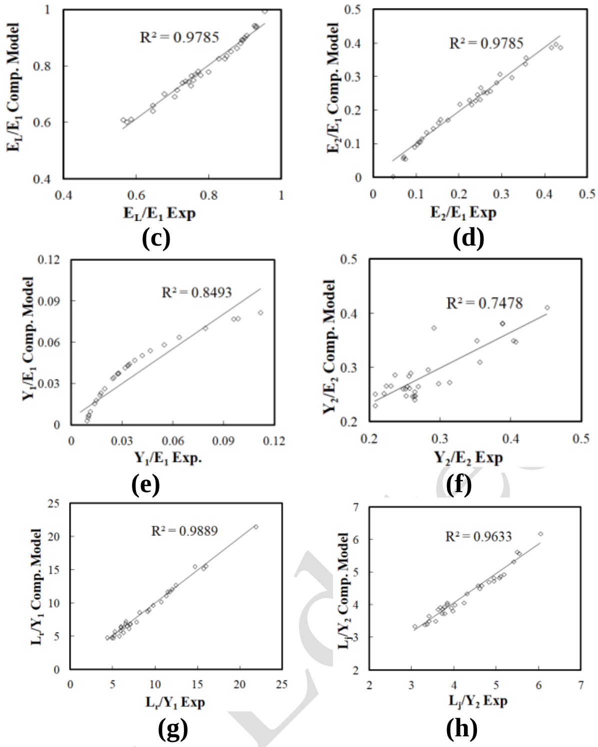


Fig. 3 (a to h). Empirical models and their linear fit (Eqns. 4 to 11) for hydraulic jump characteristics

4. Data Analysis

Literature reveals that there are only few studies reported for the analysis of hydraulic jump characteristics which involves prediction of jump length and its profile (Rajaratnam and Subramanya, 1968 and Yousefi et al., 2019). In order to observe the influence of all the eight hydraulic jump characteristics in U - shaped channel, the experimental results were used (Table 1). The variations of hydraulic jump characteristics are shown in Figures 4 (a) to (h) below.

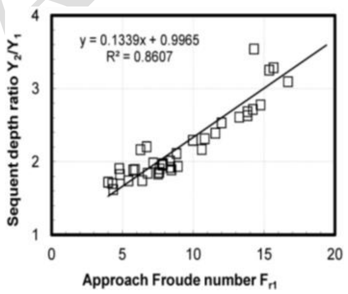
Figure 4 (a) shows a linear variation of sequent depth ratio (Y_2/Y_1) against the approach Froude number (F_{r1}) varied between 4 to 20. It shows that approximately 75 % of data lying within the range of ± 10 % of the best fit line drawn with R^2 value of 0.86 which shows scattering of data points due to development of high surface rollers causing inaccuracy in the measurements (Hu et al., 2018). Similar results were also obtained by other engineers (Bushra and Afzal, 2006 and Hager, 1989), but the observations by these researchers is slightly different because they performed the experiments on completely filled U-shaped channel, while in present study, no such condition was maintained as U-shaped channel length was not completely filled.

In Figure 4 (b), relative jump height (h_j/Y_1) varies non-linearly and tends to increase as the approach Froude number (F_{r1}) increases from 4 to 20. Author (Hager, 1989) also found a similar pattern for the relative jump height versus Froude number, although they took into account different experimental flow circumstances. Approximately 65% of the data points fall within a 10% margin of error of the best fit line, as can be seen in this image. Because of the extremely turbulent flow and the development of surface rollers, approximately 35% of the data points are scattered.

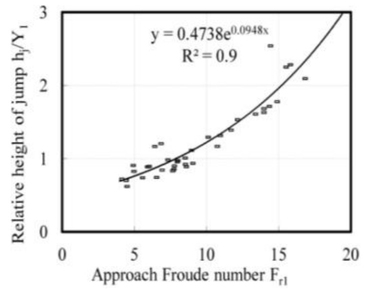
Figure 4 (c) shows non-linear increasing trend of relative energy loss (E_L/E_1) against approach Froude number (F_{r1}) ranging between 4 to 20. Approximately 90 % data points are lying within ± 10 % of the best fit line drawn. R^2 value of 0.95 indicates best logarithmic relationship between relative energy loss against approach Froude number. Figure 4 (d) shows a non-linear decreasing trend of variation of efficiency of jump (E_2/E_1) against approach Froude number (F_{r1}) varied between 4 to 20. In this figure, about 75 % data are seen lying within the range of ±10 % limit of the best fit curve drawn for observed values, rest 25 % data shows either non-uniformity or unsymmetrical high surface roller formed during the flow. A similar trend for efficiency of jump was also observed by Subramanya (2019) in his study.

Figure 4 (e) shows a non-linear power fit of the experimental data for relative pre-jump depth (Y_1/E_1) and the approach Froude number (F_{r1}) ranging between 4 to 20. Good fitting of data (R^2 = 0.99) shows a strong relationship between relative pre jump depth and approach Froude number. Figure 4 (f) shows a non-linear variation of relative post-jump depth (Y_2/E_2) against approach Froude number (F_{r1}) with a poor power fit of data points with a least value of R^2 = 0.04. Large deviation of data points indicates that relative post-jump depth is poorly related with approach Froude number between 4 to 20 under present channel conditions. Other researchers (Hager, 1989 and Stahl and Hager, 1999) have studied the relative length of jump on U-shaped channel under different channel conditions and observed deviations.

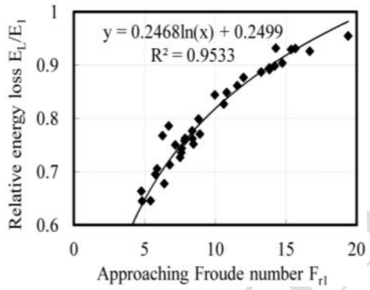
Figure 4 (g) shows a non-linear increment of relative length of roller (L_r/Y_1) against Froude number (F_{r1}) ranging from 4 to 20 with R^2 value of 0.9. It also shows that nearly 65 % of data are seen lying within the range of ± 10 % of the best fit curve drawn nearly 35 % data is found scattered. The reason to this effect can be attributed to difficulties faced in accurate judgment of position of the starting and end of the roller. Figure 4 (h) shows a non-linear variation of relative length of the jump (L_j/Y_2) against Froude number (F_{r1}) from 4 to 20 with R^2 value of 0.9. Nearly 75 % of the data points lying within the range of ± 10 % of the best fit polynomial drawn, and nearly 25 % data are seen scattered lying outside the range of ± 10 %, this again can be attributed due to high surface turbulence. Similar results have been reported by (Debabeche and Achor, 2003) using sill in U-shaped channel by fixing Y_1 and varying the approach Froude number.



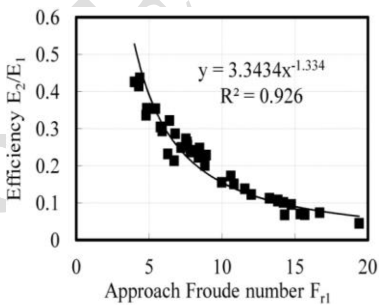
(a)



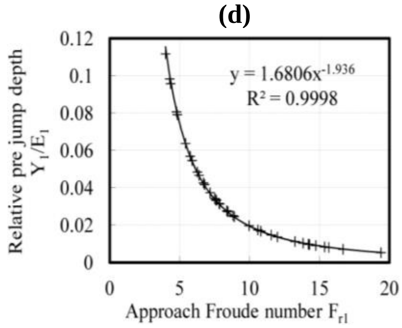
(b)



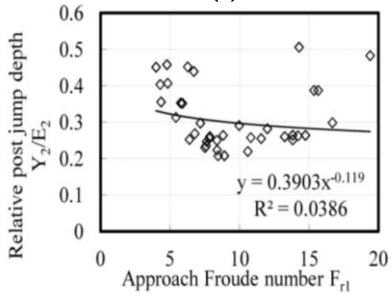
(c)



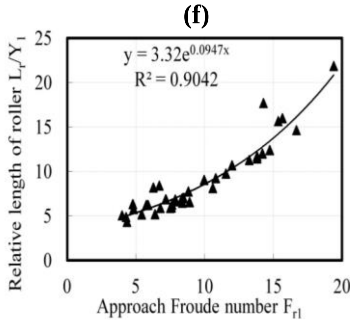
(d)



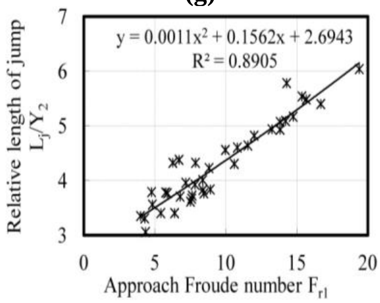
(e)



(f)



(g)



(h)

Fig. 4 (a to h). Variation hydraulic jump characteristics against Froude number

5. Testing and Comparison

***Y₂/Y₁*:** Figure 5 shows the comparison of sequent depth ratio (Y_2/Y_1) against approach Froude number (F_{r1}) for U-shaped channel between the values predicted from Eqn. (4) with the experimental data of (Debabeche and Achor, 2003 and Hager, 1989) for U-shaped channel and (Hager, 1989 and Stahl and Hager, 1999) for circular channel showing some deviation among each other. This can be attributed due to the reason that author (Debabeche and Achor, 2003) experimental data holds good for channel provided with sill as they studied the channel provided with sill and assumed that dimension of sill plays a significant role in the formation of hydraulic jump. On the other hand (Hger, 1989 and Stahl and Hager, 1999) presented their data for circular channels with the condition that $0.1 < Y_1 < 0.8$ and downstream depth is completely filled. Author (Hager, 1989) have conducted his experiment with the condition that downstream depth $Y_2 = 1$ for various value of upstream flow depth $0.1 < Y_1 < 0.8$ at all approach Froude number and semicircular part remained filled, whereas present experimentation is conducted for different upstream depth Y_1 at all approach Froude number F_{r1} with different depth at semicircular portion of the channel. Moreover obtained value of Y_2/Y_1 from present model lying between the experimental results of other authors and can be assumed to be approximately correct.

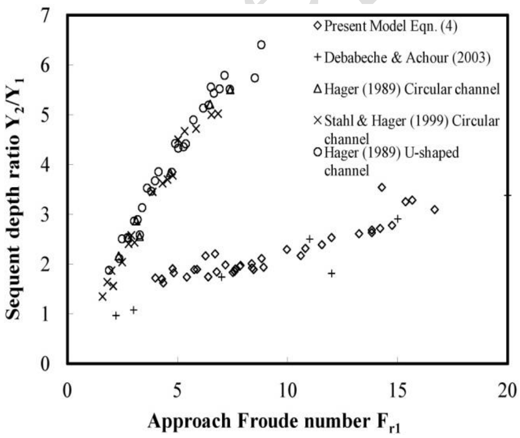


Fig. 5. Comparison between obtained values from present model (Eqn. 4) with different authors' data.

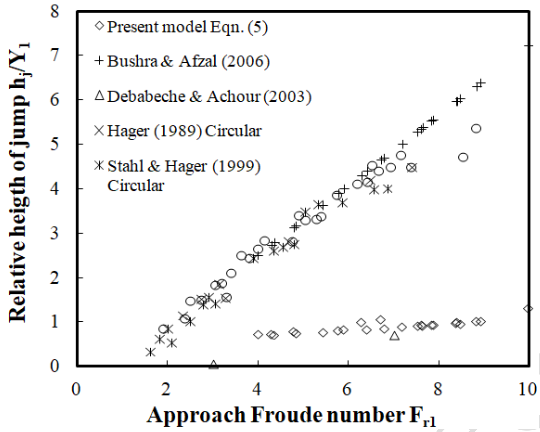


Fig. 6. Comparison between obtained values from present model (Eqn. 5) with different authors' data.

***hⱼ/Y₁*:** Figure 6 shows the comparison of values obtained for relative height of jump (h_j/Y_1) using present model (Eqn. 5) with the experimental data of (Bushra and Afzal, 2006; Debabeche and Achor 2003 and Hager 1989) for U-shaped channel and (Hager 1989, Stahl and Hager, 1999) for circular channel, which showing some deviation among each other. This again can be attributed to the same reason as explained above. Hence it is concluded that results of different authors for the present channel varying from each other due to dependence on experimental conditions. However results obtained from Eqn. (5) may be accepted for the present channel condition.

***Lⱼ/Y₂*:** Figure 7 shows the comparison of relative length of jump (L_j/Y_2) for the values obtained from present model (Eqn. 11) with the experimental data of (Debabeche and Achor 2003 and Hager 1989) for U-shaped channel, showing some deviation among each other. This can be attributed to experimental data holds good (Debabeche and Achor 2003) for channel provided with sill as their study is based on significant role of the dimension of sill on the relative length of the jump. Author (Hager 1989) have conducted his experiment by keeping downstream depth $Y_2 = 1$ (constant) for various value of upstream flow depth $0.1 < Y_1 < 0.8$ at all approach Froude number F_{r1} with semicircular part remained filled, whereas present experiment is conducted for different upstream depth Y_1 at all approach Froude number F_{r1} with different depth at semicircular portion of the channel. Moreover obtained value of L_j/Y_2 from present model Eqn. (11) lying between the experimental results of (Debabeche and Achor, 2003 and Hager, 1989) which may be assumed approximately correct to the present channel condition for the prediction of relative length of jump. The predictions are well supported by experimental results of authors (Hager, 1989 and Stahl and Hager, 1999).

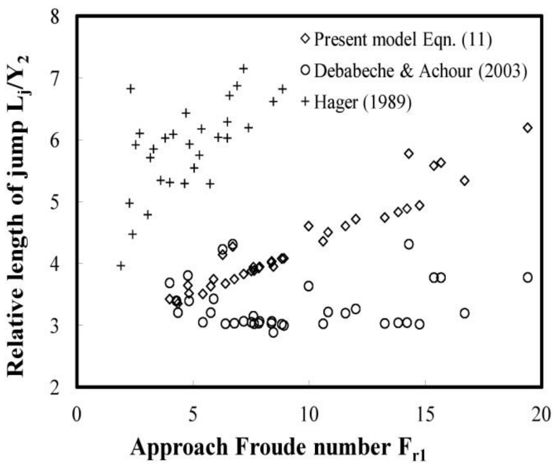


Fig. 7. Comparison of values obtained using Eqn. (11) with experimental value of other authors

***L_j/Y_1*:** Figure 8 shows the comparison of relative length of jump varying against approach Froude number. It compares the values obtained from Eqn. (4) and Eqn. (11) with the value obtained using (Bretz, 1987 and Rajaratnam and and Subramanya 1968) models. Result obtained from present model Eqn. (4) and Eqn. (11) lying below the results of other authors, which may be attributed to different experimental and channel conditions. Author (Bradley and Peterka, 1957) proposed their model on the basis of experimental results for all types of prismatic channel, moreover length of surface roller was given more preference then length of jump, according to him the length of roller L_r = 4.5 Y_2 and the length of jump is approximately L_j/L_r = 1.3. Author (Bretz, 1987) has given his empirical model for $4 < F_{r1} < 12$ with average deviation of + 5 Y_1. It is therefore concluded that result obtained from different authors are varying, some have proposed length of jump is independent of channel shape (Bushra and Afzal, 2006 and Stahl and Hager 1999) proposed length of jump is order of length of roller, hence the presented model may be accepted for prediction of relative length of jump in U-shaped horizontal channel.

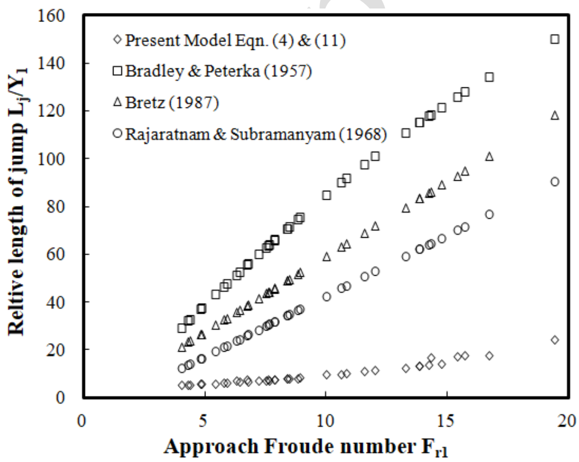


Fig. 8. Comparison of obtained relative length of jump using Eqn. (4) and Eqn. (11) with different authors

Due to limited availability of results for validation of models namely E_L/E_1, E_2/E_1, Y_1/E_1, Y_2/E_2 and L_r/Y_1 presented in section 6 were not tested. Nevertheless, the models described in Eqns. (6–10) can still be used to forecast the hydraulic jump characteristics with a high degree of accuracy as they show good R^2 value (provided in Table 2).

6. Principal Component Analysis/ Factor Analysis

A better understanding of hydraulic jump features under diverse discharge conditions can be achieved by applying multivariate statistical approaches, such as principal component analysis (PCA) and factor analysis (FA). According to authors (Hamidreza et al., 2023 and Singh et al., 2004) principal component analysis (PCA) can help identify the most important factors that characterize complete data sets, allowing for data reduction with minimal loss of original information. An effective pattern detection method that decomposes a big number of interrelated variables (Y_2/Y_1, h_j/Y_1, E_L/E_1, E_2/E_1, Y_1/E_1, Y_2/E_2, L_r/Y_1, L_j/Y_2, F_{r1}, R_{e1}, ε/Y_1) into their major components to explain their variance. According to the calculations done by Ogarekpe et al., (2022), the PC can be represented mathematically by Eqn. (12), where *a* is the component loading, *i* is the component number, *j* is the sample number, and *m* is the total number of variables (involved in hydraulic jump characteristics), *x* is the measured value of variable, *Z* is the component score.

$$Z_{ji} = a_{i1}x_{1j} + a_{i2}x_{2j} + a_{i3}x_{3j} + ...a_{im}x_{mj} \quad (12)$$

Factor analysis is a technique that is used to reduce a large number of variables (Y_2/Y_1, h_j/Y_1, E_L/E_1, E_2/E_1, Y_1/E_1, Y_2/E_2, L_r/Y_1, L_j/Y_2, F_{r1}, R_{e1}, ε/Y_1) into fewer numbers of factors according to Ogarekpe et al., 2022. By spinning the axis, it extracts a new set of variables called verifactors (VFs) and further minimizes the influence of variables that were not significant in principal component analysis (PCA). The factor analysis is represented by Eqn. (13), where, *a* is the factor loading, *e* is the residual term accounting for errors or other sources of variation, *f* is the factor score, *i* is the sample number, *j* is the variable number, m the total number of factors and *Z* is the measured value of a variable.

$$Z_{ji} = a_{f1}f_{1i} + a_{f2}f_{2i} + a_{f3}f_{3i} + ...+ a_{fm}f_{mi} + e_{fi} \quad (13)$$

Both principal component analysis (PCA) and factor analysis (FA) use similar equations to represent their respective methods. The only difference between the two is that FA uses a combination of factors to represent the measured variable whereas PCA uses a linear combination of measured variables. According to Helena et al., (2004) and Hamidreza et al., (2023), VFs in FA can contain hypothetical, latent variables that cannot be seen. Author ran PCA/FA on the correlation matrix of the observed data set. The Variance and inter-variable relationships are quantified in the correlation coefficient matrix. PCA/FA was applied to the data matrix of the U-shaped channel (Table 3). The results obtained are shown in Tables 3 and Table 4.

Table 3. Variance Explained By Two Principal Components

Rotation Sums of Squared Loadings			
Component	**Total**	**% of Variance**	**Cumulative %**
1	7.709	64.239	64.239
2	4.019	33.491	97.730
Hydraulic Jump Characteristics			
Component	**Strong +ve loading**		**Moderate +ve loading**
1	Y_2/Y_1, h_j/Y_1, E_L/E_1, E_2/E_1, L_r/Y_1, L_j/Y_2		---
2	Y_1/E_1, Y_2/E_2		E_2/E_1

Table 4. Rotated Component Matrix for Two Principal Components

Characteristics	Component	
	VF 1	**VF 2**
Y_2/Y_1	.980	-.185
h_j/Y_1	.980	-.185
E_L/E_1	.782	-.599
E_2/E_1	-.782	.599
Y_1/E_1	-.541	.826
Y_2/E_2	.497	.844
L_r/Y_1	.980	-.188
L_j/Y_2	.931	-.327

Tables reveal two principal components explaining 64.239 % and 33.491 % of the total variance. Using the procedure mentioned above, the verifactors for the different principal components are shown in Table 4. The verifactor VF1 has a strong positive loading on Y_2/Y_1, h_j/Y_1, E_L/E_1, L_r/Y_1 and L_j/Y_2 while VF2 has strong positive loadings on Y_1/E_1 and Y_2/E_2 and moderate strong positive loading on E_2/E_1. The results indicate that all eight jump variables analyzed in this study have a substantial impact on hydraulic jump in U-shaped channels. Hence, all the jump features play a crucial part in steering the hydraulic jump phenomena.

7. Conclusion

Data analysis was carried out for all the eight hydraulic jump characteristics using experimental results. The functional relationship so obtained is subjected to empirical modeling with high coefficient of determination (R^2 value). Comparisons of these models were also made to explore the merit of the empirical models for specific characteristics using the available data from literature (Bradley and Peterka, 1957; Bretz, 1987; Bushra and Afzal, 2006; Debabeche and Achor, 2003; Hager, 1989; Rajaratnam and Subramanya, 1968 and Stahl and Hager, 1999). Also, the existing literature models for U-shaped channel are tested for fitness of present experimental results.

Characteristics Y_2/Y_1, h_j/Y_1, L_j/Y_2, L_j/Y_1 were tested, validated and compared with the results of (Bushra and Afzal, 2006; Debabeche and Achor, 2003; Hager, 1989 and Stahl and Hager, 1999) for different channel conditions. Obtained values of Y_2/Y_1, h_j/Y_1 and L_j/Y_2 from present models lying between experimental results of (Debabeche and Achor, 2003 and Hager, 1989) proving their efficacy (Fig. 5 to Fig. 7). Eqn. (4) and Eqn. (11) models were validated for L_j/Y_1 with the experimental values of authors (Bradley and Peterka, 1957; Bretz, 1987 and Rajaratnam and Subramanya, 1968) and it shows little deviation from their values because their results holds for prismatic channel based on surface roller. Due to different assumptions it can be stated based on similar trend that L_j/Y_1 model can be used.

The results obtained from PCA/FA analysis can be successfully used to identify the principal hydraulic jump characteristics. Methods of multivariate statistical analysis can be used as valuable tools for identifying the key characteristics in a phenomenon and its relative significance. This method requires a huge data base too for acquiring most preferable and definite results. It is recommended to acquire huge database for applications of PCA/FA for better representation of key hydraulic jump characteristics.

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